Estimation and Direct Equalization of Doubly Selective Channels*

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Abstract

In this paper we propose estimation and direct equalization techniques for transmission over doubly selective channels. The doubly selective channel is approximated using the basis expansion model (BEM). Linear and decision feedback equalizers implemented by time-varying finite impulse response (TV FIR) filters may then be used to equalize the doubly selective channel, where the TV FIR filters are designed according to the BEM. Here we show how the BEM coefficients of the equalizer can be obtained either based on the channel BEM coefficients (or an estimated version thereof) or directly through a direct equalization technique. The proposed channel estimation and direct equalization techniques range from pilot symbol assisted modulation (PSAM) based techniques to blind and semi-blind techniques. In PSAM techniques, pilot symbols are utilized to estimate the channel or directly obtain the equalizer coefficients. The training overhead can be completely eliminated by using blind techniques or reduced by combining training based techniques with blind techniques resulting in semi-blind techniques.

I. Introduction

Over the last decade, the mobile wireless telecommunication industry has undergone tremendous changes and experienced rapid growth. The reason behind this growth is the increasing demand for bandwidth hungry multimedia applications. This demand for even higher data rates at the user's terminal is expected to continue for the coming years as more and more applications are emerging. Therefore, current cellular systems have been designed to provide date rates that range from a few megabits per second for stationary or low mobility users to a few hundred kilobits per second to high mobility users. In addition to the frequency-selectivity characteristics caused by multipath propagation, the channel often exhibits time-variant characteristics caused by the user's mobility. This result in so-called doubly selective (time- and frequency-selective) channels.

In [1], [2], linear and decision feedback equalizers have been developed for single carrier transmission over doubly selective channels. There, the time-varying channel was modeled using the basis expansion model (BEM). The BEM coefficients are then used to design the equalizer (linear or decision feedback). So far, it was assumed that the BEM coefficients are perfectly known at the receiver, and that they were obtained by a least squares (LS) fitting of the noiseless real channel. This is, however, far from realistic. A more realistic approach is to either use training symbols to estimate and/or directly equalize the channel or obtain the channel estimate and/or directly equalize the channel blindly or semi-blindly. In this paper we will focus on pilot symbol assisted modulation (PSAM)-based, blind and semi-blind techniques for channel estimation and direct equalization of rapidly time-varying channels.

PSAM techniques rely on time multiplexing data symbols and known pilot symbols at known positions, which the receiver utilizes to either estimate the channel or obtain the equalizer coefficients directly. In this paper, we first derive the optimal minimum mean-square error (MMSE) interpolation filter based channel estimation technique. Then we derive the conventional BEM channel estimation technique (based on LS fitting). It has been shown that the modeling error (between the true channel and the BEM channel model) is quite large for the case when the BEM period equals the time window [3], [4] . This case corresponds to a critical sampling of the Doppler spectrum. Reducing this modeling error can be achieved by setting the BEM period equal to a multiple of the time window. In other words, we can reduce the modeling error by oversampling the Doppler spectrum. In [5] the authors treated the first case ignoring the modeling error. However, when an oversampling of the Doppler spectrum is used, the BEM based PSAM channel estimation is sensitive to noise. Here, we show that robust PSAM based channel estimation can be obtained by combining the optimal MMSE interpolation based channel estimation with the BEM considering an oversampling rate greater than one (an oversampling rate equal to 2 appears to be sufficient). In addition, we show that the channel estimation step can be skipped and obtain the equalizer coefficients directly based on the pilot symbols. This is referred to as PSAM-based direct equalization.

The training overhead imposed on the system can be completely eliminated by using *blind techniques*. Due to the poor performance of blind techniques and their high implementation complexity, better performance and reduced complexity *semi-blind techniques* can be obtained. Semi-blind techniques are obtained by combining training-based and blind techniques.

For our blind techniques we focus on deterministic approaches. For time-invariant (TI) channels, a least-squares based deterministic channel estimation method is discussed in [6], and deterministic mutually referenced equalization

is proposed in [7], [8]. Subspace based methods have also been proposed for channel identification/equalization for TI channels [11], [12], [13], [14], [15], [16]. For doubly selective channels, deterministic blind identification/equalization techniques are proposed in [9], [10], where for a ZF FIR solution to exist, the number of sub-channels (receive antennas) is required to be greater than the number of basis functions used for BEM channel modeling. In [17], [18] blind techniques based on linear prediction are proposed for doubly selective channels, where second-order statistics of the data are used. However, these techniques also require the number of receive antennas to be greater than the number of basis functions of the BEM channel. However, for our approach we show that the ZF solution already exists when only two sub-channels (receive antennas) are used.

This paper is organized as follows. In Section II, the system model is introduced. PSAM techniques are introduced in Section III. In Section IV, blind and semi-blind techniques are investigated. Simulation results are given in Section V. Finally our conclusions are drawn in Section VI.

Notations: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, ^{H}, and † represent conjugate, transpose, Hermitian, and pseudo-inverse, respectively. Continuous-time variables (discrete-time) are denoted as $x(\cdot)$ ($x[\cdot]$). $\mathcal{E}\{\cdot\}$ denotes expectation. Finally, we denote the $N \times N$ identity matrix as $\mathbf{I}_N$.

II. System Model

We assume a single-input multiple-output (SIMO) system with N_r receive antennas, but the results can be easily extended to a multiple-input multiple-output (MIMO) system. Focusing on a baseband-equivalent description, the transmitted signal consists of discrete symbols that are pulse shaped with the transmit filter $g_{tr}(t)$ at a rate $1/T$ (the symbol rate). Hence, the baseband transmitted signal can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} x[k] g_{tr}(t - kT), \tag{1}$$

where $x[k]$ is the kth transmitted QAM symbol. The received signal, on the other hand, is filtered with the receive filter $g_{rec}(t)$. Assuming the channel time-variation is negligible over the time span of the receive filter, the input-output relationship can be written as

$$y^{(r)}(t) = \sum_{k=-\infty}^{\infty} x[k] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g^{(r)}(t;\tau) g_{tr}(t - kT - \tau - s) g_{rec}(s) \mathrm{d}s \ \mathrm{d}\tau + v^{(r)}(t), \tag{2}$$

where $g^{(r)}(t;\tau)$ is the doubly selective channel characterizing the link between the transmitter and the rth receive antenna, and $v^{(r)}(t)$ is the baseband equivalent additive noise at the rth receive antenna. The received signal is then sampled with the symbol rate $1/T$[1]. Defining $y^{(r)}[n] = y^{(r)}(nT)$, the discrete-time input-output relationship can be written as

$$\begin{aligned} y^{(r)}[n] &= \sum_{k=-\infty}^{\infty} x[k] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g^{(r)}(nT;\tau) g_{tr}((n-k)T - \tau - s) g_{rec}(s) \mathrm{d}s \ \mathrm{d}\tau + v^{(r)}(nT) \\ &= \sum_{k=-\infty}^{\infty} x[k] \int_{-\infty}^{\infty} g^{(r)}(nT;\tau) \psi((n-k)T - \tau) \mathrm{d}\tau + v^{(r)}(nT) \\ &= \sum_{k=-\infty}^{\infty} x[k] g^{(r)}[n; n-k] + v^{(r)}[n], \end{aligned} \tag{3}$$

where $\psi(t)$ is the overall impulse response of the transmit and receive filters, and $v^{(r)}[n]$ is the discrete-time additive noise at the rth receive antenna.

[1]Temporal oversampling is also possible here to obtain a SIMO system. In this paper we consider the use of multiple receive antennas. Assuming temporal oversampling, to some degree, is equivalent to using multiple receive antennas, where the number of receive antennas is equal to the temporal oversampling factor.

Assuming an ideal pulse shaping filter $\psi(t) = \mathrm{sinc}(\pi t/T)$, then $g^{(r)}[n;\nu]$ is obtained as:

$$g^{(r)}[n;\nu] = \int_{-\infty}^{\infty} g^{(r)}(nT;\tau)\mathrm{sinc}(\pi(\nu T - \tau)/T)\mathrm{d}\tau. \tag{4}$$

For large bandwidth channels, (4) can be well approximated as

$$g^{(r)}[n;\nu] = g^{(r)}(nT;\nu T), \tag{5}$$

which corresponds to sampling the time-varying channel in the time domain as well as in the lag dimension.

For causal doubly selective channels with finite maximum delay spread $\tau_{\max}$ and order $L = \lfloor \tau_{\max}/T \rfloor$, the input-output relationship (3) can be written as:

$$y^{(r)}[n] = \sum_{l=0}^{L} g^{(r)}[n;l]x[n-l] + v^{(r)}[n]. \tag{6}$$

Basis Expansion Channel Model

The mobile wireless channel can be characterized as a time-varying multipath fading channel, where each resolvable path consists of a superposition of a large number of independent scatterers (rays) that arrive at the receiver almost simultaneously. This is referred to as *Jakes' channel model* [19]. In this model the variation of each tap can be simulated as

$$g^{(r)}[n;l] = \sum_{\mu=0}^{Q_J-1} G_{l,\mu}^{(r)} e^{j2\pi f_{\max} T \cos\phi_{l,\mu}^{(r)} n}, \tag{7}$$

where Q_J is the number of scatterers, $G_{l,\mu}^{(r)}$ is the complex gain, and $\phi_{l,\mu}^{(r)}$ is the direction of arrival angle of the μth ray of the lth tap respectively. $\phi_{l,\mu}^{(r)}$ is a random variable uniformly distributed over $[0, 2\pi]$.

The channel model in (7) has a rather complex structure due to the large (possibly infinite) number of parameters to be identified, which complicates, if not prevents, the development of low complexity equalizers. This motivates the use of alternative models that have fewer parameters. This is the motivation behind the Basis Expansion Model (BEM) [9], [20], [21], [22]. In this BEM, the time-varying channel $g^{(r)}[n;l]$ over a window of N samples is expressed as a superposition of complex exponential basis functions with frequencies on a discrete grid. In other words, the time-varying channel $g^{(r)}[n,l]$ is modeled for $n \in \{0, \cdots, N-1\}$ by a BEM:

$$h^{(r)}[n;l] = \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)} e^{j2\pi qn/K}, \tag{8}$$

where Q is the number of basis functions, and K is the BEM period. Q and K should be chosen such that $Q/(KT)$ is larger than the maximum Doppler frequency, i.e. $Q/(KT) \geq f_{\max}$. Finally, $h_{q,l}^{(r)}$ is the coefficient of the qth basis of the lth tap of the TV channel characterizing the link between the transmitter and the rth receive antenna, which is kept invariant over a period of NT, but may change from block to block.

