IQ Compensation for OFDM in the Presence of IBI and Carrier Frequency-Offset

Imad Barhumi and Marc Moonen
K.U.Leuven-ESAT/SCD-SISTA, Kasteelpark Arenberg 10, B-3001 Heverlee, Belgium
Email:{imad.barhumi,marc.moonen}@esat.kuleuven.ac.be

***Abstract*— In this paper we propose a frequency-domain equalization technique for OFDM transmission over frequency-selective channels. We consider the case where the analog front-end suffers from an IQ imbalance and the local oscillator suffers from carrier frequency-offset (CFO). Besides we consider the channel delay spread is larger than the cyclic prefix (CP). This means that interlock interference is present. While the IQ imbalance results into mirroring effect, the CFO induces intercarrier interference (ICI). CFO in conjunction with IBI results into sever intercarrier interference. The frequency-domain equalizer is proposed to combat these effects. It is obtained by transferring a time-domain equalizer (TEQ) to the frequency-domain resulting into per-tone equalizer (PTEQ). Due to the presence of IQ imbalance conventional TEQ (where only one TEQ is applied to the received sequence) is not sufficient to cope with the mirroring effect. A sufficient TEQ consists of tow time-domain filters; one applied to the received sequence and the other is applied to a conjugated version of the received sequence. The TEQs are designed according the basis expansion model (BEM) which showed to be able to cope with the ICI problem. We also propose an RLS initialization scheme for direct equalization.**

I. Introduction

Orthogonal Frequency division multiplexing (OFDM) has been adopted for digital audio and video broadcasting [1] and chosen by the IEEE 802.11 standard [2] as well as by the HIPERLAN-2 standard [3] for wireless LAN (WLAN) . This is due to its robustness against multipath fading channel and its simple implementation. But OFDM is sensitive to the analog front-end imperfections; mainly the amplitude and phase imbalance (IQ imbalance) and the carrier frequency-offset (CFO). In OFDM a cyclic prefix that is equal or longer than the channel delay spread is required to maintain orthogonality between subcarriers. This is pending on the fact that ideal conditions are satisfied such as: no IQ, zero CFO, and the channel is time-invariant over the OFDM block period. In practice, it is very difficult to satisfy all these conditions. This motivates us to search for an alternative equalization techniques that are robust against these imperfectionsm mainly the IQ imbalance the CFO. The presence of IQ imbalance and CFO and the fact that the channel order is larger than the CP cause a severe degradation in performance for OFDM systems. In this paper we consider OFDM transmission over time-invariant channels with a cyclic prefix that is shorter than the channel order and the analog front-end suffers from an IQ imbalance as well as a CFO.

This research work was carried out at the ESAT laboratory of the Katholieke Universiteit Leuven, in the frame of Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office IUAP P5/22 ('Dynamical Systems and Control: Computation, Identification and Modelling') and P5/11 ('Mobile multimedia communication systems and networks'), the Concerted Research Action GOA-MEFISTO-666 (Mathematical Engineering for Information and Communication Systems Technology) of the Flemish Government, and Research Project FWO nr.G.0196.02 ('Design of efficient communication techniques for wireless time-dispersive multi-user MIMO systems'). The scientific responsibility is assumed by its authors.

I. Barhumi is Partly supported by the Palestinian European Academic Cooperation in Education (PEACE) Programme.

Different approaches have been proposed to overcome the analog front-end problems for OFDM transmission. In [4] the author proposes a training based-technique for CFO estimation assuming perfect IQ balance. A maximum likelihood (ML) frequency offset estimation is proposed in [5], also assuming perfect IQ balance. The IQ imbalance only problem is treated in [6], [7], [8] assuming zero CFO. Joint compensation of IQ imbalance and frequency-offset is treated in [9], [10]. In [9] the author assumes that the carrier frequency-offset is corrected upon perfect knowledge of the IQ imbalance parameters, and the IQ imbalance parameters can be estimated correctly in the presence of carrier frequency-offset. The assumption here is valid for small carrier frequency-offsets and small IQ imbalance parameters. In [10], the author utilizes nulled subcarrier to estimate the frequency offset by maximizing the energy on the designated subcarrier and its image. *However, in the above mentioned works, the cyclic prefix (CP) is assumed to be larger than or equal to the channel delay spread*.

