On Maximum-Likelihood Equalization of Doubly Selective Channels*

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***Abstract*— In this paper, we propose a maximum-likelihood sequence estimation (MLSE) for transmission over rapidly time-varying channels. The time-varying channel is approximated using the basis expansion model (BEM). Using the BEM allows for efficient implementation of the Viterbi algorithm (VA). This stems from the fact that using the BEM the time-varying channel can be approximated over a window of size N by a bank of time-invariant filters, and the time-variation of each filter is captured by a corresponding time-varying complex exponential basis function. Therefore, a time-invariant state diagram of each filter is first constructed and then used to update the branch metrics at every recursion. This can be applied for single carrier transmission as well as multi-carrier transmission techniques.**

I. Introduction

The wireless communication channel is mainly characterized as doubly selective channel in time and in frequency. While multipath propagation gives rise to frequency-selectivity as a result of high data rates, mobility and/or carrier frequency offset give rise to time-selectivity. The frequency-selectivity implies that echoes of the transmitted signal arrive at the receiver causing inter-symbol interference (ISI), i.e. the transmitted signal is spread in time . On the other hand, time-selectivity results in spreading the transmitted signal in frequency-domain (the so-called Doppler spread). Therefore, in order to provide reliable communication, advanced and efficient equalization techniques are necessary.

Equalizers can be classified according to their structure, namely as linear or decision feedback equalizers. Equalizers can also be classified according to the optimization criterion. In this sense, equalizers can be classified as zero-forcing (ZF), when a ZF solution is sought, or minimum mean-squared error (MMSE) when the equalizer optimizes the mean-squared error (MSE) of the symbol estimate, or maximum-likelihood (ML) when the maximum-likelihood sequence estimation (MLSE) criterion is utilized.

In the context of linear equalization of TI channels, linear MMSE and ZF equalizers are investigated in [1], [2]. Decision feedback equalizers (DFE) developed by using previously detected symbols to compensate for ISI are discussed in [3], [4]. Utilizing finite impulse response filters for DFEs is investigated in [5]. Maximum likelihood sequence estimation (MLSE) for data transmission over TI channels with ISI is introduced in [6], [7], where the MLSE is implemented using the Viterbi algorithm (VA).

For time-varying channels linear MMSE and DFE equalizers using time-varying finite impulse response (FIR) filters are introduced in [8], [9], [10]. For OFDM transmission over doubly selective channels, time-domain and frequency-domain per-tone equalizers (PTEQ) are introduced in [11], [12], [13].

Adaptive MLSE is proposed in [14], [15] to equalize time-varying channels. These techniques have difficulty to track rapidly time-varying channels. In this paper we propose an MLSE for rapidly time-varying channels. The time-varying channel is modeled using the basis expansion model (BEM). Using the BEM allows for an efficient low complexity mechanism to update the branch metrics, and therefore results in a more efficient implementation of the VA.

This paper is organized as follows. In Section II, the system model is introduced. The MLSE implemented using the VA is introduced in Section III. Our findings are confirmed by numerical simulations introduced in Section IV. Finally, our conclusions are drawn in Section V.

Notation: We use upper (lower) bold face letters to denote frequency-domain (time-domain) vectors. Matrices are also denoted using bold face upper letters, which should be distinguished from frequency-domain vectors from the context. Superscripts ^{H} represents Hermitian. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$, the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$. The unit vector with 1 at position $k+1$ is denoted as $\mathbf{e}_k$. Finally, $\mathrm{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with vector $\mathbf{x}$ on the diagonal.

II. System Model

We consider transmission over doubly selective channels. We also consider a single input single output (SISO), but the results can be easily extended to single input multiple output (SIMO), or multiple input multiple output (MIMO) systems. A data sequence $x[n]$ of length N is transmitted at a rate of $1/T$ symbols per second over the time-varying channel. The data sequence can be thought of as QAM symbols for single carrier (SC) transmission, or a time-domain equivalent of QAM frequency-domain symbols for multi-carrier (MC) transmission. The discrete time baseband equivalent descrip-