III. PSAM Techniques

A. PSAM Channel Estimation

For the sake of simplicity we assume the number of receive antennas $N_r = 1$, i.e. we assume a SISO system. This is a valid assumption because we can decouple the channel estimation problem for a SIMO system into N_r parallel SISO channel estimation problems. Using the optimal training procedure proposed in [5], the doubly selective channel of order L can be viewed as L flat fading channels on the part of the received sequence that corresponds to training. The data/training multiplexing is shown in Figure 1, where the training part consists of a training symbol surrounded by L zeros on each side. Assuming we use P such training clusters where the pilot symbols are located at positions $n_0, \ldots, n_{P-1}$, the input-output relation on the pilot positions can be written as

$$y[n_{p,l}] = g[n_{p,l};l]x[n_p] + v[n_{p,l}],$$

where $n_{p,l} = n_p + l$ for $l = 0, \ldots, L$.

In this subsection, we first derive the optimal minimum mean-squared error (MMSE) PSAM-based channel estimation, which leads to the development of the optimal interpolation filter. However, since the BEM coefficients of the TV channels are needed to design the equalizers (linear and decision feedback), the PSAM-based estimation of the BEM coefficients is also discussed and combined with PSAM-based MMSE channel estimation to enhance the LS fitting of the true channel and the estimated one.

1) MMSE Channel Estimation: A noisy estimate of the channel at the pth pilot is easily obtained by

$$\hat{g}[n_{p,l}; l] = \frac{y[n_{p,l}]}{x[n_p]} = g[n_{p,l}; l] + \tilde{v}[n_{p,l}], \text{ for } p = 0, \ldots, P-1. \tag{9}$$

where $\tilde{v}[n_{p,l}] = v[n_{p,l}]/x[n_p]$. Define $\hat{\mathbf{g}}_{t,l} = [\hat{g}[n_{0,l}; l], \ldots, \hat{g}[n_{P-1,l}; l]]^T$, which is a vector containing the noisy estimates of the channel on the pilot positions. From these noisy estimates we have to reconstruct the channel response for all $n \in \{0, \ldots, N-1\}$. In other words, we need to design a $P \times N$ interpolation matrix $\mathbf{W}$ such that

$$\hat{\mathbf{g}}_l = \mathbf{W}^H \hat{\mathbf{g}}_{t,l}, \tag{10}$$

where $\hat{\mathbf{g}}_l = [\hat{g}[0; l], \ldots, \hat{g}[N-1; l]]^T$, with $\hat{g}[n; l]$ an estimate of the lth tap of the true channel at time index n. The mean square-error (MSE) criteria can be written as

$$\begin{aligned} \mathcal{J} &= \frac{1}{N} \sum_{n=0}^{N-1} \mathcal{E}\{|\hat{g}[n; l] - g[n; l]|^2\} \\ &= \frac{1}{N} \mathcal{E}\{\|\mathbf{W}^H \hat{\mathbf{g}}_{t,l} - \mathbf{g}_l\|^2\}, \end{aligned} \tag{11}$$

where $\mathbf{g}_l = [g[0; l], \ldots, g[N-1; l]]^T$ is the channel state information at the lth tap.

The MMSE interpolation matrix $\mathbf{W}$ is then obtained by solving

$$\min_{\mathbf{W}} \mathcal{J}$$

The solution of this problem is obtained as follows [23]

$$\mathbf{W} = \left(\mathbf{R}_p + \mathbf{R}_{\tilde{v}}\right)^{-1} \mathbf{R}_h \tag{12}$$

where $\mathbf{R}_p$ is the channel correlation matrix on the pilots given by

$$\mathbf{R}_p = \begin{bmatrix} r_h[0] & \cdots & r_h[n_{P-1} - n_0] \\ r_h[n_1] & \cdots & r_h[n_{P-1} - n_1] \\ \vdots & \ddots & \vdots \\ r_h[n_{P-1}] & \cdots & r_h[0] \end{bmatrix},$$

and $\mathbf{R}_h$ is given by

$$\mathbf{R}_h = \begin{bmatrix} r_h[n_0] & \cdots & r_h[N - n_0 - 1] \\ r_h[n_1] & \cdots & r_h[N - n_1 - 1] \\ \vdots & \ddots & \vdots \\ r_h[n_{P-1}] & \cdots & r_h[N - n_{P-1} - 1] \end{bmatrix},$$

with $r_h[k] = \mathcal{E}\{g[n; l] g^*[n - |k|; l]\}$ which is independent of the tap index l assuming that the channel taps are independent identically distributed (i.i.d) random variables. $\mathbf{R}_{\tilde{v}}$ is the channel estimate error at the pilot positions covariance matrix. Both $\mathbf{R}_p$ and $\mathbf{R}_h$ are assumed to be known (assuming Jakes model, then it is only require the knowledge of the system maximum Doppler shift $f_{\max}$). Note that we used the assumption that the channel is wide sense stationary (WSS). Assuming i.i.d input symbols $x[n]$, and the training is of Kronecker delta form (i.e. $x[n_p] = 1 \ \forall p = 0, \ldots, P-1$), and white noise with with normalized power β, then $\mathbf{R}_{\tilde{v}} = \beta \mathbf{I}_P$.

2) BEM Channel Estimation: For our equalizer design we require the channel BEM coefficients. Define $\mathbf{h}_l = [h_{-Q/2,l}, \ldots, h_{Q/2,l}]^T$ as the vector containing the BEM coefficients of the lth tap of the channel. In the ideal case, where the TV channel $\mathbf{g}_l$ is perfectly known at the receiver, the channel BEM coefficients can be obtained by solving the following LS problem:

$$\min_{\mathbf{h}_l} \|\mathbf{g}_l - \boldsymbol{\mathcal{L}}\mathbf{h}_l\|^2, \tag{13}$$

where

$$\boldsymbol{\mathcal{L}} = \begin{bmatrix} 1 & \ldots & 1 \\ e^{-j2\pi\frac{Q}{2}\frac{1}{K}} & \ldots & e^{j2\pi\frac{Q}{2}\frac{1}{K}} \\ \vdots & & \vdots \\ e^{-j2\pi\frac{Q}{2}\frac{N-1}{K}} & \ldots & e^{j2\pi\frac{Q}{2}\frac{N-1}{K}} \end{bmatrix}.$$

The solution of (13) is given by

$$\mathbf{h}_l = \boldsymbol{\mathcal{L}}^{\dagger}\mathbf{g}_l.$$

In practice, only a few pilot symbols are available for channel estimation. Assuming the noisy estimates are obtained as in (9), then the channel BEM coefficients can be obtained by solving the following LS problem:

$$\min_{\mathbf{h}_l} \|\hat{\mathbf{g}}_{t,l} - \tilde{\boldsymbol{\mathcal{L}}}_l\mathbf{h}_l\|^2, \tag{14}$$

where

$$\tilde{\boldsymbol{\mathcal{L}}}_l = \begin{bmatrix} e^{-j2\pi\frac{Q}{2}\frac{n_{0,l}}{K}} & \ldots & e^{j2\pi\frac{Q}{2}\frac{n_{0,l}}{K}} \\ \vdots & & \vdots \\ e^{-j2\pi\frac{Q}{2}\frac{n_{P-1,l}}{K}} & \ldots & e^{j2\pi\frac{Q}{2}\frac{n_{P-1,l}}{K}} \end{bmatrix}.$$

The solution of (14) is obtained by

$$\mathbf{h}_l = \tilde{\boldsymbol{\mathcal{L}}}_l^{\dagger}\hat{\mathbf{g}}_t \tag{15}$$

It has been shown in [5], that when we critically sample the Doppler spectrum ($K = N$) and ignore the modeling error, the optimal training strategy consists of inserting equipowered, equispaced pilot symbols. However, critical sampling of the Doppler spectrum results in an error floor due to the large modeling error. On the other hand, oversampling the Doppler spectrum ($K = rN$, with integer $r > 1$) reduces the modeling error when the ideal case is considered [3], [24], i.e. when (13) is applied. However, this channel estimate is sensitive to noise when PSAM channel estimation is used.

A robust channel estimate can then be obtained by combining the optimal MMSE interpolation based channel estimate obtained in (10) with the BEM channel estimate obtained in (13) as follows:

- First, obtain the channel estimate $\hat{\mathbf{g}}_l$ as in (10).
- Second, obtain the LS solution of the following problem:

$$\min_{\mathbf{h}_l} \|\hat{\mathbf{g}}_l - \boldsymbol{\mathcal{L}}\mathbf{h}_l\|^2, \tag{16}$$

The solution of (16) can be obtained as

$$\mathbf{h}_l = \boldsymbol{\mathcal{L}}^{\dagger}\hat{\mathbf{g}}_l, \tag{17}$$

or equivalently in one step as

$$\mathbf{h}_l = \boldsymbol{\mathcal{L}}^{\dagger}\mathbf{W}^H\hat{\mathbf{g}}_{t,l}. \tag{18}$$

Even though this applies to critical sampling as well as to the oversampling case, little gain is obtained when combining the MMSE interpolation based channel estimate with the critically sampled BEM ($K = N$), as will be clear in Section V.

B. PSAM Direct Equalization

In this subsection we propose a PSAM-based direct equalization of doubly selective channels, where the TV FIR equalizer coefficients are obtained directly without passing through the channel estimation step. Applying the TV FIR equalizer $w^{(r)}[n;\nu]$ to the rth receive antenna sequence $y^{(r)}[n]$, an estimate of $x[n]$ for $n \in \{L', \ldots, N-1\}$ subject to some decision delay d can be obtained as

$$\hat{x}[n-d] = \sum_{r=1}^{N_r} \sum_{\nu=-\infty}^{\infty} w^{(r)}[n;\nu] y^{(r)}[n-\nu]. \tag{19}$$

Using the BEM to design the TV FIR filters such that each TV FIR equalizer $w^{(r)}[n;\nu]$ is designed to have $L'+1$ taps, where the time-variation of each tap is modeled by $Q'+1$ complex exponential basis functions with frequencies on the same DFT grid as for the channel as

$$w^{(r)}(n;\nu) = \sum_{l'=0}^{L'} \delta[\nu - l'] \sum_{q'=-Q'/2}^{Q'/2} w^{(r)}_{q',l'} e^{j2\pi q' n/K}, \tag{20}$$

Substituting (20) in (19) we obtain

$$\hat{x}[n-d] = \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q' n/K} w^{(r)}_{q',l'} y^{(r)}[n-l']. \tag{21}$$