In this paper we assume the case where the CP is not necessarily long enough to fit within the channel delay spread, and further the IQ imbalance and carrier frequency offset are present. We devise a frequency-domain per-tone equalization technique to combat the channel effect as well as the imperfection of the analog front-end. The per-tone equalizer is obtained by transferring a time-domain equalizer (TEQ) to the frequency-domain. The purpose of the TEQ is to shorten the channel to fit within the CP and eliminate the effect of the IQ imbalance effect. The proposed PTEQ is first designed assuming perfect knowledge of the channel, the IQ imbalance parameters and the CFO or they can be accurately estimated. This is, however, far from practical, since the channel estimate in the presence of the IQ imbalance and/or the CFO is not sufficiently accurate. To avoid the intermediate stage of parameters' estimation, a training-based direct RLS PTEQ is proposed.

This paper is organized as follows. In Section II, we introduce the system model. In Section III, the per-tone equalizer is proposed. Our simulations are introduced in Section IV. Finally, our conclusions are drawn in Section V.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We denote the expectation as $\mathcal{E}\{\cdot\}$ and the Kronecker product as $\otimes$. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$, the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$ and the all ones vector of length M as $\mathbf{1}_M$. The kth element of vector $\mathbf{x}$ is denoted by $[\mathbf{x}]_k$. Finally, $\mathrm{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with vector $\mathbf{x}$ on the diagonal.

II. System Model

We consider an OFDM transmission over time-invariant frequency-selective channels. We assume a single-input single-output (SISO) system, but the results can be easily extended to single-input multiple-output (SIMO) or multiple-input multiple-output (MIMO) systems.

At the transmitter the information-bearing symbols are parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed to the time-domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length ν is added to the head of each block. The time domain blocks are then serially transmitted over the channel. In the ideal case where neither IQ imbalance nor carrier frequency-offset are present, the discrete time-domain baseband equivalent description of the received signal at time index n is given by:

$$y[n] = \sum_{l=0}^{L} h_l x[n-l] + v[n],$$

where h_l is the lth tap of the multipath fading channel, $v[n]$ is the baseband equivalent noise, and $x[n]$ is the discrete time-domain sequence transmitted at a rate of $1/T$ symbols per second. The channel is assumed to be time-invariant over a period of transmitting a burst of OFDM symbols and may change independently from burst to burst. Assuming $S_k[i]$ is the QAM symbol transmitted on the kth subcarrier of the ith OFDM block, $x[n]$ can be written as:

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k[i] e^{j2\pi(m-\nu)k/N},$$

where $i = \lfloor n/(N+\nu) \rfloor$ and $m = n - i(N+\nu)$. Note that this description includes the transmission of a CP of length ν.

III. Equalization in the presence of IQ and CFO

In this section, we consider that the receiver's analog front-end suffers from an IQ imbalance and the local oscillator suffers from CFO. We further assume that the CP is shorter than the channel order, i.e. IBI is present (ICI). IBI in conjunction with CFO induces severe ICI. To eliminate IBI and ICI equalization (time- or frequency-domain) is required. Assuming that the local oscillator exhibits a CFO of $\epsilon = \Delta f T$, and the analog front end exhibits an amplitude imbalance of Δa and the phase mismatch of $\Delta\phi$. The baseband equivalent received sequence at time index n is given by:

$$r[n] = \mu e^{j2\pi\epsilon n/N} y[n] + v e^{-j2\pi\epsilon n/N} y^*[n]. \tag{1}$$

where the parameters μ and v are given by [11]:

$$\mu = \cos(\Delta\phi) + \jmath \Delta a \sin(\Delta\phi)$$
$$v = \Delta a \cos(\Delta\phi) - \jmath \sin(\Delta\phi).$$

On a block formulation, (1) can be written as:

$$\mathbf{r}[i] = \mu\alpha_i \mathbf{D}_\epsilon \mathbf{y}[i] + v\alpha_i^* \mathbf{D}_\epsilon^* \mathbf{y}^*[i], \tag{2}$$

where $\alpha_i = e^{j2\pi(i(N+\nu)+\nu+d-L')\epsilon/N}$, and $\mathbf{D}_\epsilon = \mathrm{diag}\{[1, \cdots, e^{j2\pi(N+L'-1)\epsilon/N}]^T\}$.

