Simulating Neuromorphic Systems at Scale

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1 Motivation

A major challenge in neurophysiology and neuromorphic engineering is to relate the properties of large neuronal populations to the properties of single neurons. The traditional approach has been to resort to mean-field models, but there is a large modeling gap between detailed biophysical models of the single neurons and their mean-field representations.

Designing complex neuromorphic systems requires the modulation of mean-field behavior by adjusting the parameters of neurons. Given realistic spiking models, the complexity of the dynamics describing the resulting large-scale system causes a major computational bottleneck for any attempt at *simulation* and subsequently, design. The recent work [1] proposes to ground the simulation and design of neuromorphic systems in an operator-theoretic framework. The purpose of the present abstract is to explore this operator-theoretic framework to develop a concept of simulation *at scale*. We then wish to use this concept of scale to enable a scalable framework for the simulation of populations. Such simulation at scale can be regarded as an alternative to the conventional mean-field approach.

2 Simulation at Scale

The considered neuromorphic model consists of a capacitor to describe the neuron's membrane in parallel with memristive elements of the form $g(v(t))(v(t)-v_E)$ where $g(\cdot)$ is a nonlinear relaxation element, as defined in [2], resembling a positive monotone map at a given time.

The operator theoretic framework of [1] is used within the context of this research. It expresses the solution of the circuit equations as the problem of finding the zero of an operator mapping voltage signals to current signals. Mathematically speaking,

$$\mathrm{F}(v) - i^* = 0 \tag{1}$$

where $\mathrm{F}(v)$ describes the equations governing the dynamics of the circuit and i^* is the external current trajectory.

The highly parallel structure of the neuromorphic circuit leads to a natural splitting of the operator in a large family of elementary "memristive" operators.

To simulate the circuit at scale, we exploit both the structure of the operator and the structure of the signal space. The signal space is a vector space spanned by real exponentials: growing exponentials in the past, and decaying exponentials in the future. The past and future signal space event solutions are thus linear subspaces of the Hardy space $H2^{\perp}$ and $H2$, respectively. The rate of each exponential is regarded as a scale parameter. Restricting the range of the scale parameter determines the choice of an approximating subspace in which an approximated solution can be looked for.

To solve the circuit in a particular subspace, we exploit the property that the operator F can be expressed as a sum of a differential operator and gradient operators. Each of those operators is well-defined on any subspace spanned by real exponentials. The solution of the zero-finding problem *at scale* is simply the input-output behavior of the neuromorphic behavior in a subspace determined by a given scale.

We will illustrate the versatility of this modelling approach and argue that it provides a novel approach to mean-field modelling and simulation of neuromorphic systems.

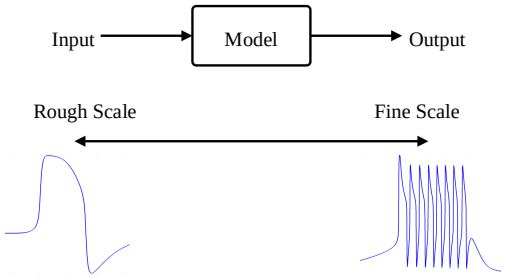

Figure 1: Choice of the scale dictates the level of detail captured through the simulation and the computational cost

References

[1] An Operator-Theoretic Framework to Simulate Neuromorphic Circuits, Shahhosseini, A., Chaffey, T. and Sepulchre Rodolphe, CDC 2024

[2] Relaxation systems, Sepulchre, R., Chaffey, T., Forni, F. and Huo, Y., CDC 2024

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