Define $\mathbf{w}^{(r)} = [\mathbf{w}^{(r)T}_{-Q'/2}, \ldots, \mathbf{w}^{(r)T}_{Q'/2}]^T$ with $\mathbf{w}^{(r)}_{q'} = [w^{(r)}_{q',0}, \ldots, w^{(r)}_{q',L'}]^T$, then a block level formulation of (21) is

$$\begin{aligned} \hat{\mathbf{x}}^T_* &= \sum_{r=1}^{N_r} \mathbf{w}^{(r)T} \boldsymbol{\mathcal{Y}}^{(r)} \\ &= \mathbf{w}^T \boldsymbol{\mathcal{Y}}, \end{aligned} \tag{22}$$

where $\hat{\mathbf{x}}_* = [\hat{x}[L'-d], \ldots, \hat{x}[N-d-1]]^T$ with $\hat{x}[n]$ is an estimate of $x[n]$, $\mathbf{w} = [\mathbf{w}^{(1)T}, \ldots, \mathbf{w}^{(N_r)T}]^T$, and $\boldsymbol{\mathcal{Y}} = [\boldsymbol{\mathcal{Y}}^{(1)T}, \ldots, \boldsymbol{\mathcal{Y}}^{(N_r)T}]^T$, with $\boldsymbol{\mathcal{Y}}^{(r)}$ a $(Q'+1)(L'+1) \times N$ matrix containing the time- and frequency-shifts of the received sequence given by $\boldsymbol{\mathcal{Y}}^{(r)} = [\mathbf{y}^{(r)}_{-Q'/2,0}, \ldots, \mathbf{y}^{(r)}_{-Q'/2,L'}, \ldots, \mathbf{y}^{(r)}_{Q'/2,L'}]^T$. The q' frequency-shifted and l' time-shifted received sequence related to the rth receive antenna is given by

$$\mathbf{y}^{(r)}_{q',l'} = \mathbf{D}_{q'} \mathbf{Z}_{l'} \mathbf{y}^{(r)},$$

with $\mathbf{Z}_{l'}$ and $\mathbf{D}_{q'}$ defined as

$$\begin{aligned} \mathbf{Z}_{l'} &= [\mathbf{0}_{(N-L')\times(L'-l')}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times l'}] \\ \mathbf{D}_{q'} &= \mathrm{diag}\{[1, \ldots, e^{j2\pi q'(N-L'-1)/K}]^T\}. \end{aligned}$$

Assume that we have P pilot symbols collected in the vector $\mathbf{x}_t = [x[n_0], \ldots, x[n_{P-1}]]^T$. Note that for direct equalization, the optimal training strategy is unknown. Therefore, we assume that the pilot symbols are inserted at positions $n_0, \ldots, n_{P-1}$ and that the pilot symbols are not necessarily surrounded with zeros on each side. Define $\boldsymbol{\mathcal{Y}}_t$ as the collection of columns of $\boldsymbol{\mathcal{Y}}$ that corresponds to the training symbol positions subject to some decision delay. Define $[\boldsymbol{\mathcal{Y}}]_i$ as the ith column of the matrix $\boldsymbol{\mathcal{Y}}$, and define $\boldsymbol{\mathcal{Y}}_t = [[\boldsymbol{\mathcal{Y}}]_{d+n_0}, \ldots, [\boldsymbol{\mathcal{Y}}]_{d+n_{P-1}}]$. To find the TV FIR coefficients $\mathbf{w}$ the following cost function is then to be minimized

$$\mathcal{J}(\mathbf{w}) = \|\mathbf{w}^T \boldsymbol{\mathcal{Y}}_t - \mathbf{x}^T_t\|^2.$$

The LS solution is then given as

$$\mathbf{w} = \left(\boldsymbol{\mathcal{Y}}^*_t \boldsymbol{\mathcal{Y}}^T_t\right)^{-1} \boldsymbol{\mathcal{Y}}^*_t \mathbf{x}_t. \tag{23}$$

A ZF TV FIR filter can be obtained with (23) if $P \geq N_r(Q'+1)(L'+1)$, provided that $N_r(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$ (see [1]).

IV. BLIND AND SEMI-BLIND TECHNIQUES

A. Channel estimation

In this section we focus on the problem of channel estimation. We first discuss a deterministic blind channel estimation procedure. In blind methods the channel is estimated up to a scalar ambiguity and e.g. computed from the singular value decomposition (eigenvalue decomposition) of a large matrix. To resolve the scalar ambiguity, a blind technique combined with a training based technique is favorable resulting in a semi-blind technique, which is discussed in a second subsection.

1) Blind Channel Estimation: Here we discuss a deterministic subspace based blind channel estimation [25]. It operates on time- and frequency-shifted versions of the received sequence. Assume that $Q'+1$ frequency-shifts and $L'+1$ time-shifts of the received sequence related to the rth receive antenna are stored in an $(Q'+1)(L'+1)\times N$ matrix $\boldsymbol{\mathcal{Y}}^{(r)}$.

Approximating the doubly selective channel using the BEM, we can write the received vector at the rth receive antenna $\mathbf{y}^{(r)}=[y^{(r)}[0],\ldots,y^{(r)}[N-1]]^T$ as

$$\mathbf{y}^{(r)}=\sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2}h_{q,l}^{(r)}\bar{\mathbf{D}}_q\bar{\mathbf{Z}}_l\mathbf{x}+\mathbf{v}^{(r)}, \tag{24}$$

where $\bar{\mathbf{D}}_q=\mathrm{diag}\{[1,\ldots,e^{j2\pi q(N-1)/K}]^T\}$, $\bar{\mathbf{Z}}_l=[\mathbf{0}_{N\times(L-l)},\mathbf{I}_N,\mathbf{0}_{N\times l}]$, $\mathbf{x}=[x[-L],\ldots,x[N-1]]^T$, and $\mathbf{v}^{(r)}$ is defined similar to $\mathbf{y}^{(r)}$. Hence, $\mathbf{y}_{q',l'}^{(r)}$ can be written as

$$\mathbf{y}_{q',l'}^{(r)}=\sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2}e^{j2\pi q(L'-l')/K}h_{q,l}^{(r)}\bar{\mathbf{D}}_{q+q'}\tilde{\mathbf{Z}}_{l+l'}\mathbf{x}+\mathbf{v}_{q',l'}^{(r)}, \tag{25}$$

where $\tilde{\mathbf{Z}}_k=[\mathbf{0}_{(N-L')\times(L+L'-k)},\mathbf{I}_{N-L'},\mathbf{0}_{(N-L')\times k}]$, and $\mathbf{v}_{q',l'}^{(r)}$ is similarly defined as $\mathbf{y}_{q',l'}^{(r)}$.

Define $\boldsymbol{\mathcal{X}}=[\mathbf{x}_{-(Q'+Q)/2,0},\ldots,\mathbf{x}_{-(Q'+Q)/2,(L+L'+1)},\ldots,\mathbf{x}_{(Q'+Q)/2,(L+L'+1)}]^T$ with $\mathbf{x}_{p,k}$ obtained as

$$\mathbf{x}_{p,k}=\mathbf{D}_p\tilde{\mathbf{Z}}_k\mathbf{x}.$$

A relationship between $\boldsymbol{\mathcal{Y}}^{(r)}$ and the transmitted sequence can be obtained by substituting (24) in $\boldsymbol{\mathcal{Y}}^{(r)}$ resulting in

$$\boldsymbol{\mathcal{Y}}^{(r)}=\boldsymbol{\mathcal{H}}^{(r)}\boldsymbol{\mathcal{X}}+\boldsymbol{\mathcal{V}}^{(r)} \tag{26}$$

where $\boldsymbol{\mathcal{H}}^{(r)}$ is a $(Q'+1)(L'+1)\times(Q+Q'+1)(L+L'+1)$ given as

$$\boldsymbol{\mathcal{H}}^{(r)}=\begin{bmatrix}\boldsymbol{\Omega}_{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(r)} & \cdots & \boldsymbol{\Omega}_{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(r)} & \mathbf{0}\\ & \ddots & & \ddots\\ \mathbf{0} & \boldsymbol{\Omega}_{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(r)} & \cdots & \boldsymbol{\Omega}_{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(r)}\end{bmatrix}, \tag{27}$$

where $\boldsymbol{\Omega}_q=\mathrm{diag}\{[e^{-j2\pi qL'/K},\ldots,1]^T\}$, and $\boldsymbol{\mathcal{H}}_q^{(r)}$ is given by

$$\boldsymbol{\mathcal{H}}_q^{(r)}=\begin{bmatrix}h_{q,0}^{(r)} & \cdots & h_{q,L}^{(r)} & 0\\ & \ddots & & \ddots\\ 0 & & h_{q,0}^{(r)} & \cdots & h_{q,L}^{(r)}\end{bmatrix}.$$

The noise matrix $\boldsymbol{\mathcal{V}}^{(r)}$ is similarly defined as $\boldsymbol{\mathcal{Y}}^{(r)}$.

Stacking the N_r resulting matrices $\boldsymbol{\mathcal{Y}}=[\boldsymbol{\mathcal{Y}}^{(1)T},\ldots,\boldsymbol{\mathcal{Y}}^{(N_r)T}]^T$, we obtain

$$\boldsymbol{\mathcal{Y}}=\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{X}}+\boldsymbol{\mathcal{V}}, \tag{28}$$

where $\boldsymbol{\mathcal{H}}=[\boldsymbol{\mathcal{H}}^{(1)T},\ldots,\boldsymbol{\mathcal{H}}^{(N_r)T}]^T$ and $\boldsymbol{\mathcal{V}}=[\boldsymbol{\mathcal{V}}^{(1)T},\ldots,\boldsymbol{\mathcal{V}}^{(N_r)T}]^T$.

Let us assume the following:

A1) $\boldsymbol{\mathcal{H}}$ has full column rank $(Q+Q'+1)(L+L'+1)$ (see [1]).

A2) $\boldsymbol{\mathcal{X}}$ has full row rank $(Q+Q'+1)(L+L'+1)$.
A3) $N \geq N_r(Q'+1)(L'+1)$.

Under these assumptions, the matrix $\boldsymbol{\mathcal{Y}}$ has $I = N_r(Q'+1)(L'+1) - (Q+Q'+1)(L+L'+1)$ zero singular values in the noiseless case. Suppose that $\mathbf{u}_1, \ldots, \mathbf{u}_I$ are the I left singular vectors corresponding to the I zero singular values. Then we can write

$$\mathbf{u}_i^H \boldsymbol{\mathcal{H}} = \mathbf{0}_{1\times(Q+Q'+1)(L+L'+1)}, \ \forall i \in \{1, \ldots, I\}. \tag{29}$$