We start from the time-domain equalization. In the time-domain we apply two TEQs, one to the received sequence and the other to a conjugated version of the received sequence. The output of the TEQs in the ith OFDM block can be written as:

$$\mathbf{z}[i] = \mathbf{W}_1^H[i]\mathbf{r}[i] + \mathbf{W}_2^H[i]\mathbf{r}^*[i], \tag{3}$$

The time dependency of the TEQs here stems from the fact that the presence of CFO also imposes time-variations on the system, and hence, the TEQs may change from block to block.

The purpose of the TEQs is to eliminate IBI and ICI and to compensate for the IQ imbalance. In other words, to restore orthogonality between subcarriers that was destroyed by the IBI/ICI and compensate for the mirroring effect induced by the IQ imbalance. Hence, the TEQ is required to shorten the channel into a target impulse response whose order fits within the CP and eliminate ICI (eliminate the time-variations imposed on the system due to the CFO). Designing and optimizing the performance of the TEQ is beyond the scope of this paper. However, we are targeting frequency-domain per-tone equalization techniques, which is obtained by transferring the TEQs' operations to the frequency-domain as will be clear later.

It was shown in [12], [13] that modeling the TEQ using the basis expansion model (BEM) is an efficient way to tackle the problem of IBI/ICI for OFDM systems. Using the BEM to model the TEQs, $\mathbf{W}_1[i]$ and $\mathbf{W}_2[i]$ can be written as:

$$\mathbf{W}_p[i] = \sum_{q=-Q/2}^{Q/2} \mathbf{W}_{p,q}[i]\tilde{\mathbf{D}}_q^*, \qquad \text{for } p = 1, 2, \tag{4}$$

where $\tilde{\mathbf{D}}_q = \mathrm{diag}\{[1, \cdots, e^{j2\pi q(N-1)/N}]^T\}$, and $\mathbf{W}_{p,q}[i]$ is an $(N+L') \times N$ Toeplitz matrix with first column $[w_{p,q,L'}[i], \ldots, w_{p,q,0}[i], \mathbf{0}_{1\times(N-1)}]^T$ and first row $[w_{p,q,L'}[i], \mathbf{0}_{1\times(N-1)}]$. In conjunction with the time-domain filtering, a 1-tap FEQ is applied in the frequency-domain to recover the transmitted QAM symbols. Hence, the estimate of the QAM symbol transmitted on the $k$the subcarrier in the ith OFDM block can be written as:

$$\begin{aligned}\hat{S}_k[i] &= \left(\mathcal{F}^{(k)} \sum_q \tilde{\mathbf{D}}_q \mathbf{W}_{1,q}^H[i]\mathbf{r}[i] + \mathcal{F}^{(k)} \sum_q \tilde{\mathbf{D}}_q \mathbf{W}_{2,q}^H[i]\mathbf{r}^*[i] \right)/d_k \\ &= \left(\sum_q \mathcal{F}^{(k-q)} \mathbf{W}_{1,q}^H[i]\mathbf{r}[i] + \sum_q \mathcal{F}^{(k-q)} \mathbf{W}_{2,q}^H[i]\mathbf{r}^*[i] \right)/d_k \end{aligned} \tag{5}$$

A PTEQ is then can be obtained by transferring the TEQ operation to the frequency-domain. Doing so we obtain the estimate of the transmitted QAM symbol at the kth subcarrier in the ith OFDM symbol as:

$$\hat{S}_k[i] = \sum_{q=-Q/2}^{Q/2} \tilde{\mathbf{w}}_{1,q}^{(k)H}[i] \tilde{\boldsymbol{\mathcal{F}}}^{(k-q)} \mathbf{r}[i] + \sum_{q=-Q/2}^{Q/2} \tilde{\mathbf{w}}_{2,q}^{(k)H}[i] \tilde{\boldsymbol{\mathcal{F}}}^{(k-q)} \mathbf{r}^*[i] \tag{6}$$

where $\tilde{\mathbf{w}}_{p,q}^{(k)}[i] = [w_{p,q,0}^{(k)}[i], \ldots, w_{p,q,L'}^{(k)}[i]]^T/d_k$ for $p = 1, 2$. Using the following properties:

