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Time-domain and frequency-domain per-tone equalization for OFDM over doubly selective channels ☆

Imad Barhumi[a,*,1], Geert Leus[b,2], Marc Moonen[a]

[a] *Katholieke Universiteit Leuven, ESAT/SCD-SISTA, B-3001 Leuven, Belgium*
[b] *Delft University of Technology, 2628 CD Delft, The Netherlands*

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Abstract

In this paper, we propose new time- and frequency-domain per-tone equalization techniques for orthogonal frequency division multiplexing (OFDM) transmission over time- and frequency-selective channels. We present one mixed time- and frequency-domain equalizer (MTFEQ) and one frequency-domain per-tone equalizer. The MTFEQ consists of a one-tap time-varying (TV) time-domain equalizer (TEQ), which converts the doubly selective channel into a purely frequency-selective channel, followed by a one-tap frequency-domain equalizer (FEQ), which then equalizes the resulting frequency-selective channel in the frequency-domain. The frequency-domain per-tone equalizer (PTEQ) is then obtained by transferring the TEQ operation to the frequency-domain. While the one-tap TEQ of the MTFEQ optimizes the performance on all subcarriers in a joint fashion, the PTEQ optimizes the performance on each subcarrier separately. This results into a significant performance improvement of the PTEQ over the MTFEQ, at the cost of a slight increase in complexity. Through computer simulations we show that the MTFEQ suffers from an early and high error floor, while the PTEQ outperforms the MMSE equalizer for OFDM over purely frequency-selective channels, it

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* Corresponding author.
E-mail addresses: imad.barhumi@esat.kuleuven.ac.be (I. Barhumi), leus@cas.et.tudelft.nl (G. Leus), marc.moonen@esat.kuleuven.ac.be (M. Moonen).

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can approach the performance of the block MMSE equalizer. An important feature of the proposed techniques is that no bandwidth expansion or redundancy insertion is required except for the cyclic prefix.
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1. Introduction

Wireless communication systems are currently designed to provide high-data rates at high terminal speeds. High-data rates give rise to so-called intersymbol interference (ISI) due to multipath fading. Such an ISI channel is called *frequency-selective*. On the other hand, due to mobility and/or carrier frequency offsets the received signal is subject to frequency shifts (Doppler shifts) and hence time-variation. The Doppler effect in conjunction with ISI gives rise to a so-called *doubly selective* channel (frequency- and time-selective). In this paper, we present new equalization techniques for orthogonal frequency division multiplexing (OFDM) transmission over such a challenging channel.

OFDM has attracted a lot of attention, due to its simple implementation and robustness against frequency-selective channels. However, in a doubly selective channel the channel variation over an OFDM block destroys the orthogonality between the subcarriers resulting into so-called inter-carrier interference (ICI).

Different approaches for reducing ICI have been proposed, including frequency-domain equalization and time-domain windowing. In [8,11] the authors propose matched-filter, least-squares (LS) and minimum mean-square error (MMSE) receivers incorporating all subcarriers. Receivers considering only the dominant adjacent subcarriers have been presented in [7]. For multiple-input multiple-output (MIMO) OFDM over doubly selective channels, a frequency-domain ICI mitigation technique is proposed in [18]. A time-domain windowing (linear pre-processing) approach to restrict ICI support in conjunction with iterative MMSE estimation is presented in [16,17]. However, these works assume perfect knowledge of the time varying (TV) channel at the receiver, which is hard (if not impossible) to obtain in practice. In this work, we approximate the TV channel by using the basis expansion model (BEM) and only assume the BEM coefficients are known at the receiver, which is more realistic and easier to obtain in practice [14]. In addition to the above methods, ICI self-cancellation schemes have also been proposed in [3,23]. There redundancy is added to enable self-cancellation, which implies a substantial reduction in bandwidth efficiency. To avoid this rate loss, partial response encoding in conjunction with maximum-likelihood sequence detection to mitigate ICI in OFDM systems is studied in [22]. However, the performance of such an approach is not satisfactory.

In this paper, we focus on new time-domain and frequency-domain per-tone ICI mitigation techniques. We consider a mixed time- and frequency-domain equalizer (MTFEQ) and a frequency-domain per-tone equalizer (PTEQ). The MTFEQ consists of a one-tap TV time-domain equalizer (TEQ), which converts the doubly selective channel into a purely frequency-selective channel, followed by a one-tap frequency-domain equalizer (FEQ), which then equalizes the resulting frequency-selective channel in the frequency-domain. The PTEQ is then obtained through transferring the TEQ operation into the frequency-domain. This allows us to develop a more general architecture that unifies and extends previously proposed frequency-domain techniques.

A time-invariant (TI) TEQ has been traditionally used to shorten a purely frequency-selective channel when its delay spread is larger than the cyclic extension [2]. In this paper, we consider the case where the channel has a delay spread that fits within the cyclic extension, but on the other hand has a high Doppler spread. Hence, in a dual fashion, a one-tap TV TEQ is applied at the receiver. The purpose of this one-tap TV TEQ is to

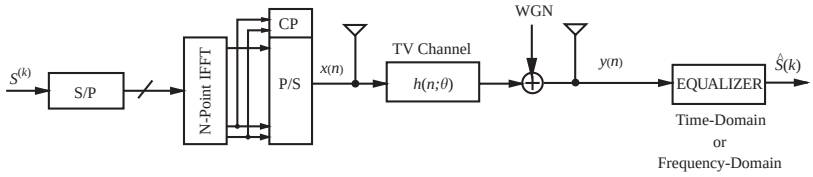


Fig. 1. System model.

convert the doubly selective channel into a purely frequency-selective channel. Then, this channel is equalized in the frequency-domain by means of a one-tap FEQ to recover the frequency-domain symbols. Note that, the one-tap TV TEQ is reminiscent of the time-domain windowing approach introduced in [16,17]. However, while the time-domain window of [16,17] is designed based on the channel statistics, we design our one-tap TV TEQ based on the observed channel realization. This leads to a more reliable window, at the cost of slight increase in complexity. Similar to the case of a purely frequency-selective channel where the TI TEQ is transferred to the frequency-domain resulting into a so-called PTEQ that treats each tone separately [1], we now transfer the one-tap TV TEQ to the frequency-domain resulting in the PTEQ. This new structure enhances the performance at the cost of a slight increase in complexity.

This paper is organized as follows. The system model is described in Section 2. A brief description of ICI is presented in Section 3. The proposed MTFEQ is introduced in Section 4. In Section 5, we discuss how the PTEQ can be obtained. In Section 6, we show through computer simulations the performance of the proposed equalizers. Finally, our conclusions are drawn in Section 7.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, $^{\mathrm{T}}$, and $^{\mathrm{H}}$ represent conjugate, transpose, and Hermitian, respectively. We denote the Kronecker delta as $\delta(n)$. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. Finally, diag{$\mathbf{x}$} denotes the diagonal matrix with $\mathbf{x}$ on the diagonal.