Define $\mathbf{u}_i = [\mathbf{u}_i^{(1)T}, \ldots, \mathbf{u}_i^{(N_r)T}]^T$, and define $\mathbf{u}_i^{(r)} = [\mathbf{u}_{i,-Q'/2}^{(r)T}, \ldots, \mathbf{u}_{i,Q'/2}^{(r)T}]^T$ with $\mathbf{u}_{i,q'}^{(r)} = [u_{i,q',0}^{(r)}, \ldots, u_{i,q',L'}^{(r)}]^T$, then (29) can be equivalently written as

$$\boldsymbol{\mathcal{U}}_i^H \mathbf{h} = \mathbf{0}_{1\times(Q+Q'+1)(L+L'+1)}, \ \forall i \in \{1, \ldots, I\}, \tag{30}$$

where $\mathbf{h} = [\mathbf{h}^{(1)T}, \ldots, \mathbf{h}^{(N_r)T}]^T$ with $\mathbf{h}^{(r)} = [\mathbf{h}_{-Q/2}^{(r)T}, \ldots, \mathbf{h}_{Q/2}^{(r)T}]^T$, and $\mathbf{h}_q^{(r)} = [h_{q,0}^{(r)}, \ldots, h_{q,L}^{(r)}]^T$. In (30), $\boldsymbol{\mathcal{U}}_i = [\boldsymbol{\mathcal{U}}_i^{(1)^T}, \ldots, \boldsymbol{\mathcal{U}}_i^{(N_r)T}]^T$, where $\boldsymbol{\mathcal{U}}_i^{(r)}$ is defined as

$$\boldsymbol{\mathcal{U}}_i^{(r)} = \begin{bmatrix} \boldsymbol{\Omega}_1^{-Q/2}\boldsymbol{\mathcal{U}}_{i,-Q'/2}^{(r)}\boldsymbol{\Omega}_2^{Q/2} & \ldots & \boldsymbol{\Omega}_1^{-Q/2}\boldsymbol{\mathcal{U}}_{i,Q'/2}^{(r)}\boldsymbol{\Omega}_2^{Q/2} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \boldsymbol{\Omega}_1^{Q/2}\boldsymbol{\mathcal{U}}_{i,-Q'/2}^{(r)}\boldsymbol{\Omega}_2^{-Q/2} & \ldots & \boldsymbol{\Omega}_1^{Q/2}\boldsymbol{\mathcal{U}}_{i,Q'/2}^{(r)}\boldsymbol{\Omega}_2^{-Q/2} \end{bmatrix},$$

with $\boldsymbol{\mathcal{U}}_{i,q'}^{(r)}$ an $(L+1)\times(L'+L+1)$ Toeplitz matrix given by

$$\boldsymbol{\mathcal{U}}_{i,q'}^{(r)} = \begin{bmatrix} u_{i,q',0}^{(r)} & \ldots & u_{i,q',L'}^{(r)} & & 0 \\ & \ddots & & \ddots & \\ 0 & & u_{i,q',0}^{(r)} & \ldots & u_{i,q',L'}^{(r)} \end{bmatrix},$$

and $\boldsymbol{\Omega}_1 = \mathrm{diag}\{[1, \ldots, e^{j2\pi L/K}]^T\}$ and $\boldsymbol{\Omega}_2 = \mathrm{diag}\{[1, \ldots, e^{j2\pi L'/K}]^T\}$.

Stacking the I left singular vectors we obtain

$$\boldsymbol{\mathcal{U}}^H \mathbf{h} = \mathbf{0}_{I(Q+Q'+1)(L+L'+1)\times 1},$$

where $\boldsymbol{\mathcal{U}} = [\boldsymbol{\mathcal{U}}_1, \ldots, \boldsymbol{\mathcal{U}}_I]$, from which $\mathbf{h}$ can be computed up to a scalar ambiguity. In the presence of noise, we compute the I left singular vectors of $\boldsymbol{\mathcal{Y}}$ corresponding to the I smallest singular values. We denote these vectors as $\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_I$, and obtain the corresponding $\hat{\boldsymbol{\mathcal{U}}}$ in a similar fashion to computing $\boldsymbol{\mathcal{U}}$. The channel estimate is then obtained as

$$\min_{\mathbf{h}} \|\hat{\boldsymbol{\mathcal{U}}}^H \mathbf{h}\|^2.$$

The solution is obtained by the singular vector of $\hat{\boldsymbol{\mathcal{U}}}$ corresponding to the smallest singular value.

2) Semi-Blind Channel Estimation: In blind methods, the channel is estimated up to a scalar multiplication. To resolve the scalar ambiguity, training symbols are used along with the blind technique resulting in the so-called semi-blind technique. In semi-blind techniques, the channel estimate is obtained by minimizing a cost function formed of two parts. The first part corresponds to the training, and the second part corresponds to the blind estimation.

First, let us consider the channel estimate that relies on known symbols. To facilitate channel estimation, we write the input-output relationship as

$$\mathbf{y}^T = \mathbf{h}^T(\mathbf{I}_{N_r} \otimes \boldsymbol{\mathcal{X}}_{sb}) + \mathbf{v}^T, \tag{31}$$

where $\mathbf{y} = [\mathbf{y}^{(1)T}, \ldots, \mathbf{y}^{(N_r)T}]^T$, $\mathbf{v} = [\mathbf{v}^{(1)T}, \ldots, \mathbf{v}^{(N_r)T}]^T$, and the $(Q'+1)(L'+1)\times N$ matrix $\boldsymbol{\mathcal{X}}_{sb} = [\mathbf{x}_{-Q/2,0}^T, \ldots, \mathbf{x}_{Q/2,L}^T]$ with the qth frequency-shift and lth time-shift of the transmitted sequence $\mathbf{x}_{q,l}$ given by

$$\mathbf{x}_{q,l} = \bar{\mathbf{D}}_q \bar{\mathbf{Z}}_l \mathbf{x}.$$

Let us assume that N_t symbols are used for training, and the remaining symbols are data symbols. Collecting the received symbols that correspond to training in one vector $\mathbf{y}_t$, and the training symbols in a matrix $\boldsymbol{\mathcal{X}}_{sb,t}$, we can write the received sequence corresponding to training as

$$\mathbf{y}_t^T = \mathbf{h}^T(\mathbf{I}_{N_r} \otimes \boldsymbol{\mathcal{X}}_{sb,t}) + \mathbf{v}_t.$$

A LS channel estimate $\hat{\mathbf{h}}_{tr}$ is then computed based on the training symbols as

$$\hat{\mathbf{h}}_{tr} = \left(\mathbf{I}_{N_r} \otimes \boldsymbol{\mathcal{X}}_{sb,t}^T\right)^\dagger \mathbf{y}_t.$$

To avoid the under-determined case it is required that the number of training symbols be $N_t \geq N_r(Q+1)(L+1)$. To have non-overlapping data and training the optimal training strategy again consists of $N_r(Q+1)$ clusters of $2L+1$ training symbols. Each cluster consists of a training symbol and L surrounding zeros on each side [5]. Therefore, the training overhead is actually $N_r(Q+1)(2L+1)$, and the non-overlapping part is $N_t = N_r(Q+1)(L+1)$. This training overhead can be greatly reduced by combining the training with a blind estimation techniques resulting in a semi-blind technique.

The semi-blind channel estimate can be obtained by using the following cost function

$$\hat{\mathbf{h}}_{sb} = \arg\min_{\mathbf{h}} \left\{ \alpha \mathbf{h}^T \hat{\boldsymbol{\mathcal{U}}}^* \hat{\boldsymbol{\mathcal{U}}}^T \mathbf{h}^* + \|\mathbf{y}_t^T - \mathbf{h}^T(\mathbf{I}_{N_r} \otimes \boldsymbol{\mathcal{X}}_{sb,t})\|^2 \right\} \tag{32}$$

where $\alpha > 0$ is a weighting factor. In (32) the first part corresponds to blind estimation while the second part corresponds to training. If α is large, then the blind method is emphasized, whereas the LS training based estimation is emphasized for small α.

The solution for the semi-blind channel estimation is then obtained as

$$\hat{\mathbf{h}}_{sb} = \left(\alpha \hat{\boldsymbol{\mathcal{U}}} \hat{\boldsymbol{\mathcal{U}}}^H + \mathbf{I}_{N_r} \otimes (\boldsymbol{\mathcal{X}}_{sb,t} \boldsymbol{\mathcal{X}}_{sb,t}^H)^T\right)^{-1} (\mathbf{I}_{N_r} \otimes \boldsymbol{\mathcal{X}}_{sb,t}^H)^T \mathbf{y}_t. \tag{33}$$

B. Direct Equalization

In direct equalization the equalizer coefficients are obtained directly without passing through the channel estimation stage. There are many techniques that can be applied to obtain directly the equalizer coefficients for the case of frequency-selective channels. These techniques range from stochastic to deterministic. However, due the fact that we assume the BEM channel model, and the fact that the channel BEM coefficients may change from block to block, stochastic techniques cannot be applied. In this section we will rely on deterministic direct equalization techniques. We first discuss a deterministic blind direct equalization that relies on the so-called mutually referenced equalization (MRE). MRE has been successfully applied to the case of TI channels [7], [8]. In MRE the idea is to tune a number of equalizers, where the output of these tuned equalizers are used to train the other equalizers in a mutual fashion. For the case of TV channels, the same idea can be applied, but taking into account the time- as well as the frequency-shifts of the received signal. A semi-blind algorithm is again obtained by combining the training based LS method and the blind MRE method.

1) Blind Direct Equalization: The idea of MRE-based blind direct equalization is to tune various equalizers associated with reconstructing the transmitted signal subject to a time- and frequency shift. Define $\mathbf{w}_{p,k}^T$ as the TV FIR equalizer that reconstructs the pth frequency-shifted and kth time-shifted (delayed) version of the received sequence in the noiseless case as

$$\mathbf{w}_{p,k}^T \boldsymbol{\mathcal{Y}} = \mathbf{x}^T \tilde{\mathbf{Z}}_k^T \mathbf{D}_p. \tag{34}$$

In order to have mutually referenced equalizers training each other for frequency-shifts $p \in \{-(Q+Q')/2, \ldots, (Q+Q')/2\}$ and time-shifts (delays) $k \in \{0, \ldots, L+L'\}$, we set $\mathbf{x} = [\mathbf{0}_{1\times(L+L')}, \mathbf{x}_*^T, \mathbf{0}_{1\times(L+L')}]^T$, with $\mathbf{x}_*$ a data vector of length $M = N - L - 2L'$.

Define $\boldsymbol{\mathcal{Y}}_{p,k} = \boldsymbol{\mathcal{Y}} \mathbf{D}_{-p} \breve{\mathbf{Z}}_k$, with $\breve{\mathbf{Z}}_k = [\mathbf{0}_{M\times k}, \mathbf{I}_M, \mathbf{0}_{M\times(L+L'-k)}]^T$. Hence, we can write (34) as

$$\mathbf{w}_{p,k}^T \boldsymbol{\mathcal{Y}}_{p,k} = \mathbf{x}_*^T. \tag{35}$$

In order that (34) is satisfied in the noiseless case (i.e. a ZF solution exists) we require that:

A1) the matrix $\boldsymbol{\mathcal{H}}$ is of full column rank $(Q+Q'+1)(L+L'+1)$.
A2) the inequality $N_r(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$ is satisfied, which requires at least two receive antennas.
A3) the data length $M > (Q+Q'+1)(L+L'+1)$
A4) the input $x[n]$ is persistently exciting of order at least $(Q+Q'+1)(L+L'+1)$, i.e. rank $\boldsymbol{\mathcal{X}}$ equals $(Q+Q'+1)(L+L'+1)$ [8].