$$\tilde{\boldsymbol{\mathcal{F}}}^{(k)} \mathbf{r}[i] = \mathbf{T}^{(k)} \begin{bmatrix} R^{(k)}[i] \\ \Delta\mathbf{r}[i] \end{bmatrix} \begin{matrix} \updownarrow 1 \times 1 \\ \updownarrow L' \times 1 \end{matrix}, \tag{7a}$$

and

$$\tilde{\boldsymbol{\mathcal{F}}}^{(k)} \mathbf{r}^*[i] = \mathbf{T}^{(k)} \begin{bmatrix} R^{(N-k)*}[i] \\ \Delta\mathbf{r}^*[i] \end{bmatrix} \begin{matrix} \updownarrow 1 \times 1 \\ \updownarrow L' \times 1 \end{matrix}, \tag{7b}$$

and defining $\mathbf{v}_{1,q}^{(k)H}[i] = \tilde{\mathbf{w}}_{1,q}^{(k)H}[i]\mathbf{T}^{(k-q)}$ and $\mathbf{v}_{2,q}^{(k)H}[i] = \tilde{\mathbf{w}}_{2,q}^{(k)H}[i]\mathbf{T}^{(k-q)}$, (6) can now be written as:

$$\begin{aligned}\hat{S}_k[i] = &\sum_{q=-Q/2}^{Q/2} \mathbf{v}_{1,q}^{(k)H}[i] \begin{bmatrix} R^{(k-q)}[i] \\ \Delta\mathbf{r}[i] \end{bmatrix} \\ &+ \sum_{q=-Q/2}^{Q/2} \mathbf{v}_{2,q}^{(k)H}[i] \begin{bmatrix} R^{(N-k+q)*}[i] \\ \Delta\mathbf{r}^*[i] \end{bmatrix}\end{aligned} \tag{8}$$

Define $\mathbf{v}_1^{(k)}[i] = [\mathbf{v}_{1,-Q/2}^{(k)T}[i], \ldots, \mathbf{v}_{1,Q/2}^{(k)T}[i]]^T$ and $\mathbf{v}_2^{(k)}[i] = [\mathbf{v}_{2,-Q/2}^{(k)T}[i], \ldots, \mathbf{v}_{2,Q/2}^{(k)T}[i]]^T$. Define the selection matrix $\mathbf{J}$ as:

$$\mathbf{J} = \begin{bmatrix} \mathbf{I}_{Q+1} \otimes \tilde{\mathbf{I}}_{L'+1} \\ \mathbf{0}_{L'\times(Q+1)(L'+1)} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{(Q+1)\times(Q+1)(L'+1)} \\ \mathbf{1}_{Q+1}^T \otimes \tilde{\mathbf{I}}_{L'+1} \end{bmatrix},$$

where $\tilde{\mathbf{I}}_{L'+1}$ is the first row of the matrix $\mathbf{I}_{L'+1}$ and $\tilde{\tilde{\mathbf{I}}}_{L'+1}$ is the matrix constitute of the last L' rows of the matrix $\mathbf{I}_{L'+1}$. Finally, define $\mathbf{u}_1^{(k)}[i] = \mathbf{J}\mathbf{v}_1^{(k)}[i]$ and $\mathbf{u}_2^{(k)}[i] = \mathbf{J}\mathbf{v}_2^{(k)}[i]$, (8) can now be written as:

$$\hat{S}_k[i] = \mathbf{u}_1^{(k)H}[i] \begin{bmatrix} R^{(k+Q/2)}[i] \\ \vdots \\ R^{(k-Q/2)}[i] \\ \Delta\mathbf{r}[i] \end{bmatrix} + \mathbf{u}_2^{(k)H}[i] \begin{bmatrix} R^{(N-k-Q/2)*}[i] \\ \vdots \\ R^{(N-k+Q/2)*}[i] \\ \Delta\mathbf{r}^*[i] \end{bmatrix} \tag{9}$$