2. System model

We assume a single-input single-output (SISO) system (see Fig. 1), but the results can be easily extended to a single-input multiple-output (SIMO) or a MIMO system. At the transmitter, the incoming bit sequence is parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed into a time-domain sequence using an N-point IFFT. To avoid inter-block interference (IBI), a cyclic prefix (CP) of length v equal to or larger than the channel order L is inserted at the head of each block. The time-domain blocks are then serially transmitted over a multipath fading channel. The channel is assumed to be TV. Focusing only on the baseband-equivalent description and assuming symbol rate sampling, the received sequence can be written as

$$y(n) = \sum_{\theta=-\infty}^{\infty} h(n;\theta)x(n-\theta) + \eta(n), \tag{1}$$

where $x(n)$ is the transmitted time-domain sequence, $\eta(n)$ is additive noise, and $h(n;\theta)$ is the baseband-equivalent doubly selective (time- and frequency-selective) channel, which includes the physical channel as well as the transmit and receive filters. Suppose $S^{(k)}[i]$ is the QAM symbol transmitted on the kth subcarrier of the ith OFDM block. The transmitted time-domain sequence $x(n)$ can then be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S^{(k)}[i] \mathrm{e}^{j2\pi mk/N},$$

where $i = \lfloor n/(N+v) \rfloor$, and $m = n - i(N+v) - v$ (this definition is applicable throughout the paper). Note that this description includes the transmission of a CP of length v.

3. Inter-carrier interference analysis

Due to the time-variation of the channel, the orthogonality between subcarriers is destroyed, and hence ICI is introduced. ICI is the amount of energy on a specific subcarrier leaked from neighboring subcarriers. This energy leakage is proportional to the channel Doppler spread. In this section, we will give a brief analysis of the ICI introduced by the TV channel. This analysis will help us to understand the mechanism of ICI, and hence to develop an ICI suppression technique. Assuming the channel delay spread is bounded by $\tau_{\max}$, (1) can be written as:

$$y(n) = \sum_{l=0}^{L} h(n;l)x(n-l) + \eta(n), \tag{2}$$

where $L = \lfloor \tau_{\max}/T \rfloor + 1$ is the channel order and T is the sampling time.

Defining $Y^{(k)}[i]$, as the frequency response of the received sequence after removing the CP at the kth subcarrier in the ith OFDM block:

$$Y^{(k)}[i] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y(n) e^{-j2\pi km/N}. \tag{3}$$

Substituting (2) in (3) we obtain

$$\begin{aligned} Y^{(k)}[i] &= \sum_{r=0}^{N-1} S^{(r)}[i] \sum_{l=0}^{L} H_l^{(r-k)}[i] e^{-j2\pi lr/N} + \Xi^{(k)}[i] \\ &= S^{(k)}[i] \sum_{l=0}^{L} H_l^{(0)}[i] e^{-j2\pi lk/N} + \sum_{\substack{r=0 \\ r\neq k}}^{N-1} S^{(r)}[i] \\ &\quad \times \sum_{l=0}^{L} H_l^{(k-r)}[i] e^{-j2\pi lr/N} + \Xi^{(k)}[i], \end{aligned} \tag{4}$$

where $H_l^{(t)}[i]$ is given by

$$H_l^{(t)}[i] = \frac{1}{N} \sum_{m=0}^{N-1} h(n;l) e^{-j2\pi mt/N}, \tag{5}$$

and $\Xi^{(k)}[i]$ is the frequency response of the noise at subcarrier k in the ith OFDM block. In (4), $S^{(r)}[i] \sum_{l=0}^{L} H_l^{(k-r)}[i] e^{-j2\pi lr/N}$ represents the amount of interference induced by subcarrier r on subcarrier k when $r \neq k$.

Note that, when the channel is TI for at least one OFDM block (i.e., $h(n;l) = h_l[i]$, $\forall n \in \{i(N+v)+v, ..., (i+1)(N+v)-1\}$), (4) can be written as

$$Y^{(k)}[i] = S^{(k)}[i] H^{(k)}[i] + \Xi^{(k)}[i], \tag{6}$$

where $H^{(k)}[i]$ is the frequency response of the channel at the kth subcarrier in the ith OFDM block. For this case, we see that no ICI is present and orthogonality between subcarriers is preserved.

In this paper we use the BEM to approximate the doubly selective channel [9,12,15,19], which has been shown to accurately model realistic channels. In this BEM, the channel is modeled as a TV FIR filter, where each tap is expressed as a superposition of complex exponential basis functions with frequencies on a DFT grid. Assuming the channel Doppler spread is bounded by $f_{\max}$, it is possible to accurately model the doubly selective channel $h(n;\theta)$ for $n \in \{i(N+v)+v, ..., (i+1)(N+v)-1\}$ as

$$h(n;\theta) = \sum_{l=0}^{L} \delta(\theta - l) \sum_{q=-Q/2}^{Q/2} h_{q,l}[i] e^{j2\pi qn/K}, \tag{7}$$

where $h_{q,l}[i]$ is the coefficient of the lth tap and qth basis function of the channel in the ith OFDM block, which is kept invariant over a period of NT. Q is the number of TV basis functions. K determines the BEM frequency-resolution, and is assumed to be larger than or equal to the number of subcarriers, i.e., $K \geqslant N$. The parameter Q should be selected such that $Q/(2KT) \geqslant f_{\max}$. Substituting (7) in (5), we obtain the following:

$$H_l^{(t)}[i] = \frac{1}{N} \sum_{q=-Q/2}^{Q/2} \tilde{h}_{q,l}[i] \sum_{m=0}^{N-1} e^{-j2\pi m(t/N - q/K)}, \tag{8}$$

where $\tilde{h}_{q,l}[i] = h_{q,l}[i] e^{j2\pi q(i(N+v)+v)/K}$. In this paper, we will assume that K is an integer multiple of the block size N, i.e., $K = PN$, where P is an integer greater than or equal to 1. Then, $H_l^{(t)}[i]$ can be written as

$$H_l^{(t)}[i] = \sum_{q=-Q/2}^{Q/2} \tilde{h}_{q,l}[i] \frac{e^{-j\pi\phi}}{e^{-j\pi\phi/N}} \frac{\mathrm{sinc}(\phi)}{\mathrm{sinc}(\phi/N)}\bigg|_{\phi = t - q/P}, \tag{9}$$

where $\mathrm{sinc}(\phi) = \sin(\pi\phi)/\pi\phi$.

Note that for $P=1$, the ICI support is limited to Q neighboring subcarriers. For $P>1$, the ICI support comes from all subcarriers but the main part is still related to the Q neighboring subcarriers. For this reason, conventional frequency-domain equalization techniques combine a number of neighboring subcarriers to remove the ICI. However, this approach can be generalized as discussed in the next sections, where we develop novel time-domain and frequency-domain per-tone ICI mitigation techniques. We first discuss a MTFEQ and then a frequency-domain PTEQ.