Taking the 0th frequency-shift and the 0th time-shift equalizer $\mathbf{w}_{0,0}$ as a reference equalizer and collecting the different equalizer coefficients in one vector $\mathbf{w} = [\mathbf{w}_{0,0}^T, \mathbf{w}_{-(Q+Q')/2,0}^T, \ldots, \ldots, \mathbf{w}_{-1,L+L'}^T, \mathbf{w}_{0,1}^T, \ldots, \mathbf{w}_{(Q+Q')/2,L+L'}^T]^T$, we arrive at the following:

$$\mathbf{w}^T \breve{\boldsymbol{\mathcal{Y}}} = \mathbf{0}_{1\times M(Q+Q'+1)(L+L'+1)}, \tag{36}$$

where

$$\breve{\boldsymbol{\mathcal{Y}}} = \begin{bmatrix} \boldsymbol{\mathcal{Y}}_{0,0} & \boldsymbol{\mathcal{Y}}_{0,0} & \cdots & \boldsymbol{\mathcal{Y}}_{0,0} \\ -\boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,0} & \mathbf{0} & & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,1} & & \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & -\boldsymbol{\mathcal{Y}}_{(Q+Q')/2,L+L'} \end{bmatrix}.$$

Note that in the noiseless case, it can be proven that the rank of $\breve{\boldsymbol{\mathcal{Y}}}$ is $(Q+Q'+1)^2(L+L'+1)^2 - 1$.

The different $\mathbf{w}_{p,k}$'s are linearly independent and cannot be obtained from each other. The different equalizers can be used as rows of a $(Q+Q'+1)(L+L'+1) \times N_r(Q'+1)(L'+1)$ matrix $\boldsymbol{\mathcal{W}}$. Based on the ZF conditions we obtain the following relation:

$$\boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{H}} = \alpha \mathbf{I}_{(Q+Q'+1)(L+L'+1)}, \tag{37}$$

where α is some scalar ambiguity satisfying:

$$\mathbf{w}_{0,0}^T \boldsymbol{\mathcal{Y}}_{0,0} = \mathbf{w}_{p,k}^T \boldsymbol{\mathcal{Y}}_{p,k} = \alpha \mathbf{x}_*^T, \ \forall p,k \ p \neq 0, \ k \neq 0. \tag{38}$$

We can solve (36) either by using LS or by subspace methods [8]. For the LS solution we constrain the first entry of $\mathbf{w}$ to 1 and solve for the normalized $\bar{\mathbf{w}}$ as:

$$\bar{\mathbf{w}}_{LS}^T = \left(\bar{\breve{\boldsymbol{\mathcal{Y}}}}^H \bar{\breve{\boldsymbol{\mathcal{Y}}}} \right)^{-1} \bar{\breve{\boldsymbol{\mathcal{Y}}}}^H \bar{\mathbf{y}},$$

where $\bar{\breve{\boldsymbol{\mathcal{Y}}}}$ is the matrix obtained after removing the first row of $\breve{\boldsymbol{\mathcal{Y}}}$ and $\bar{\mathbf{y}}$ is this row multiplied by -1. The subspace approach is obtained by setting the $\|\mathbf{w}\|^2 = 1$, and then $\mathbf{w}$ is found as the left singular vector corresponding to the minimum singular value of $\breve{\boldsymbol{\mathcal{Y}}}$.

Note that, if channel estimation is required, then using (37) the channel can be estimated subject to some scalar ambiguity.

2) Semi-Blind Direct Equalization: The MRE blind algorithm estimates the transmitted signal up to a scalar ambiguity α (see (38)). In addition, the blind MRE is very complex. These two difficulties with the blind MRE can be resolved by combining training with the blind MRE method resulting in a so-called semi-blind direct equalization. The proposed semi-blind approach consists of a combination of the training-based least-squares (LS) method [26] and the blind MRE method [7], [8], both well-known for frequency-selective channels, but here applied to doubly selective channels. The basic idea is that we consider different serial linear equalizers (SLES) that detect different time- and frequency-shifted versions of the transmitted sequence (blind MRE part discussed earlier in this paper). While during training periods, the training symbols are used to train all equalizers. During data transmission periods, each equalizer output is used to train the other equalizers.

Starting from (35), we assume that N_t symbols in $\mathbf{x}_*$ are training symbols and the remaining $N_d = M - N_t$ symbols in $\mathbf{x}_*$ are data symbols. Let us then collect the training symbols of $\mathbf{x}_*$ in $\mathbf{x}_{*,t}$ and the data symbols of $\mathbf{x}_*$ in $\mathbf{x}_{*,d}$. Let us further collect the corresponding columns of $\boldsymbol{\mathcal{Y}}_{p,k}$ in $\boldsymbol{\mathcal{Y}}_{p,k,t}$ and $\boldsymbol{\mathcal{Y}}_{p,k,d}$, respectively. Splitting

(35) into its training part and data part and stacking the results for $p \in \{-(Q+Q')/2, \ldots, (Q+Q')/2\}$ and $k \in \{0, \ldots, L+L'\}$ we arrive at the following:

$$\mathbf{w}^T[\underline{\boldsymbol{\mathcal{Y}}}_t, \underline{\boldsymbol{\mathcal{Y}}}_d] = [\mathbf{x}_{*,t}^T \underline{\mathbf{I}}_{N_t}, \mathbf{x}_{*,d}^T \underline{\mathbf{I}}_{N_t}],$$

where $\underline{\boldsymbol{\mathcal{Y}}}_t$ and $\underline{\boldsymbol{\mathcal{Y}}}_d$ are defined as

$$\underline{\boldsymbol{\mathcal{Y}}}_t = \begin{bmatrix} \boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,0,t} & & & & \\ & \ddots & & & \\ & & \boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,L+L',t} & & \\ & & & \ddots & \\ & & & & \boldsymbol{\mathcal{Y}}_{(Q+Q')/2,L+L',t} \end{bmatrix},$$

$$\underline{\boldsymbol{\mathcal{Y}}}_d = \begin{bmatrix} \boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,0,d} & & & & \\ & \ddots & & & \\ & & \boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,L+L',d} & & \\ & & & \ddots & \\ & & & & \boldsymbol{\mathcal{Y}}_{(Q+Q')/2,L+L',d} \end{bmatrix}$$

and

$$\underline{\mathbf{I}}_{N_t} = \mathbf{1}_{1\times R} \otimes \mathbf{I}_{N_t},$$
$$\underline{\mathbf{I}}_{N_d} = \mathbf{1}_{1\times R} \otimes \mathbf{I}_{N_d}.$$

where $R = (Q+Q'+1)(L+L'+1)$.

In the noisy case, we then have to solve

$$\min_{\mathbf{w}, \mathbf{x}_{*,d}} \{\|\mathbf{w}^T[\underline{\boldsymbol{\mathcal{Y}}}_t, \underline{\boldsymbol{\mathcal{Y}}}_d] - [\mathbf{x}_{*,t}^T \underline{\mathbf{I}}_{N_t}, \mathbf{x}_{*,d}^T \underline{\mathbf{I}}_{N_d}]\|^2\}. \tag{39}$$

The solution for $\mathbf{x}_{*,d}$ is given by

$$\hat{\mathbf{x}}_{*,d}^T = \mathbf{w}^T \underline{\boldsymbol{\mathcal{Y}}}_d R^{-1} \underline{\mathbf{I}}_{N_d}^T. \tag{40}$$

Substituting (40) in (39), we obtain

$$\min_{\underline{\mathbf{w}}} \{\|\underline{\mathbf{w}}^T[\underline{\boldsymbol{\mathcal{Y}}}_t, \underline{\boldsymbol{\mathcal{Z}}}_d] - [\mathbf{x}_{*,t}^T \underline{\mathbf{I}}_{N_t}, \mathbf{0}_{1\times N_d R}]\|^2\}, \tag{41}$$

where $\underline{\boldsymbol{\mathcal{Z}}}_d$ is given by

$$\underline{\boldsymbol{\mathcal{Z}}}_d = R^{-1} \begin{bmatrix} (R-1)\boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,0,d} & \cdots & -\boldsymbol{\mathcal{Y}}_{-(Q+Q')/2,0,d} \\ \vdots & \ddots & \vdots \\ -\boldsymbol{\mathcal{Y}}_{(Q+Q')/2,L+L',d} & \cdots & (R-1)\boldsymbol{\mathcal{Y}}_{(Q+Q')/2,L+L',d} \end{bmatrix} \tag{42}$$

In this equation, the left and right part respectively correspond to the training-based LS method [26] and the blind MRE method [7], [8], now applied to doubly selective channels. So far in our analysis we considered all possible time- and frequency-shits which exhibits similar complexity to the blind technique. Due to the existence of the training part, we can limit the number of time- and frequency-shifts resulting in a much lower complexity semi-blind technique. Therefore, we can redo the above analysis for time-shifts $k \in \{0, \ldots, K_1\}$ for $K_1 \leq (L+L')$ and frequency-shifts $p \in \{-K_2, \ldots, K_2\}$ for $K_2 \leq (Q+Q')/2$. In other words, by the aid of training the number of tuned equalizers can be greatly reduced resulting in a much lower complexity than the blind techniques. In contrast, for blind techniques, when a ZF solution is to be found, we require to tune the equalizers corresponding to all possible time- and frequency-shifts.

V. Simulation Results

In this section, we evaluate the performance of the proposed channel estimation and direct equalization techniques. We consider a rapidly time-varying channel simulated according to Jakes' model with $f_{\max} = 100\ Hz$, and sampling time $T = 25\mu sec$. The channel order is considered as $L = 3$. The channel autocorrelation function is given by $r_h[k] = \sigma_h^2 J_0(2\pi f_{\max} kT)$, where J_0 is the zero-th order Bessel function and $\sigma_h^2 = 1$ denotes the variance of the channel. We consider a window size of $N = 800$ symbols. For the BEM, we consider the critically sampled Doppler spectrum $K = N$, as well as the oversampled Doppler spectrum with oversampling rate 2 (i.e. $K = 2N$). The number of basis functions is, therefore, chosen to be $Q = 4$ for the critical sampling case, and $Q = 8$ for the oversampling case.

A. PSAM Techniques

• *PSAM-Based Channel Estimation:*

We use PSAM to estimate the channel. We consider equipowered and equispaced pilot symbols with D the spacing between the pilots. The number of pilots is then computed as $P = \lfloor N/D \rfloor + 1$. Since we adhere to the optimal training [5], the optimal training consists of P-clusters, and each cluster consists of a training symbol and L surrounding zeros from each side as explained in Figure 1. This means that the training overhead is $\frac{P(2L+1)}{N-d}\%$.

First, we consider the normalized channel MSE versus SNR. We evaluate the performance of the different estimation techniques, in particular, a BEM with $K = N$, a combined BEM and MMSE with $K = N$, a BEM with $K = 2N$, a combined BEM and MMSE with $K = 2N$, and finally the MMSE channel estimate. We consider the case when the spacing between pilot symbols is $D = 160$ which corresponds to $P = 5$ pilot symbols dedicated for channel estimation. This choice is well suited for the case of $K = N$, where the number of BEM coefficients to be estimated is $Q + 1$. As shown in Figure 2, all the MSE channel estimates suffer from an early error floor. However, combining the critically sampled BEM with the MMSE results in a slightly better performance. We further consider the case when the spacing between the pilot symbols is $D = 95$ which corresponds to $P = 9$ pilot symbols dedicated for channel estimation. This is well suited for the case when $K = 2N$. As shown in Figure 3, the performance of the BEM with $K = N$ suffers from an early error floor, which means that increasing the number of pilot symbols does not enhance the channel estimation technique. For the case when $K = 2N$, the MSE curves do not suffer from an early error floor. However, the oversampled BEM channel estimate is sensitive to noise. A significant improvement is obtained when the combined BEM and MMSE method is used, where a gain of $9\ dB$ at $MSE = 10^{-2}$ is obtained over the conventional BEM method, when the oversampling rate is 2. Note also that the performance of the combined BEM and MMSE method when $K = 2N$ coincides with the performance of the MMSE only.