Due to the IQ imbalance the mirroring effect arises, and hence a judicious equalizer takes into account this effect by estimating the transmitted symbol on subcarrier k and its mirror subcarrier $N-k$ in a joint fashion. Writing a similar equation for the estimated symbol on the $(N-k)$th subcarrier and combining it with (9) we arrive at:

$$\underbrace{\begin{bmatrix} \hat{S}_k[i] \\ \hat{S}_{N-k}^*[i] \end{bmatrix}}_{\tilde{\mathbf{s}}_k[i]} = \underbrace{\begin{bmatrix} \mathbf{u}_1^{(k)H}[i] & \mathbf{u}_2^{(k)H}[i] \\ \mathbf{u}_2^{(N-k)T}[i] & \mathbf{u}_1^{(N-k)T}[i] \end{bmatrix}}_{\mathbf{U}_k^H[i]} \underbrace{\begin{bmatrix} R^{(k+Q/2)}[i] \\ \vdots \\ R^{(k-Q/2)}[i] \\ \Delta\mathbf{r}[i] \\ R^{(N-k-Q/2)*}[i] \\ \vdots \\ R^{(N-k+Q/2)*}[i] \\ \Delta\mathbf{r}^*[i] \end{bmatrix}}_{\tilde{\mathbf{r}}_k[i]}. \tag{10}$$

From (10) the estimate of the QAM transmitted symbols can be obtained by applying the time-varying PTEQ that varies from block to block. However, the implementation/computation of the TV PTEQ can be significantly reduced by splitting the PTEQ into two parts: a time-invariant part that is fixed as long as the channel is stationary and a time-varying part that can be compensated for by means of a complex rotation applied to all subcarriers in the same OFDM block as will be clear later on.

Note that $\tilde{\mathbf{r}}_k[i]$ can now be written as:

$$\tilde{\mathbf{r}}_k[i] = \underbrace{\begin{bmatrix} \mathbf{F}^{(k)} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^{(N-k)*} \end{bmatrix}}_{\tilde{\mathbf{F}}_k} \begin{bmatrix} \mathbf{r}[i] \\ \mathbf{r}^*[i] \end{bmatrix} \tag{11}$$

where $\mathbf{F}^{(k)}$ is given by:

$$\mathbf{F}^{(k)} = \left[\begin{array}{ccc} \mathbf{0}_{1\times L'} & & \mathcal{F}^{(k+Q/2)} \\ & \vdots & \\ \mathbf{0}_{1\times L'} & & \mathcal{F}^{(k-Q/2)} \\ \hline \bar{\mathbf{I}}_{L'} & \mathbf{0}_{L'\times(N-L')} & -\bar{\mathbf{I}}_{L'} \end{array}\right]$$

Hence we can write the received sequence and its conjugate as in (12) shown on the top of next page. To solve for $\mathbf{U}_k[i]$, we define the following MMSE cost function:

$$\mathcal{J}(\mathbf{U}_k[i]) = \mathcal{E}\left\{ \|\tilde{\mathbf{s}}_k[i] - \mathbf{U}_k^H[i]\tilde{\mathbf{r}}_k[i]\|^2 \right\}.$$

Hence, the MMSE solution of $\mathbf{U}_k[i]$ can be obtained by setting $\partial\mathcal{J}(\mathbf{U}_k[i])/\partial\mathbf{U}_k[i] = \mathbf{0}$ as:

$$\mathbf{U}_k[i] = \left(\tilde{\mathbf{F}}_k \left(\mathbf{G}\mathbf{\Lambda}[i]\mathbf{R}_s\mathbf{\Lambda}^H[i]\mathbf{G}^H + \mathbf{R}_{\tilde{\eta}}\right)\tilde{\mathbf{F}}_k^H\right)^{-1} \tilde{\mathbf{F}}_k\mathbf{G}\mathbf{\Lambda}[i]\mathbf{R}_s \times [\mathbf{e}_k\mathbf{e}_k]. \tag{13}$$

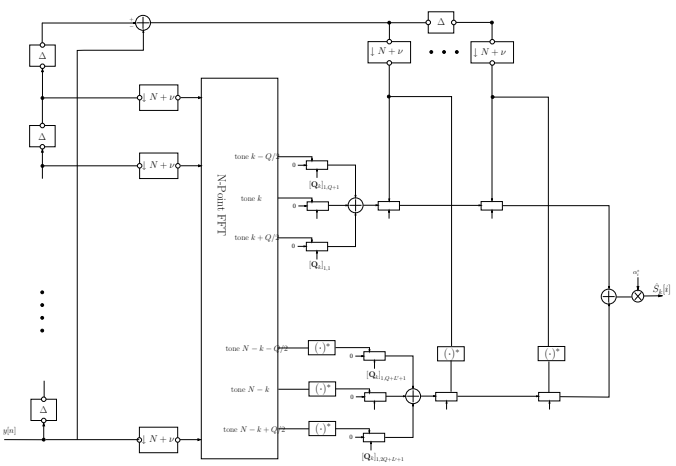


Fig. 1. PTEQ for OFDM with IQ imbalance and CFO.