4. Mixed time- and frequency-domain equalization

In this section, we introduce a MTFEQ for OFDM over doubly selective channels. We assume that the channel order fits within the CP. The proposed MTFEQ consists of a one-tap TV TEQ followed by a one-tap FEQ.

The purpose of the one-tap TV TEQ is to convert the doubly selective channel into a purely frequency-selective channel.[3] In other words, it transforms the doubly selective channel with order $L\leqslant v$, into a TI target impulse response (TIR) with order also $L''\leqslant v$ (see Fig. 2). The one-tap FEQ then equalizes the TIR in the frequency-domain.

Since we approximate the doubly selective channel using the BEM, it is convenient to also parameterize the one-tap TV TEQ using the BEM. In other words, we parameterize the time variation of the one-tap TV TEQ by $Q'+1$ complex exponential basis functions. Thereby constraining its design and limiting the allowable degrees of freedom to $Q'+1$ rather than N. Hence, we can write the one-tap TV TEQ[4] $g(n;0)$ for $n\in$ $\{i(N+v)+v,...,(i+1)(N+v)-1\}$ as

$$g(n;0)=\sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]e^{j2\pi q'n/K}. \tag{10}$$

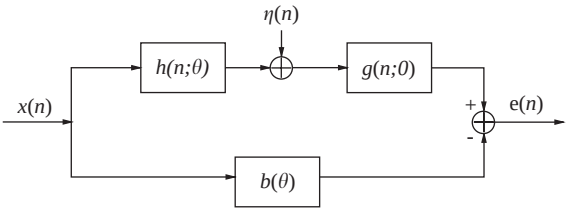


Fig. 2. Time-domain equalizer.

We can then write the output of the one-tap TV TEQ at the receiver as

$$\begin{aligned} z(n)&=\sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]e^{j2\pi q'n/K}y(n)\\ &=\sum_{q'=-Q'/2}^{Q'/2}\sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} g_{q'}[i]e^{j2\pi q'n/K}h_{q,l}[i]e^{j2\pi qn/K}x(n-l)\\ &\quad+\sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]e^{j2\pi q'n/K}\eta(n). \end{aligned} \tag{11}$$

It is more convenient at this point to switch to a block level description. We assume from now on that the CP equals the channel order, i.e., $v=L$. Define $\mathbf{z}[i]=[z(i(N+v)+v),...,z((i+1)(N+v)-1)]^{\mathrm{T}}$, $\mathbf{x}[i]=[x(i(N+v)+v),...,x((i+1)(N+v)-1)]^{\mathrm{T}}$ and $\boldsymbol{\eta}[i]=[\eta(i(N+v)+v),...,\eta((i+1)(N+v)-1)]^{\mathrm{T}}$. Then (11) can be written on the block level as

$$\begin{aligned} \mathbf{z}[i]&=\sum_{q'=-Q'/2}^{Q'/2}\sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} g_{q'}[i]h_{q,l}[i]\mathbf{D}_{q'}[i]\mathbf{D}_{q}[i]\mathbf{Z}_l\mathbf{x}[i]\\ &\quad+\sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]\mathbf{D}_{q'}[i]\boldsymbol{\eta}[i], \end{aligned} \tag{12}$$

where $\mathbf{D}_{q'}[i]=\mathrm{diag}\{[e^{j2\pi q'(i(N+v)+v)/K},...,e^{j2\pi q'((i+1)(N+v)-1)/K}]^{\mathrm{T}}\}$ and $\mathbf{Z}_l$ is an $N\times N$ circulant matrix with the first column $[\mathbf{0}_{1\times l},1,\mathbf{0}_{1\times(N-l-1)}]^{\mathrm{T}}$.

[3] As we will see later, a perfect shortening is not possible in the case of SISO systems. However, the purpose here is to convert the doubly selective channel into a frequency-selective channel in the MMSE sense.

[4] A TV FIR TEQ with delay spread larger than zero has been treated in [5].

Defining $r = q' + q$, we can write (12) as

$$\mathbf{z}[i] = \sum_{r=-(Q'+Q)/2}^{(Q'+Q)/2} \sum_{l=0}^{L} f_{r,l}[i]\mathbf{D}_r[i]\mathbf{Z}_l\mathbf{x}[i] + \sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]\mathbf{D}_{q'}[i]\boldsymbol{\eta}[i], \quad (13)$$

where the 2-D function $f_{r,l}[i]$ is given by

$$f_{r,l}[i] = \sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]h_{r-q',l}[i]. \quad (14)$$

Defining $\mathbf{f}[i] = [f_{-(Q'+Q)/2,0}[i], ..., f_{(Q'+Q)/2,0}[i], ..., f_{(Q'+Q)/2,L}[i]]^{\mathrm{T}}$, we can further rewrite (13) as

$$\mathbf{z}[i] = (\mathbf{f}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i] + (\mathbf{g}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{B}[i]\boldsymbol{\eta}[i], \quad (15)$$

where $\mathbf{g}[i] = [g_{-Q'/2}[i], ..., g_{Q'/2}[i]]^{\mathrm{T}}$, and $\mathbf{A}[i]$ and $\mathbf{B}[i]$ are given by

$$\mathbf{A}[i] = \begin{bmatrix} \mathbf{D}_{-(Q'+Q)/2}[i]\mathbf{Z}_0 \\ \vdots \\ \mathbf{D}_{(Q'+Q)/2}[i]\mathbf{Z}_0 \\ \vdots \\ \mathbf{D}_{(Q'+Q)/2}[i]\mathbf{Z}_L \end{bmatrix}, \quad \mathbf{B}[i] = \begin{bmatrix} \mathbf{D}_{-Q'/2}[i] \\ \vdots \\ \mathbf{D}_{Q'/2}[i] \end{bmatrix}.$$

Defining $\mathscr{H}_l[i]$ as

$$\mathscr{H}_l[i] = \begin{bmatrix} h_{-Q/2,l}[i] & \cdots & h_{Q/2,l}[i] & \mathbf{0} \\ & \ddots & \ddots & \\ \mathbf{0} & h_{-Q/2,l}[i] & \cdots & h_{Q/2,l}[i] \end{bmatrix},$$

and $\mathscr{H}[i] = [\mathscr{H}_0[i], ..., \mathscr{H}_L[i]]$, we can also derive from (14) the following linear relationship:

$$\mathbf{f}^{\mathrm{T}}[i] = \mathbf{g}^{\mathrm{T}}[i]\mathscr{H}[i]. \quad (16)$$

As mentioned earlier, the purpose of the one-tap TV TEQ is to convert the doubly selective channel into a TIR that is purely frequency-selective of order $L'' \leqslant v$. Note that a perfect shortening TEQ, a.k.a. a zero-forcing (ZF) solution, doesn't exist for the case of SISO systems. In this paper, we are therefore aiming at designing an MMSE TV TEQ. Conditions for the existence of the ZF solution for the case of SIMO systems will be discussed at the end of this section.