Second, the estimated channel BEM coefficients are used to design TV FIR equalizers serial and decision feedback. We consider here a single-input multiple-output (SIMO) system with $N_r = 2$ receive antennas. We consider the MMSE-SLE [1] as well as the MMSE serial decision feedback equalizer (MMSE-SDFE) [2]. For the case of the MMSE-SLE, the SLE equalizer is designed to have order $L' = 12$ and the number of TV basis functions $Q' = 12$. For the case of the MMSE-SDFE, the TV FIR feedforward filter is designed to have order $L' = 12$ and the number of TV basis functions $Q' = 12$, while the TV FIR feedback filter is designed to have order $L'' = L$ and $Q'' = Q$. The SLE coefficients as well as the SDFE coefficients are computed as explained in [1] for the MMSE-SLE, and in [2] for the MMSE-SDFE. The BEM resolution of the TV FIR equalizer matches that of the channel. QPSK signaling is assumed. We define the SNR as $SNR = \sigma_h^2(L+1)E_s/\sigma_n^2$, where E_s is the QPSK symbol power. As shown in Figure 4, for the case of MMSE-SLE, the BER curve experiences an error floor when $D = 165$ for the different scenarios. For the case of $D = 95$, we experience an SNR loss of $11.5\ dB$ for the case of $K = 2N$ compared to the case when perfect channel state information (CSI) is known at $BER = 10^{-2}$, while the SNR loss is reduced to $6\ dB$ for the case of combined BEM and MMSE when $K = 2N$. For $K = N$, both cases (BEM and combined BEM and MMSE) suffer from an error floor. Similar observations can be made for the case of MMSE-SDFE as shown in Figure 5.

Finally, we measure the MSE of the channel estimation techniques as a function of the maximum Doppler frequency. We design the system to have a maximum target Doppler frequency of $f_{\max} = 100\ Hz$ (used to design $\mathbf{W}$). We then examine the performance of the channel estimation techniques for different maximum Doppler frequencies at a fixed $SNR = 25\ dB$. The results are shown in Figure 6 for the case when $P = 5$ pilot symbols are used for channel estimation, and Figure 7 when $P = 9$ pilot symbols are used. For either case, the channel estimation techniques maintain a low MSE channel estimate as long as the channel maximum Doppler frequency is smaller than the target maximum Doppler frequency.

• *PSAM Direct Equalization*:

We use here the same channel setup. For this setup, the training overhead required for the ZF solution to exist is measured to be 50%, i.e. inserting a pilot symbol every second symbol. The simulation results are shown in Figure 8 for window size $K = N$ as well as for window size $K = 2N$. In PSAM direct equalization the SNR loss is about 9dB for $K = N$ compared to perfect CSI at $BER = 10^{-2}$. For the case of window size $K = 2N$ the equalizer fails.

B. Blind and Semi-blind Techniques

• *Blind channel estimation:*

For the case of channel estimation, we first plot a realization of one of the taps of the true channel and its estimate as a function of time at $SNR = 16\ dB$. For this case, we consider the BEM resolution $K = N$. The complex scalar ambiguity is resolved by means of LS fitting the estimated channel with the true channel.

Considering the BEM channel as the true channel, the mismatch between the BEM channel and its estimate is shown in Figure 9. Considering the Jakes' channel as the true channel, the mismatch between the true channel and its estimate is shown in Figure 10. Assuming the true channel obeys the BEM, we see that the mismatch is considerably smaller for this channel setup. However, the mismatch is clearly larger when the true channel obeys Jakes' model.

We also consider the MSE of the channel estimation versus SNR. We consider the case where the true channel obeys the BEM as well as the case where the true channel obeys Jakes' model. When the true channel obeys the BEM, the MSE channel is computed as

$$MSE = \frac{1}{N_{ch} N_r N(L+1)} \sum_{i=1}^{N_{ch}} \sum_{r=1}^{N_r} \sum_{n=0}^{N-1} \sum_{\nu=0}^{L} |\hat{h}^{(r)}[n;\nu] - h^{(r)}[n;\nu]|^2$$

The MSE channel estimate is computed as

$$MSE = \frac{1}{N_{ch} N_r N(L+1)} \sum_{i=1}^{N_{ch}} \sum_{r=1}^{N_r} \sum_{n=0}^{N-1} \sum_{\nu=0}^{L} |\hat{h}^{(r)}[n;\nu] - g^{(r)}[n;\nu]|^2$$

when the true channel obeys Jakes' model. N_{ch} is the number of channel realizations.

As shown in Figure 11, the MSE is a monotonically decreasing function in SNR when the true channel obeys the BEM and $K = N$, whereas the MSE channel estimate suffers from an error floor at $MSE = 4 \times 10^{-2}$ when the true channel obeys Jakes' model and the BEM resolution is taken as $K = N$. For $K = 2N$, the blind technique fails to estimate the channel for either channel models.

Direct Equalization

• *Blind direct equalization:*

For the case of blind equalization and for feasibility and complexity reasons we consider the following channel setup:

- A SIMO system with $N_r = 2$ receive antennas.
- The channel is doubly selective according to Jakes' model with order $L = 1$ and maximum Doppler frequency $f_{max} = 100\ Hz$.
- The equalizer order $L' = 3$ and the number of TV basis functions $Q' = 2$.

– The channel is approximated using the BEM over a period of $N = 37$, and $M = 32$ symbols, the sampling time $T = 25$ μsec, and hence the number of TV basis functions $Q = 2$.
– The BEM resolution $K = N$.

For this channel setup we can easily prove that the ZF solution exists in the noiseless case. However, in the noisy case, in order to obtain good results we require a large Q' and L' such that it is very difficult to obtain the SVD of $\breve{\mathbf{\mathcal{Y}}}$. Therefore, we restrict ourselves to the noiseless case.

As shown in Figure 12, the blind technique retrieves the transmitted symbols subject to a complex scalar ambiguity. This scalar ambiguity can be resolved by combining the training based technique with the blind technique.

• *Semi-blind direct equalization:*

For the semi-blind technique we consider a SIMO system with $N_r = 4$ receive antennas. We assume a doubly selective channel with Doppler spread of $f_{\max} = 100$ and order $L = 3$. We use QPSK signaling. We assume the data sequence and the additive noises are mutually uncorrelated and white.

We consider a time-window of $NT = 200T$. As already mentioned, when $NT \leq 1/(2f_{\max})$, which is the case here, an accurate channel model can be obtained by taking $Q = 2$. We insert a pilot symbol after every three data symbols. We consider three SLE designs: ideal design where perfect channel state information is assumed to be known at the receiver (see [1], [3]), the direct training-based design where the equalizer coefficients are obtained directly based on PSAM, and the direct semi-blind design proposed in this paper. For all designs, we assume $Q' = 2$, $L' = 3$, and $d = (L + L')/2 = 3$. For the direct semi-blind design we take $K_1 = L$ and $K_2 = Q/2$, i.e. we consider the time-shifts $k \in \{0, \ldots, L\}$, and frequency-shifts $p \in \{-Q/2, \ldots, Q/2\}$. For the ideal design, we first fit a BEM to the true doubly selective channel over the time window of $NT = 200T$, and use the obtained BEM coefficients to design the BEM coefficients of the SLE. From Figure 13, we can observe that the direct semi-blind design clearly outperforms the direct training-based design, and is not too far from the performance of the ideal design.

VI. Conclusions

In this paper, we have proposed estimation and direct equalization techniques for transmission over doubly selective channels. In particular, we have proposed PSAM, blind and semi-blind techniques. In PSAM techniques we rely on pilot symbols for channel parameters estimation or direct equalization. For channel parameters estimation, we consider the case when the Doppler spectrum is critically sampled ($K = N$) as well as when the Doppler spectrum is oversampled ($K \geq rN$ with integer $r > 1$). While in the first case, the estimation scheme suffers from an early error floor due to the large modeling error, the estimation is sensitive to noise in the oversampled case. It has been shown through computer simulations that combining the MMSE interpolation based channel estimate with the oversampled BEM significantly improves the channel estimation. We also show that the channel estimation step can be skipped to perform direct equalization based on PSAM. In blind techniques, no training overhead is used to estimate or directly equalize the doubly selective channel. Semi-blind techniques, on the other hand, are obtained by combining the training based techniques with the blind techniques. Doing so, the scalar ambiguity of the blind techniques are resolved, and the complexity may be greatly reduced especially for the case of direct equalization. For the channel estimation part we show through computer simulations that the blind and semi-blind techniques perform well when the true channel obeys the BEM channel, and noticeably when the BEM resolution equals the window size. For BEM resolutions larger than the window size the different algorithms fail to identify and/or directly equalize the doubly selective channel.

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Fig. 1. Optimal training for doubly selective channels