For white source $\mathbf{R}_s = \sigma_s^2\mathbf{I}$ (13) can be written as:

$$\begin{aligned} \mathbf{U}_k[i] &= \left(\tilde{\mathbf{F}}_k \left(\mathbf{G}\mathbf{G}^H + \sigma_s^{-2}\mathbf{R}_{\tilde{\eta}}\right)\tilde{\mathbf{F}}_k^H\right)^{-1} \tilde{\mathbf{F}}_k\mathbf{G}\mathbf{\Lambda}[i][\mathbf{e}_{N+k+1}\mathbf{e}_{4N+k+1}] \\ &= \underbrace{\left(\tilde{\mathbf{F}}_k \left(\mathbf{G}\mathbf{G}^H + \sigma_s^{-2}\mathbf{R}_{\tilde{\eta}}\right)\tilde{\mathbf{F}}_k^H\right)^{-1} \tilde{\mathbf{F}}_k\mathbf{G}[\mathbf{e}_{N+k+1}\mathbf{e}_{4N+k+1}]}_{\mathbf{Q}_k} \\ &\quad \times \underbrace{\begin{bmatrix} \alpha_i & 0 \\ 0 & \alpha_i^* \end{bmatrix}}_{\tilde{\mathbf{\Lambda}}[i]} \end{aligned} \tag{14}$$

The MMSE PTEQ obtained in (14) for the case of white input source consists of two parts. A time-invariant part denoted by $\mathbf{Q}_k$ and a time-varying part captured by the complex rotation matrix $\tilde{\mathbf{\Lambda}}[i]$. the time-invariant part is fixed as long as the channel, the IQ imbalance parameters and the CFO are fixed. The complex rotation, on the other hand, changes from block to block. Note that, the complex rotation within the same OFDM block is fixed over all subcarriers. Exploiting this structure we can significantly reduce the computational complexity by computing the time-invariant part of the PTEQ and compensate for the time-variations by means of the complex rotation afterwards. On the implementation level, the PTEQ is explained in Figure 1.

RLS Initialization

In order to obtain the PTEQ, the channel, the IQ parameters and the CFO must be known/estimated. In this section, we devise a training-based RLS direct equalization/estimation of the time-invariant part of the PTEQ assuming that the CFO only known at the receiver. Assuming that I OFDM symbols are available for training, the optimal $\mathbf{Q}_k$ for the kth subcarrier can be computed as:

$$\min_{\mathbf{Q}_k} \mathcal{J}(\mathbf{Q}_k) = \min_{\mathbf{Q}_k} \sum_{i=0}^{I-1} \left\| \tilde{\mathbf{\Lambda}}\tilde{\mathbf{s}}_k[i] - \mathbf{Q}_k^H\tilde{\mathbf{r}}_k[i] \right\|^2 \tag{15}$$

The RLS algorithm to compute the time-invariant part of the PTEQ is listed in Algorithm 1.

IV. Simulations

In this section we present some of the simulation results of the proposed equalization technique for OFDM transmission over TI channels. We consider an OFDM system with $N = 64$ subcarriers. The TI channel is of order $L = 6$. The channel taps are simulated as i.i.d complex Gaussian random variables. The channel is kept fixed over a burst of transmission, and may change from burst to burst