As shown in Fig. 2, we require to design a TEQ $\mathbf{g}[i]$, and a TIR $\mathbf{b}[i] = [b_0[i], ..., b_{L''}[i]]^{\mathrm{T}}$ such that the error between the upper and lower branch is minimized. Defining the error vector in the ith OFDM block as $\mathbf{e}[i] = [e(i(N+v)+v), ..., e((i+1)(N+v)-1)]^{\mathrm{T}}$, we can express $\mathbf{e}[i]$ as

$$\begin{aligned}\mathbf{e}[i] &= (\mathbf{f}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i] + (\mathbf{g}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{B}[i]\boldsymbol{\eta}[i] \\ &\quad - \sum_{l''=0}^{L''} b_{l''}[i]\mathbf{Z}_{l''}\mathbf{x}[i] \\ &= (\mathbf{f}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i] + (\mathbf{g}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{B}[i]\boldsymbol{\eta}[i] \\ &\quad - (\tilde{\mathbf{b}}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i], \end{aligned} \quad (17)$$

where the augmented vector $\tilde{\mathbf{b}}[i]$ is $\tilde{\mathbf{b}}[i] = \mathbf{C}\mathbf{b}[i]$, with $\mathbf{C}$ given by

$$\mathbf{C} = \mathbf{T} \otimes \begin{bmatrix} \mathbf{0}_{(Q+Q')/2} \\ 1 \\ \mathbf{0}_{(Q+Q')/2} \end{bmatrix}, \quad (18)$$

where $\mathbf{T}$ is given by

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{L''+1} \\ \mathbf{0}_{(L-L'')\times(L''+1)} \end{bmatrix}.$$

Hence, we can write the mean-square error cost function as

$$\begin{aligned}\mathscr{J} &= \mathscr{E}\{\mathbf{e}^{\mathrm{H}}[i]\mathbf{e}[i]\} \\ &= \mathrm{tr}\{(\mathbf{f}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^{\mathrm{H}}[i](\mathbf{f}^{*}[i] \otimes \mathbf{I}_N)\} \\ &\quad + \mathrm{tr}\{(\mathbf{g}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{B}[i]\mathbf{R}_\eta\mathbf{B}^{\mathrm{H}}[i](\mathbf{g}^{*}[i] \otimes \mathbf{I}_N)\} \\ &\quad + \mathrm{tr}\{(\tilde{\mathbf{b}}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^{\mathrm{H}}[i](\tilde{\mathbf{b}}^{*}[i] \otimes \mathbf{I}_N)\} \\ &\quad - 2\mathrm{tr}\{\Re\{(\mathbf{f}^{\mathrm{T}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{R}_x \\ &\quad \times \mathbf{A}^{\mathrm{H}}[i](\tilde{\mathbf{b}}^{*}[i] \otimes \mathbf{I}_N)\}\}, \end{aligned} \quad (19)$$

where $\mathbf{R}_x$ and $\mathbf{R}_\eta$ are the input covariance matrix and the noise covariance matrix, respectively.

Let us now introduce the following properties:

$$\mathrm{tr}\{(\mathbf{v}^{\mathrm{T}} \otimes \mathbf{I}_N)\mathbf{V}\} = \mathbf{v}^{\mathrm{T}}\mathrm{subtr}\{\mathbf{V}\},$$
$$\mathrm{tr}\{(\mathbf{v}^{\mathrm{T}} \otimes \mathbf{I}_N)\mathbf{V}(\mathbf{v}^{*} \otimes \mathbf{I}_N)\} = \mathbf{v}^{\mathrm{T}}\mathrm{subtr}\{\mathbf{V}\}\mathbf{v}^{*},$$

where subtr$\{\cdot\}$ splits the matrix into $N \times N$ submatrices and replaces each submatrix by its

trace.[5] Hence, the cost function in (19) reduces to

$$\mathscr{J} = \mathbf{g}^{\mathrm{T}}[i]\big(\mathscr{H}[i]\mathbf{R}_{\tilde{A}}[i]\mathscr{H}^{\mathrm{H}}[i] + \mathbf{R}_{\tilde{B}}[i]\big)\mathbf{g}^{*}[i] + \tilde{\mathbf{b}}^{\mathrm{T}}[i]\mathbf{R}_{\tilde{A}}[i]\tilde{\mathbf{b}}^{*}[i] - 2\Re\{\mathbf{g}^{\mathrm{T}}[i]\mathscr{H}[i]\mathbf{R}_{\tilde{A}}[i]\tilde{\mathbf{b}}^{*}[i]\}, \quad (20)$$

where $\mathbf{R}_{\tilde{A}}[i] = \mathrm{subtr}\{\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^{\mathrm{H}}[i]\}$, and $\mathbf{R}_{\tilde{B}}[i] = \mathrm{subtr}\{\mathbf{B}[i]\mathbf{R}_\eta\mathbf{B}^{\mathrm{H}}[i]\}$. In order to avoid the trivial solution (zero vector $\mathbf{g}[i]$ and zero vector $\mathbf{b}[i]$) when minimizing the cost function in (20), a non-triviality constraint is needed, e.g., a unit tap constraint, $b_0[i] = 1$; a unit-norm constraint, $\|\mathbf{b}[i]\|^2 = 1$ or $\|\mathbf{g}[i]\|^2 = 1$; or a unit-energy constraint, $\tilde{\mathbf{b}}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{A}}[i]\tilde{\mathbf{b}}[i] = 1$ or $\mathbf{g}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}[i]\mathbf{g}[i] = 1$. More details about these constraints for TI channels can be found in [2] for the unit-tap and unit-norm constraints, and in [21] for the unit-energy constraint. To find $\mathbf{g}[i]$ and $\mathbf{b}[i]$ we solve the following:

$$\min_{\mathbf{g}[i],\mathbf{b}[i]} \mathscr{J} \quad \text{subject to non-triviality constraint}$$

In Table 1, we show the different constraints and the corresponding solutions, considering only the unit-norm and the unit-energy constraints.