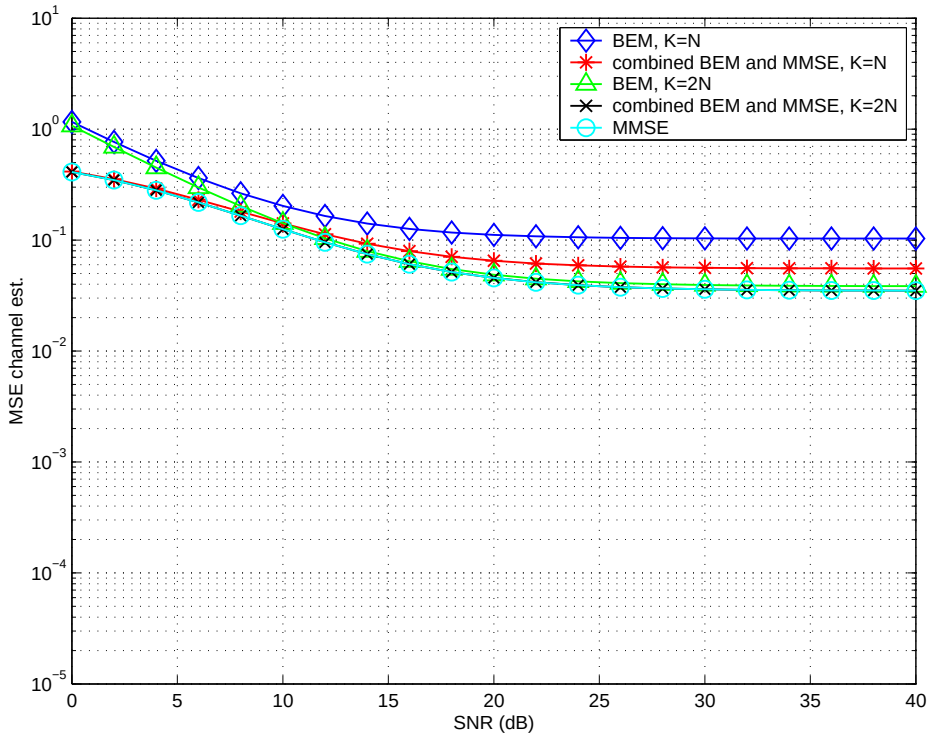


Fig. 2. MSE vs. SNR for $P = 5$, $D = 160$

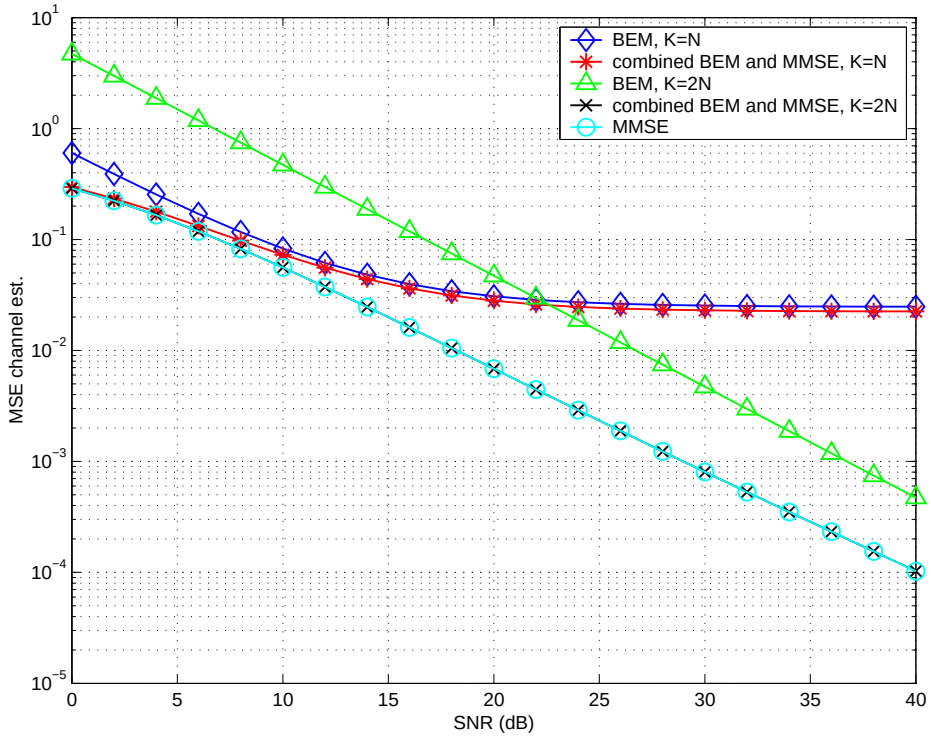


Fig. 3. MSE vs. SNR for $P = 9$, $D = 95$

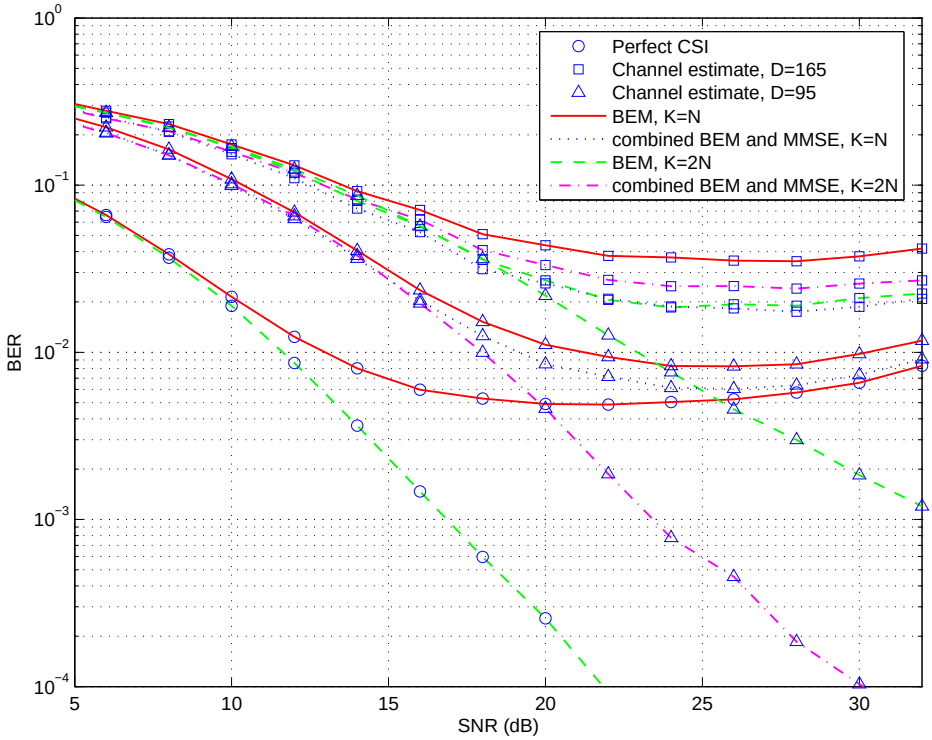


Fig. 4. BER vs. SNR using the MMSE-SLE.

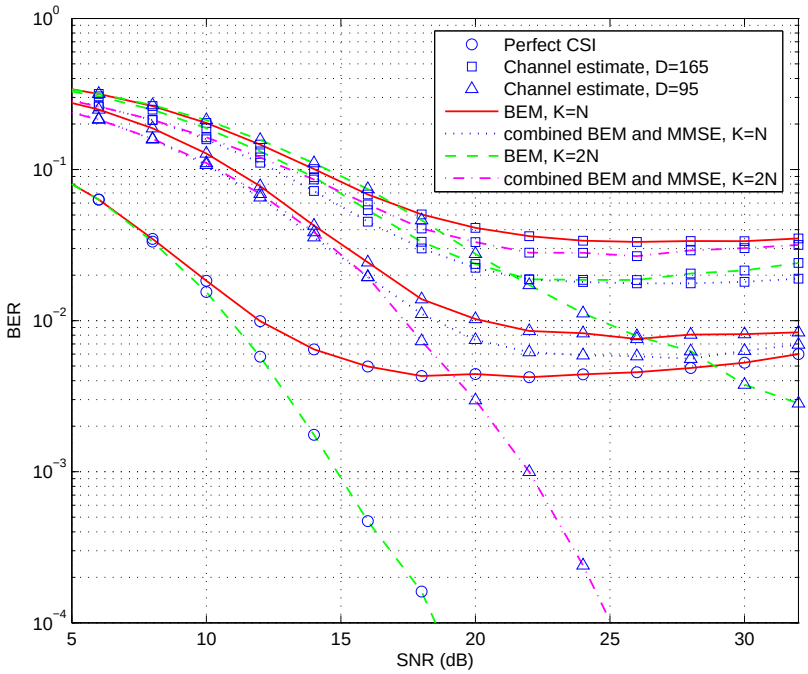


Fig. 5. BER vs. SNR using the MMSE-SDFE.

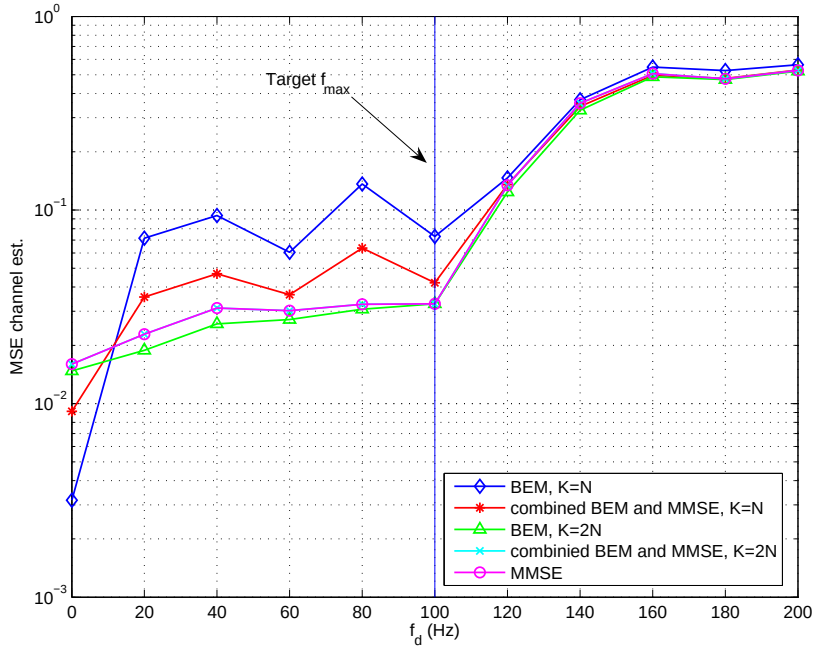


Fig. 6. MSE vs. $f_{\max}$ for $P = 5$, $D = 160$, and $SNR = 25\ dB$

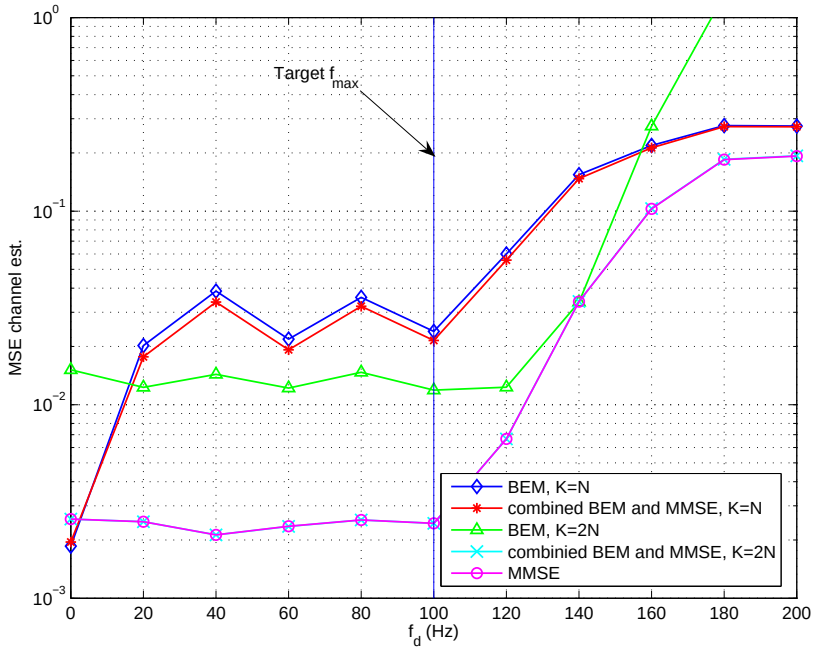


Fig. 7. MSE vs. $f_{\max}$ for $P = 9$, $D = 95$, and $SNR = 25\ dB$

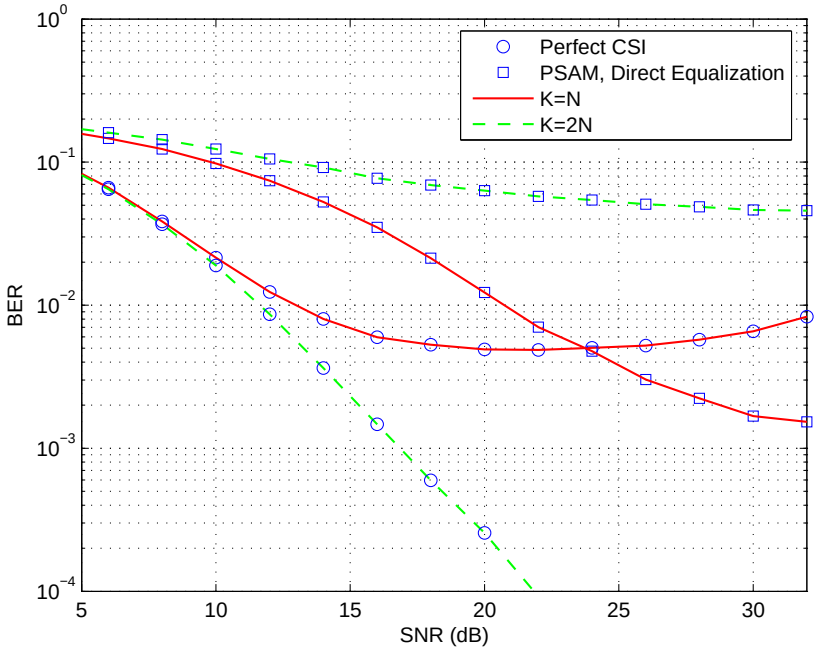


Fig. 8. BER vs SNR for PSAM-based direct equalization.