$$\begin{bmatrix}\mathbf{r}[i]\\ \mathbf{r}^*[i]\end{bmatrix} = \underbrace{\begin{bmatrix}\mu\mathbf{D}_\epsilon\tilde{\mathbf{H}}\left(\mathbf{I}_3\otimes\mathbf{P}\right)\left(\mathbf{I}_3\otimes\boldsymbol{\mathcal{F}}^H\right) & v\mathbf{D}_\epsilon^*\tilde{\mathbf{H}}^*\left(\mathbf{I}_3\otimes\mathbf{P}\right)\left(\mathbf{I}_3\otimes\boldsymbol{\mathcal{F}}^H\right)\\ v^*\mathbf{D}_\epsilon\tilde{\mathbf{H}}\left(\mathbf{I}_3\otimes\mathbf{P}\right)\left(\mathbf{I}_3\otimes\boldsymbol{\mathcal{F}}^H\right) & \mu^*\mathbf{D}_\epsilon^*\tilde{\mathbf{H}}^*\left(\mathbf{I}_3\otimes\mathbf{P}\right)\left(\mathbf{I}_3\otimes\boldsymbol{\mathcal{F}}^H\right)\end{bmatrix}}_{\mathbf{G}}\underbrace{\begin{bmatrix}\alpha_i\mathbf{I}_{3N} & \mathbf{0}_{3N\times 3N}\\ \mathbf{0}_{3N\times 3N} & \alpha_i^*\mathbf{I}_{3N}\end{bmatrix}}_{\boldsymbol{\Lambda}[i]}\begin{bmatrix}\mathbf{s}[i-1]\\ \mathbf{s}[i]\\ \mathbf{s}[i+1]\\ \mathbf{Z}_1\mathbf{s}^*[i-1]\\ \mathbf{Z}_1\mathbf{s}^*[i]\\ \mathbf{Z}_1\mathbf{s}^*[i+1]\end{bmatrix} + \tilde{\boldsymbol{\eta}}[i]. \quad (12)$$

Algorithm 1 RLS Direct Equalization with CFO

for $k \in \{1,\ldots,N2/-1\}$ **do**
  initialize $\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[-1] = \delta^{-1}\mathbf{I}$
  initialize $\mathbf{Q}_k[-1] = \mathbf{0}_{2(L'+1)(Q+1)\times 2}$
end for
for $i = 0$ to $I-1$ **do**
  for $k \in \{1,\ldots,N2/-1\}$ **do**
    $\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i] \leftarrow \mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i-1] - \frac{\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i-1]\tilde{\mathbf{r}}_k[i]\tilde{\mathbf{r}}_k^H[i]\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i-1]}{1+\tilde{\mathbf{r}}_k^H[i]\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i-1]\tilde{\mathbf{r}}_k[i]}$
    $\mathbf{e}[i] = \tilde{\mathbf{s}}_k[i] - \mathbf{Q}_k^H[i-1]\tilde{\mathbf{r}}_k[i]$
    $\mathbf{Q}_k[i] \leftarrow \mathbf{Q}_k[i-1] + \frac{\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i-1]\tilde{\mathbf{r}}_k[i]}{1+\tilde{\mathbf{r}}_k^H[i]\mathbf{R}_{\tilde{\mathbf{r}}}^{k^{-1}}[i-1]\tilde{\mathbf{r}}_k[i]}\mathbf{e}[i]^H$
  end for
end for
$\mathbf{Q}_k = \mathbf{Q}_k[I-1]$

independently. The analog front-end of the OFDM receiver suffers from an IQ imbalance and CFO. We consider an OFDM receiver with IQ phase imbalance $\Delta\phi = 10^o$ and IQ amplitude imbalance $\Delta a = 0.1$. The CFO is assumed to be $\epsilon = 0.2$ (we consider the non-integer part of the CFO), since the integer part of the CFO results into a cyclic shift and does not contribute to the ICI.

We focus on the frequency-domain PTEQ. The PTEQ is obtained by transferring the TEQ operations to the frequency-domain. The TEQ is designed according to the BEM of order $L' = 10$ and the number of basis functions is $Q = 8$. The number of basis functions reflect the span of the ICI that is considered in the equalization process (which is in this case 4 neighboring subcarriers on each side).

In the first setup, we consider that the channel, the IQ imbalance parameters and the CFO are known at the receiver. As shown in Figure 2, the proposed PTEQ approaches the performance of ideal OFDM transmission over TI channels.

In the second setup, we develop a training-based RLS direct equalization to obtain the time-invariant part of the PTEQ. The CFO is assumed to be known at the receiver. For the RLS algorithm we consider that $I = 150$ and $I = 250$ OFDM symbols are available for training. As shown in Figure 2, The RLS based direct equalization suffers from an error floor at $BER = 9\times 10^{-1}$, while the performance coincides with the performance when perfect knowledge is assumed for $I = 250$.