For white additive noise and white input (i.e., $\mathbf{R}_\eta = \sigma_\eta^2\mathbf{I}$ and $\mathbf{R}_x = \sigma_x^2\mathbf{I}$), $\mathbf{R}_{\tilde{A}}[i]$ can be written as

$$\mathbf{R}_{\tilde{A}}[i] = \sigma_x^2\mathbf{J}[i] \otimes \mathbf{I}_{L+1},$$

where $\mathbf{J}[i]$ is a $(Q+Q'+1)\times(Q+Q'+1)$ matrix with:

$$[\mathbf{J}[i]]_{r,r'} = \begin{cases} N, & r = r', \\ \mathrm{e}^{j2\pi(r-r')(i(N+\nu)+\nu)/K}\,\frac{1-\mathrm{e}^{j2\pi(r-r')N/K}}{1-\mathrm{e}^{j2\pi(r-r')/K}}, & r \neq r', \end{cases}$$

[5] Let $\mathbf{A}$ be the $pN \times qN$ matrix: $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \dots & \mathbf{A}_{1q} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{p1} & \dots & \mathbf{A}_{pq} \end{bmatrix}$, where $\mathbf{A}_{ij}$ is the (i,j)th $N \times N$ submatrix of $\mathbf{A}$. The $p \times q$ matrix $\mathrm{subtr}\{\mathbf{A}\}$ is then defined as: $\mathrm{subtr}\{\mathbf{A}\} = \begin{bmatrix} \mathrm{tr}\{\mathbf{A}_{11}\} & \dots & \mathrm{tr}\{\mathbf{A}_{1q}\} \\ \vdots & \ddots & \vdots \\ \mathrm{tr}\{\mathbf{A}_{p1}\} & \dots & \mathrm{tr}\{\mathbf{A}_{pq}\} \end{bmatrix}$.

Table 1
Constraints of the TEQ

1. $\|\mathbf{b}[i]\|^2 = 1$ Unit norm constraint
$\mathbf{g}^{\mathrm{T}}[i] = \tilde{\mathbf{b}}^{\mathrm{T}}[i](\mathscr{H}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\mathscr{H}[i] + \mathbf{R}_{\tilde{A}}^{-1}[i])^{-1}\mathscr{H}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}^{-1}[i]$
$\mathbf{b}[i] = \mathrm{eig}_{\min}(\mathbf{R}^{\perp}[i])^{\mathrm{a}}$
$\mathbf{R}^{\perp}[i] = \mathbf{C}^{\mathrm{T}}(\mathscr{H}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\mathscr{H}[i] + \mathbf{R}_{\tilde{A}}^{-1}[i])^{-1}\mathbf{C}$

2. $\tilde{\mathbf{b}}^{\mathrm{T}}[i]\mathbf{R}_{\tilde{A}}[i]\tilde{\mathbf{b}}[i] = 1$ Unit energy constraint
$\mathbf{g}^{\mathrm{T}}[i] = \tilde{\mathbf{b}}^{\mathrm{T}}[i](\mathscr{H}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\mathscr{H}[i] + \mathbf{R}_{\tilde{A}}^{-1}[i])^{-1}\mathscr{H}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}^{-1}[i]$
$\mathbf{b}[i] = \mathrm{eig}_{\max}(\tilde{\mathbf{R}}^{\perp}[i])^{\mathrm{a}}$
$\tilde{\mathbf{R}}^{\perp}[i] = \mathbf{C}^{\mathrm{T}}(\mathscr{H}^{\mathrm{H}}[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\mathscr{H}[i] + \mathbf{R}_{\tilde{A}}^{-1}[i])^{-1}\mathscr{H}^{\mathrm{H}}\mathbf{R}_{\tilde{B}}^{-1}[i]\mathscr{H}\mathbf{R}_{\tilde{A}}[i]\mathbf{C}$

[a] $\mathrm{eig}_{\min}(\mathbf{A})$ ($\mathrm{eig}_{\max}(\mathbf{A})$) is the eigenvector corresponding to the minimum (maximum) eigenvalue of matrix $\mathbf{A}$.

and the $(Q'+1)\times(Q'+1)$ matrix $\mathbf{R}_{\tilde{B}}[i]$ is given by

$$[\mathbf{R}_{\tilde{B}}[i]]_{q',q''} = \sigma_\eta^2\begin{cases} N, & q' = q'', \\ \mathrm{e}^{j2\pi(q'-q'')(i(N+\nu)+\nu)/K}\,\frac{1-\mathrm{e}^{j2\pi(q'-q'')N/K}}{1-\mathrm{e}^{j2\pi(q'-q'')/K}}, & q' \neq q''. \end{cases}$$

Now that we have designed $\mathbf{g}[i]$, we can filter the received sequence by the one-tap TEQ $g(n;0)$ and remove the CP. In conjunction with the one-tap FEQ, an estimate of the frequency-domain symbol at the kth subcarrier of the ith OFDM block can then be obtained as

$$\hat{S}^{(k)}[i] = \frac{1}{d^{(k)}[i]\sqrt{N}}\sum_{m=0}^{N-1} z(n)\mathrm{e}^{-j2\pi mk/N} = \frac{1}{d^{(k)}[i]\sqrt{N}}\sum_{m=0}^{N-1}\sum_{q'=-Q'/2}^{Q'/2} g_{q'}[i]\mathrm{e}^{j2\pi nq'/K}y(n)\mathrm{e}^{-j2\pi mk/N}, \quad (21)$$

where $d^{(k)}[i]$ is the one-tap FEQ coefficient corresponding to the frequency response of the TIR at the kth subcarrier of the ith OFDM block.

In the SIMO case with $N_r > 1$ receive antennas, we can easily show that a necessary condition for a perfect shortening TV TEQ (ZF solution) is $N_r(Q'+1) \geqslant (Q+Q'+1)(L+1)$. This implies that we require at least $(L+2)$ receive antennas for the ZF solution to exist. For more details on the existence of the ZF solution see [4,6].

5. Frequency-domain per-tone equalization

In Section 4, a mixed time- and frequency-domain ICI mitigation technique is proposed, consisting of a one-tap TV TEQ and a one-tap FEQ. The one-tap TV TEQ optimizes the performance on all subcarriers in a joint fashion. In this section, we propose a frequency-domain per-tone ICI mitigation technique, which is obtained by transferring the TEQ operation to the frequency-domain.

From (21) we can see that it is possible to transfer the TEQ operation to the frequency-domain resulting into a frequency-domain per-tone equalizer (PTEQ) that allows us to optimize the equalizer coefficients for each subcarrier separately. Let us now discuss this in more detail. As mentioned earlier, we assume that K is an integer multiple of the block size N, i.e., $K = PN$, where P is an integer greater than or equal to 1. Defining $\mathbb{Q} = \{-Q'/2, \cdots, Q'/2\}$, and $\mathbb{Q}_p = \{q \in \mathbb{Q} \,|\, |q| \bmod P = p\}$ for $p = 0, \cdots, P-1$, we can write (21) as

$$\begin{aligned}\hat{S}^{(k)}[i] &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \\ &\quad \times g_{q_p}[i]/d^{(k)}[i] \mathrm{e}^{j2\pi q_p n/K} y(n) \mathrm{e}^{-j2\pi mk/N} \\ &= \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} w_{p,l_p}^{(k)}[i] \frac{1}{\sqrt{N}} \\ &\quad \sum_{m=0}^{N-1} \underbrace{\mathrm{e}^{j2\pi pn/K} y(n)}_{\tilde{y}_p(n)} \mathrm{e}^{-j2\pi m(k-l_p)/N} \\ &= \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} w_{p,l_p}^{(k)}[i] \tilde{Y}_p^{(k-l_p)}[i], \end{aligned} \tag{22}$$