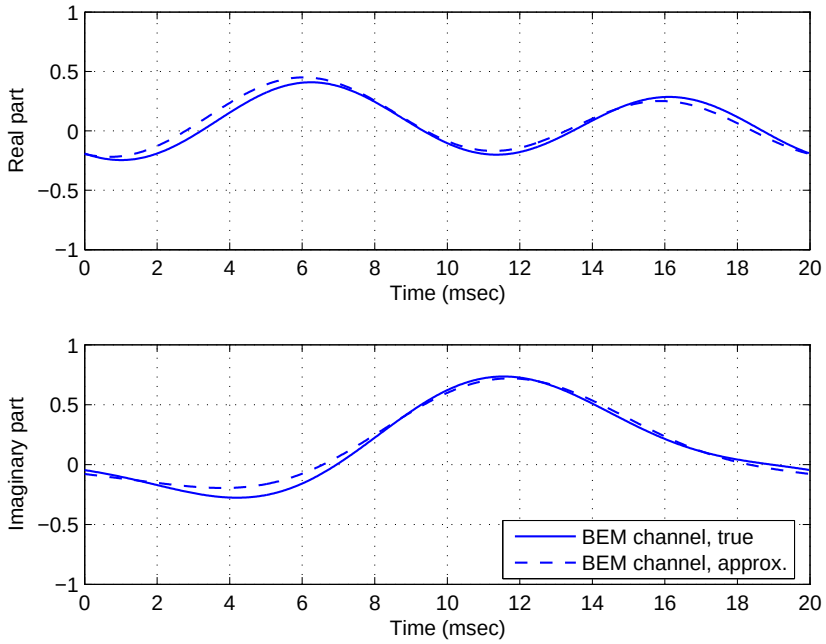


Fig. 9. Mismatch between the true BEM channel and its estimate for $SNR = 16\ dB$ using blind channel estimation technique.

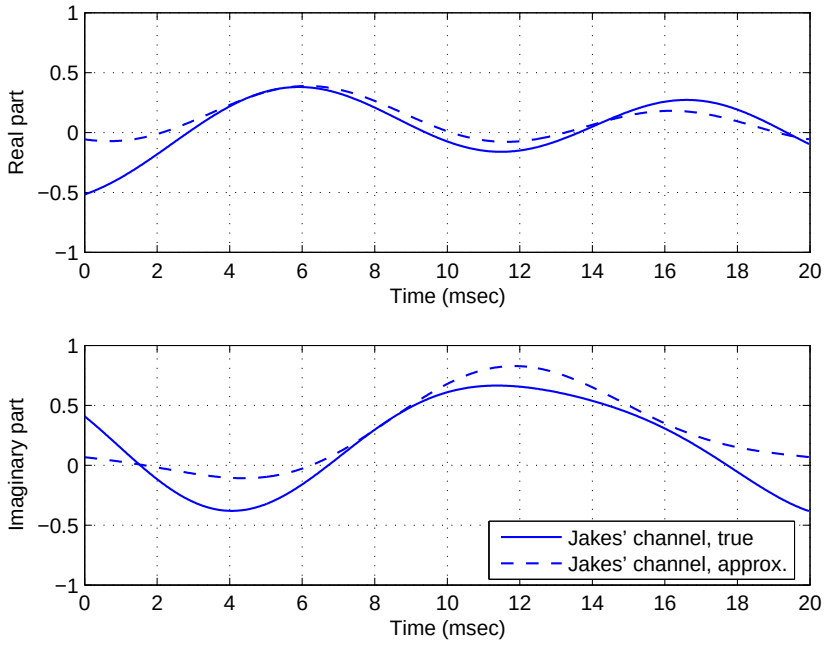


Fig. 10. Mismatch between the true Jakes' channel and its estimate for $SNR = 16\ dB$ using blind channel estimation technique.

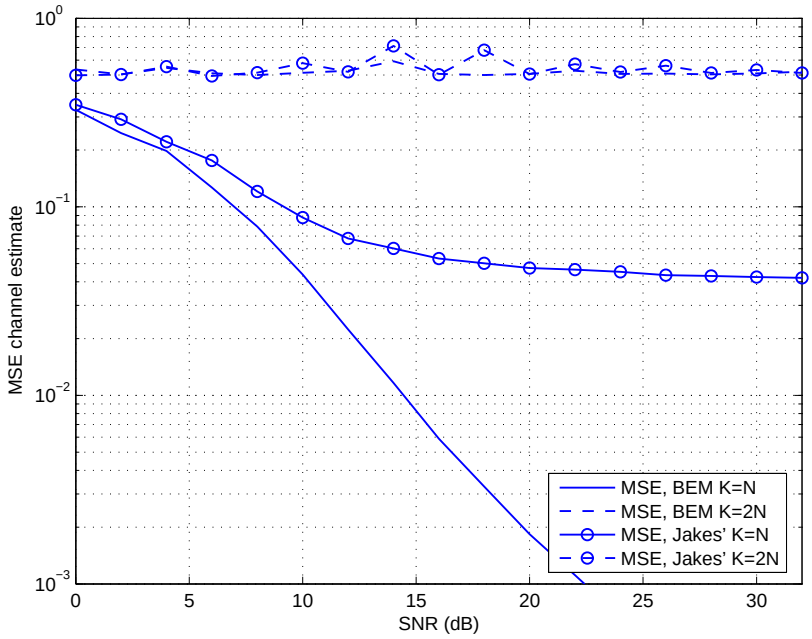
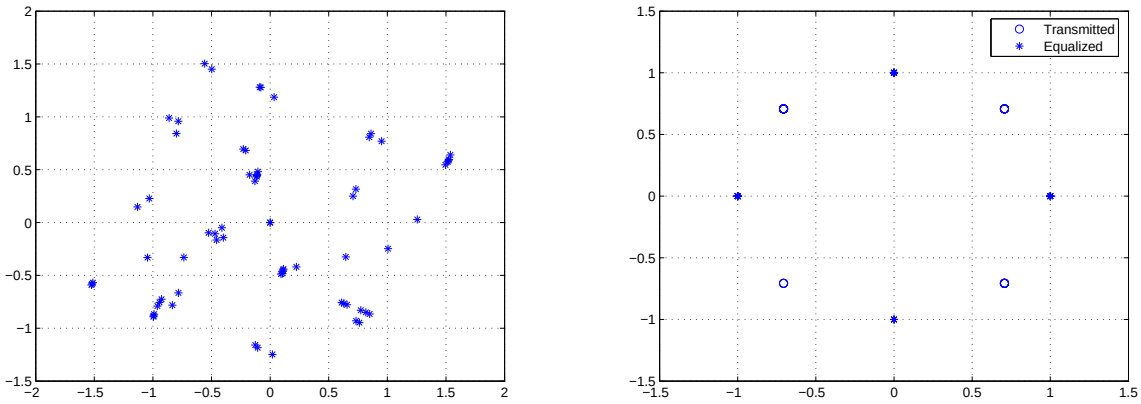


Fig. 11. MSE vs. SNR for the blind channel estimation technique.



(a) Before equalization

(b) After equalization

Fig. 12. Received and equalized symbols.

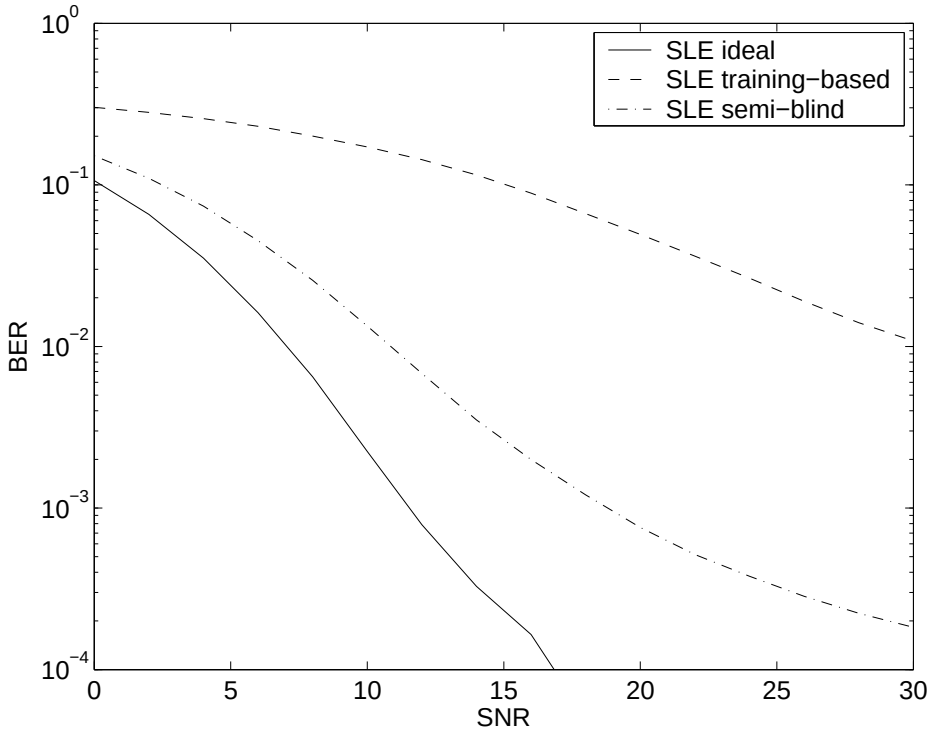


Fig. 13. Comparison of different SLE designs for doubly selective channels.