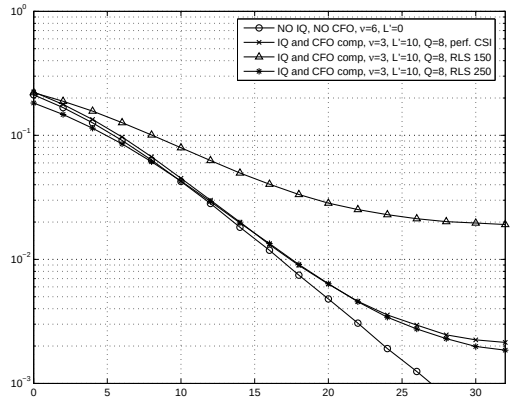


Fig. 2. IQ and CFO direct Equalization

V. Conclusions

In this paper we propose a frequency-domain PTEQ for OFDM transmission over TI channels with imperfect analog front-end receivers. The CP is assumed to be less than the channel order, which means that IBI is present. We show that analog front-end imperfections degrade the performance significantly, and therefore equalization/compensation is crucial. For the case of IQ imbalance and CFO, the PTEQ is time-varying. However, it was shown that the time-varying can be split into two parts. A time-invariant part, and a time-varying part captured by means of complex rotation. In all cases a training-based RLS scheme can be used to design the time-invariant part of the PTEQ.

References

[1] "ETSI Digital Video Broadcasting (DVB); Framing Structure, Channel Coding and Modulation for Digital Terrestrial Television," Aug. 2001.

[2] "IEEE 802.11 Local and Metropolitan Area Networks: Wireless LAN Medium Acess Control (MAC) and Physical (PHY) Specifications.," 1999.

[3] "ETSI Broadband radio access networks (BRAN); HIPERLAN type 2 technical specification; Physical layer," Aug. 1999.

[4] P. H. Moose, "A Technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction," *IEEE Trans. Commun.*, vol. 42, pp. 2908–2914, Oct. 1994.

[5] J. H. Yu and Y. T. Su, "Pilot-Assisted Maximum-Likelihood Frequency-Offset Estimation for OFDM Systems," *IEEE Trans. Commun.*, vol. 52, pp. 1997–2008, Nov. 2004.

[6] J. Tubbax, B. Come, L. Van der Perre, L. Deneire,S. Donnay, S and M. Engels, "Compensation of IQ Imbalance in OFDM Systems," in *IEEE Int. Conf. on Communications*, (Anchorage, Alaska USA), May, 11-15 2003.

[7] C.-L. Liu, "Impacts of I/Q Imbalance on QPSK-OFDM-QAM Detection," *IEEE Trans. Consumer Electron.*, vol. 44, pp. 984–989, Aug. 1998.

[8] A. Targhiat and A. H. Sayed, "On the Baseband Compensation of IQ Imbalance in OFDM Systems," in *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, (Montreal, Canada), May 2004.

[9] J. Tubbax,A. Fort, L. Van der Perre, S. Donnay, M. Engels, M. Moonen, and H. De Man, "Joint Compensation of IQ Imbalance and Frequency Offset in OFDM Systems," in *IEEE Global Communications Conference*, (San Francisco, CA USA), December, 1-5 2003.

[10] S. Fouladifard and H. Shafiee, "Frequency Offset Estimation in OFDM Systems in Presence of IQ Imbalance," in *ICCS*, (Singapore), Nov. 2002.

[11] M. Valkama and V.Koivunen, "Advanced Methods for I/Q Imbalance Compensation in Communication Receivers," *IEEE Trans. Signal Processing*, vol. 49, pp. 2335–2344, Oct. 2001.

[12] I. Barhumi, G. Leus and M. Moonen, "Per-Tone Equalization for OFDM over Doubly-Selective Channels," in *IEEE Int. Conf. on Communications*, (Paris, France), June 2004.

[13] I. Barhumi, G. Leus, and M. Moonen, "Time-Domain and Frequency-Domain Per-Tone Equalization for OFDM over Doubly-Selective Channels," *Signal Processing*, vol. 84/11, pp. 2055–2066, 2004. Special Section Signal Processing in Communications.