where $l_p = (q_p - p)/P$. Here $w_{p,l_p}^{(k)}[i] = g_{q_p}[i] \mathrm{e}^{j2\pi l_p (i(N+v)+v)/N}/d^{(k)}[i]$ is the coefficient operating on the $(k - l_p)$th subcarrier of the pth FFT (see below) to compute the symbol $S^{(k)}[i]$ transmitted on the kth subcarrier. $\tilde{Y}_p^{(k-l_p)}[i]$ is the frequency response of $\tilde{y}_p(n)$ on the $(k - l_p)$th subcarrier. From (22), the receiver structure can be realized as depicted in Fig. 3. The proposed ICI mitigation technique is simply achieved by taking P FFTs of different modulated versions $\tilde{y}_p(n)$ of the received sequence $y(n)$. Note that this actually corresponds to oversampling the received sequence in the frequency-domain by a factor of P. To detect a symbol on the kth subcarrier of the ith OFDM block, $|\mathbb{Q}_p|$ neighboring subcarriers are combined at the output of the pth FFT. The resulting outputs are subsequently combined to obtain the symbol transmitted on that subcarrier. Note that (22) allows us to optimize the equalizer coefficient $w_{p,l_p}^{(k)}[i]$ for each subcarrier k without taking into account the specific relation between $w_{p,l_p}^{(k)}[i]$, $g_{q_p}[i]$ and $d^{(k)}[i]$.

For each subcarrier, we can find the MMSE equalizer coefficients by minimizing the following cost function:

$$\begin{aligned}&\min_{w_{p,l_p}^{(k)}[i]} J(w_{p,l_p}^{(k)}[i]) \\ &\quad = \min_{w_{p,l_p}^{(k)}[i]} \mathscr{E}\{\|\hat{S}^{(k)}[i] - S^{(k)}[i]\|^2\} \\ &\quad = \min_{w_{p,l_p}^{(k)}[i]} \mathscr{E}\{\|\mathbf{w}^{(k)\mathrm{T}}[i]\mathbf{F}^{(k)}[i]\mathbf{y}[i] - S^{(k)}[i]\|^2\}, \end{aligned} \tag{23}$$

where $\mathbf{w}^{(k)}[i] = [\mathbf{w}_0^{(k)\mathrm{T}}[i], ..., \mathbf{w}_{P-1}^{(k)\mathrm{T}}[i]]^{\mathrm{T}}$, $\mathbf{w}_p^{(k)}[i] = [..., w_{p,-1}^{(k)}[i], w_{p,0}^{(k)}[i], w_{p,1}^{(k)}[i], ...]^{\mathrm{T}}$, and $\mathbf{F}^{(k)}[i] = [\mathbf{F}_0^{(k)\mathrm{T}}[i], ..., \mathbf{F}_{P-1}^{(k)\mathrm{T}}[i]]^{\mathrm{T}}$, where

$$\mathbf{F}_p^{(k)}[i] = \begin{bmatrix} \vdots \\ \mathscr{F}^{(k-1)}\mathbf{D}_p[i] \\ \mathscr{F}^{(k)}\mathbf{D}_p[i] \\ \mathscr{F}^{(k+1)}\mathbf{D}_p[i] \\ \vdots \end{bmatrix}$$

with $\mathscr{F}^{(m)}$ the mth row of the unitary DFT matrix $\mathscr{F}$. The received vector $\mathbf{y}[i] = [y(i(N+v)+v), ..., y((i+1)(N+v)-1)]^{\mathrm{T}}$ is given by

$$\mathbf{y}[i] = \mathbf{H}[i]\mathscr{F}^{\mathrm{H}}\mathbf{s}[i] + \boldsymbol{\eta}[i],$$

where $\mathbf{H}[i]$ is the TV channel matrix given by

$$\mathbf{H}[i] = \sum_{q=-Q/2}^{Q/2} \mathbf{D}_q[i]\mathbf{H}_q[i], \tag{24}$$

with $\mathbf{H}_q[i]$ a circulant matrix with $[h_{q,0}[i], ..., h_{q,L}[i], \mathbf{0}_{N-L-1\times 1}]^{\mathrm{T}}$ as its first column. Furthermore, $\mathbf{s}[i]$ is the vector of the frequency-domain transmitted symbols $\mathbf{s}[i] = [S^{(0)}[i], ..., S^{(N-1)}[i]]^{\mathrm{T}}$, and $\boldsymbol{\eta}[i]$ is the noise vector similarly defined as $\mathbf{y}[i]$. Solving

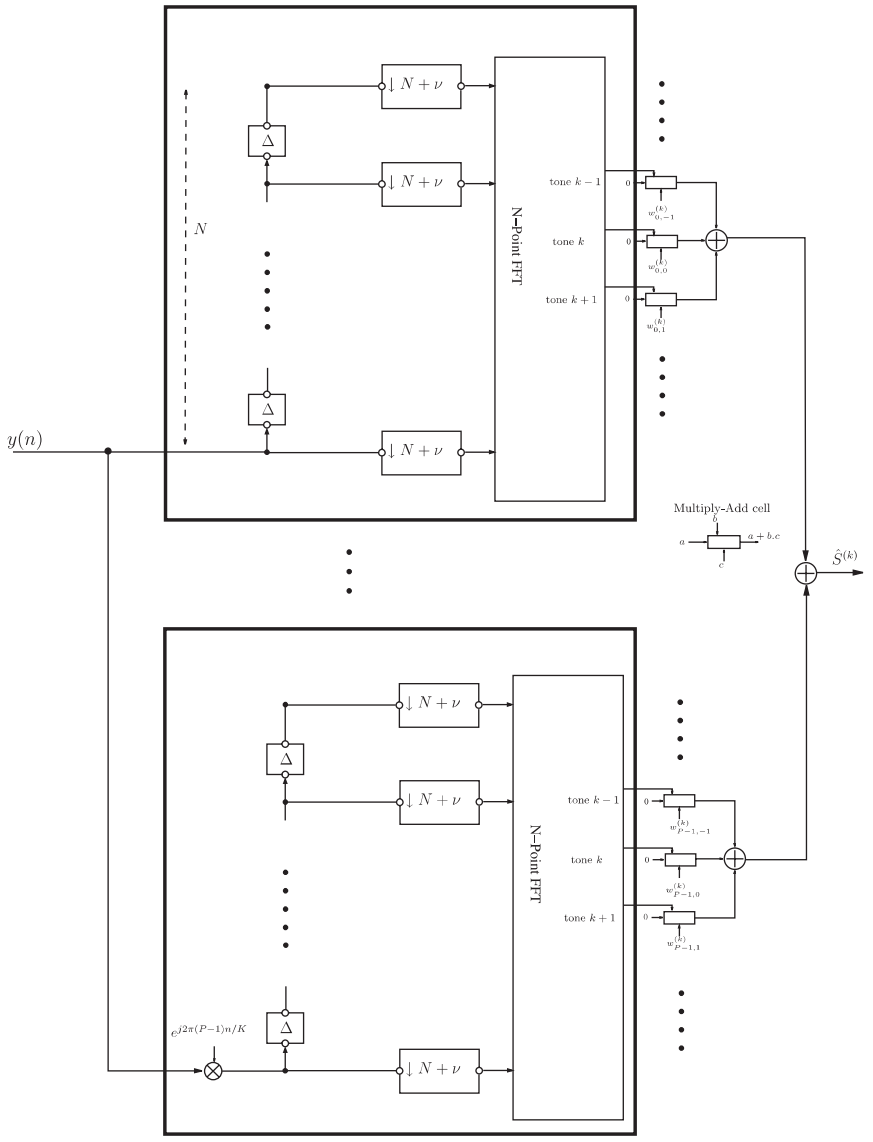


Fig. 3. Receiver structure with the PTEQ.

for $\mathbf{w}^{(k)}[i]$ in (23) we obtain

$$\mathbf{w}^{(k)\mathrm{T}}[i] = \mathbf{e}^{(k)\mathrm{T}}\mathbf{R}_s\mathscr{F}\mathbf{H}^{\mathrm{H}}[i]\mathbf{F}^{(k)\mathrm{H}}[i] \times(\mathbf{F}^{(k)}[i]\mathbf{H}[i]\mathscr{F}^{\mathrm{H}}\mathbf{R}_s\mathscr{F}\mathbf{H}^{\mathrm{H}}[i]\mathbf{F}^{(k)\mathrm{H}}[i] + \mathbf{F}^{(k)}[i]\mathbf{R}_\eta\mathbf{F}^{(k)\mathrm{H}}[i])^{-1}, \quad (25)$$

where $\mathbf{e}^{(k)}$ is the $N \times 1$ unit vector with a 1 in position $k+1$, and $\mathbf{R}_s$ and $\mathbf{R}_\eta$ are the data and the noise covariance matrices, respectively. For white data and white noise with variances σ_s^2 and σ_η^2, respectively (i.e., $\mathbf{R}_s = \sigma_s^2\mathbf{I}_N$ and $\mathbf{R}_\eta = \sigma_\eta^2\mathbf{I}_N$), (25) reduces to

$$\mathbf{w}^{(k)\mathrm{T}}[i] = \mathscr{F}^{(k)}\mathbf{H}^{\mathrm{H}}[i]\mathbf{F}^{(k)\mathrm{H}}[i] \times\left(\mathbf{F}^{(k)}[i]\mathbf{H}[i]\mathbf{H}^{\mathrm{H}}[i]\mathbf{F}^{(k)\mathrm{H}}[i] + \frac{\sigma_\eta^2}{\sigma_s^2}\mathbf{F}^{(k)}[i]\mathbf{F}^{(k)\mathrm{H}}[i]\right)^{-1}. \quad (26)$$

It is worth noting that our PTEQ solution (26) unifies and extends earlier proposed techniques for both TI and TV channels.

(1) In the TI case with $P=1$, $Q=Q'=0$: the solution in (26) then boils down to the MMSE solution obtained in [20]:

$$w^{(k)}[i]=\left(H^{(k)*}[i]H^{(k)}[i]+\frac{\sigma_\eta^2}{\sigma_s^2}\right)^{-1}H^{(k)*}[i], \quad (27)$$

where $H^{(k)}[i]$ is the frequency response of the TI channel on the kth subcarrier in the ith OFDM block.

(2) In the TV case with $P=1$, $Q\neq 0$: the solution in (26) then boils down to the MMSE solution obtained in [7], where only one FFT is applied and Q' adjacent subcarriers are used to cancel interference.

Note that, to implement the PTEQ we require P-FFTs in addition to $Q'+1$ multiply–add (MA) operations per subcarrier. Whereas, to implement the MTFEQ we require 1 FFT in addition to 2 multiplications per subcarrier. On the other hand, the design complexity of the PTEQ is higher than the design complexity of the MTFEQ. We can easily show that the design complexity of the MTFEQ requires $\mathcal{O}((Q+Q'+1)^3(L+1)^3)$ MA operations, while it requires $\mathcal{O}((Q'+1)^2N^2)$ MA operations per OFDM symbol for the PTEQ. Note that, computing $\mathbf{F}^{(k)}[i]\mathbf{H}[i]$ requires $(Q'+1)(L+1)N$ MA operations and $\mathbf{F}^{(k)}[i]\mathbf{H}[i]\mathbf{H}^{\mathrm{H}}[i]\mathbf{F}^{(k)\mathrm{H}}[i]$ requires $(Q'+1)^2N$ MA operations. Hence, the overall design complexity of the PTEQ requires $\mathcal{O}((Q'+1)^2N^2)$ MA operations per OFDM symbol. The complexity associated with inverting a $(Q'+1)\times(Q'+1)$ matrix requires $\mathcal{O}((Q'+1)^3)$ MA operations, which is small compared to the above complexity. This complexity is much less than the design complexity associated with the block MMSE equalizer, which requires $\mathcal{O}(N^3)$ MA operations per OFDM symbol (assuming that $Q'\ll N$, which is the case in practice and in our simulations). The design complexity of the MTFEQ is mainly due to a matrix inversion of size $(Q+Q'+1)(L+1)\times(Q+Q'+1)(L+1)$. The complexity associated with computing the max (min) eigenvector of an $(L''+1)\times(L''+1)$ matrix which requires $\mathcal{O}((L''+1)^2)$ MA operations [10], is negligible compared to the above matrix inversion.

6. Simulation results

In this section, we show some simulation results for the proposed ICI mitigation techniques. We consider a SISO system with a doubly selective channel of order $L=6$ and a maximum Doppler spread $f_{\max}=100\,\mathrm{Hz}$. The channel taps are simulated as i.i.d., correlated in time with a correlation function according to Jakes' model $\mathscr{E}\{h(n_1;l_1)h^*(n_2;l_2)\}=\sigma_h^2J_0(2\pi f_{\max}T(n_1-n_2))\delta(l_1-l_2)$, where J_0 is the zeroth-order Bessel function of the first kind, T is the sampling time, and σ_h^2 denotes the variance of the channel. We consider $N=128$ subcarriers, and a CP of length $v=6$. The sampling time is $T=50\,\mu\mathrm{s}$, the total OFDM symbol duration is 6.7 ms. QPSK signaling is assumed. We define the signal-to-noise ratio (SNR) as $\mathrm{SNR}=\sigma_h^2(L+1)E_\mathrm{s}/\sigma_\eta^2$, where E_s is the QPSK symbol power.

We use the BEM to approximate the channel. The channel BEM resolution is determined by $K=PN$, where $P=1,2$. The number of complex exponentials is then determined by $Q=2$ for $P=1$ and $Q=4$ for $P=2$. Note that for both values of P, $Q/(2KT)\geqslant f_{\max}$ is satisfied. We only assume the knowledge of the BEM coefficients of the channel at the receiver, and not the knowledge of the true Jakes' channel, which is rather difficult to obtain in practice. For the lth tap of the channel, in the ith OFDM block, the BEM coefficient vector $\mathbf{h}_l[i]=[h_{-Q/2,l}[i],\cdots,h_{Q/2,l}[i]]^{\mathrm{T}}$ is obtained by

$$\mathbf{h}_l[i]=\mathscr{L}^{\dagger}[i]\mathbf{h}_l^{(\mathrm{Jakes})}[i],$$

where $\mathbf{h}_l^{(\mathrm{Jakes})}[i]=[h_l^{(\mathrm{Jakes})}(i(N+v)+v),\ldots,h_l^{(\mathrm{Jakes})}((i+1)(N+v)-1)]^{\mathrm{T}}$ is the lth tap of the TV channel modeled by Jakes' model over N symbol periods, and $\mathscr{L}[i]$ is an $N\times(Q+1)$ matrix with the $(q+Q/2+1)$th column given by $[\mathrm{e}^{j2\pi q(i(N+v)+v)/K},\cdots,\mathrm{e}^{j2\pi q((i+1)(N+v)-1)/K}]^{\mathrm{T}}$. The BEM coefficients of the approximated channel are used

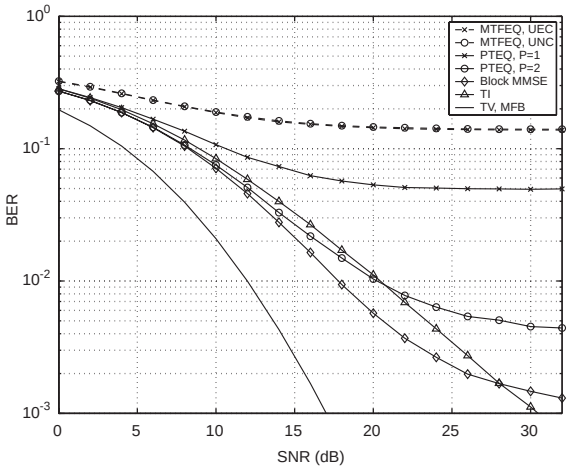


Fig. 4. BER vs. SNR for MTFEQ and PTEQ.

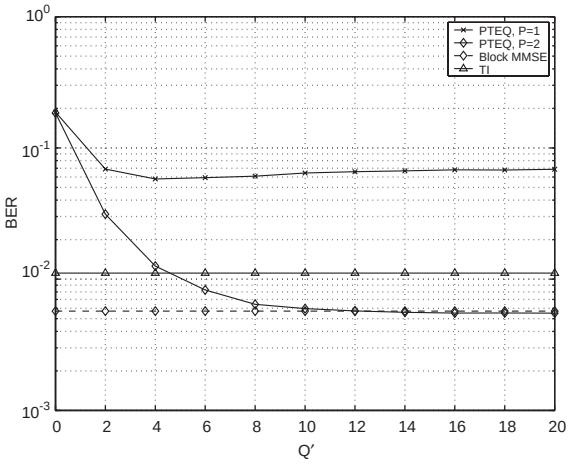


Fig. 5. BER vs. Q' for $P = 1, 2$ for PTEQ at SNR = 20 dB.

to design the MTFEQ (considering the unit-norm constraint (UNC) and the unit-energy constraint (UEC)), and the PTEQ. The block MMSE equalizer for OFDM over doubly selective channel used here is obtained based on a full channel knowledge. For the MTFEQ and the PTEQ equalizers we consider $Q' = 6$ TV complex exponentials. The TIR order is always chosen to be equal to the CP, i.e., $L'' = v$. The proposed MTFEQ and PTEQ are used to equalize the true Jakes' channel. The performance is measured in terms of BER vs. SNR.

As shown in Fig. 4, the performance of the proposed MTFEQ for $P = 2$ suffers from an early and high error floor for both the UNC and UEC, which exhibits almost the same performance. The performance of the PTEQ is also plotted, for different BEM resolutions $P = 1, 2$. For $P = 1$, we see that the BER curve suffers from an early error floor at 5×10^{-2}. This result coincides with the result obtained in [7]. On the other hand, for $P = 2$ the performance is significantly improved, and the error floor is clearly reduced to 5×10^{-3}. Note that Q' is the same for $P = 1, 2$. This means that the frequency spread of the equalizer is twice as large for $P = 1$ than for $P = 2$, but still it does not give a better performance. In other words, oversampling the received sequence in the frequency-domain is really crucial. We can also see that the proposed PTEQ slightly outperforms the MMSE equalizer for OFDM over TI channels when $P = 2$ for low SNR (SNR $\leqslant$ 20 dB). We also consider the performance of the block MMSE equalizer. We can see that, the PTEQ experiences an SNR loss of 3 dB compared to the block MMSE at BER $= 10^{-2}$ when $P = 2$. As a benchmark, we also show the matched-filter bound (see [7,13]), which shows that, both the PTEQ and the block MMSE are far from achieving the matched filter bound.

Finally, we measure the performance of the proposed PTEQ as a function of Q' at a fixed SNR of 20 dB. As shown in Fig. 5, a significant gain can be obtained by increasing Q' up to a certain threshold value ($Q' = 2$ for $P = 1$ and $Q' = 10$ for $P = 2$) after which almost no gain is obtained. Again, we clearly observe that oversampling the received sequence in the frequency-domain really pays off. In addition, the performance of the PTEQ outperforms the MMSE equalizer for OFDM over TI channels when $Q' \geqslant 6$ for $P = 2$. Finally, while almost no gain is obtained by increasing Q' for $P = 1$, we can approach the performance of the block MMSE when $P = 2$ ($Q' = 10$ appears to be enough to approach the performance of the block MMSE for this channel setup).

7. Conclusions

In this paper, we propose new time- and frequency-domain equalization techniques for

multi-carrier systems over time- and frequency-selective channels. We present one mixed time- and frequency-domain equalizer (MTFEQ) and one full frequency-domain equalizer. The MTFEQ consists of a one-tap time-varying (TV) time-domain equalizer (TEQ) applied to the time-domain received symbols, and a one-tap FEQ applied to the frequency-domain received symbols. The full frequency-domain per-tone equalizer (PTEQ) is then obtained by transferring the TEQ operation to the frequency-domain. While the MTFEQ optimizes the performance on all subcarriers in a joint fashion, the PTEQ optimizes the performance of each subcarrier separately. This results into a significant enhancement in performance. The resulting PTEQ uses adjacent subcarriers to mitigate ICI on a specific subcarrier. By increasing the BEM resolution beyond the size of the FFT (i.e., oversampling the received sequence in the frequency-domain), we can outperform the MMSE equalizer for OFDM over purely frequency-selective channels.

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