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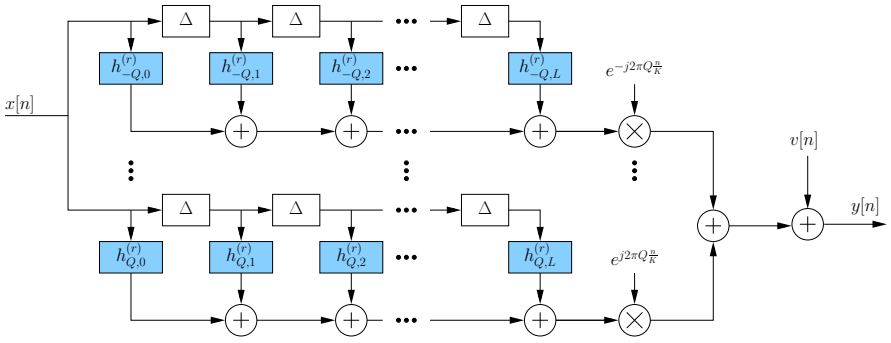


Fig. 1. Input-output relationship using the BEM to approximate the doubly selective channel

tion of the received sequence at time index n is given by:

$$y[n] = \sum_{l=0}^{L} g[n;l]x[n-l] + v[n], \tag{1}$$

where $g[n;\theta]$ is the discrete time equivalent baseband representation of the time-varying frequency-selective channel taking into account the multi-path physical channel and the transmitter and receiver pulse shaping filters. L is the channel order $L = \lfloor \tau_{\max}/T \rfloor + 1$ with $\tau_{\max}$ is the channel maximum delay spread. $v[n]$ is the discrete time additive white noise (AWN).

Using the basis expansion model (BEM) [16], [17], [18] to approximate the doubly selective channel $g[n;l]$, where the doubly selective channel is modeled as a time-varying FIR filter over a window of size N. For $n \in \{0,\ldots,N-1\}$, each tap of the time-varying FIR filter is expressed as a superposition of time-varying complex exponential basis functions with frequencies on a DFT grid as

$$h[n;l] = \sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/K} h_{q,l}, \tag{2}$$

where Q is the number of time-varying basis functions satisfying $Q/(2KT) \geq f_{\max}$, with $f_{\max}$ is the channel maximum Doppler spread, and $K \geq N$ is the BEM resolution. $h_{q,l}$ is the coefficient of the qth basis of the lth tap, which is kept invariant over a period of the transmission of a block of N symbols, and may change from block to block independently.

Substituting (2) in (1), we arrive at the following input-output relationship

$$y[n] = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} e^{j2\pi qn/K} h_{q,l} x[n-l] + v[n]. \tag{3}$$

The input-output relationship (3) is depicted in Figure 1.

Defining $\mathbf{y} = [y[0],\ldots,y[N-1]]^T$, on a block level formulation the received vector $\mathbf{y}$, can be written as

$$\mathbf{y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \mathbf{D}_q \mathbf{Z}_l \mathbf{x} + \mathbf{v}, \tag{4}$$

where $\mathbf{D}_q$ is a diagonal matrix with the qth time-varying basis function components on its diagonal $\mathbf{D}_q = \text{diag}\{[1, e^{j2\pi q/K},\ldots,e^{j2\pi q(N-1)/K}]^T\}$. $\mathbf{Z}_l$ is an $N \times (N+L)$ Toeplitz matrix defined as $\mathbf{Z}_l = [\mathbf{0}_{N\times(L-l)}, \mathbf{I}_N, \mathbf{0}_{N\times l}]$. The transmitted symbols vector $\mathbf{x}$ is defined as $\mathbf{x} = [x[-L],\ldots,x[N-1]]^T$, and finally the additive noise vector $\mathbf{v}$ is similarly defined as $\mathbf{y}$.

III. Maximum-Likelihood Sequence Estimation

For channels with memory where intersymbol interference (ISI) occurs, the MLSE searches for the sequence $\mathcal{X}^{(j)}$ that maximizes the conditional probability $p(\mathbf{y}|\mathcal{X}^{(j')})$. The maximum-likelihood rule can therefore be written as

$$\mathcal{X}^{(j')}) = \max_{j} p(\mathbf{y}|\mathcal{X}^{(j)}). \tag{5}$$

where the sequence $\mathcal{X}^{(j)} = \{x_0^{(j)},\ldots,x_{N-1}^{(j)}\}$ with $x_i^{(j)} \in \mathcal{X}$ the set of the input alphabet with cardinality $|\mathcal{X}| = M$. The optimal sequence that maximizes the conditional probability can be obtained by performing an exhaustive search over the M^N different sequences. For large block size and/or large alphabet sizes, this exhaustive search may be prohibitive for practical applications. For ISI channels with finite memory L (described as finite state machine (FSM)), the Viterbi algorithm (VA) can be invoked to find the most-likely state transition sequence in a state diagram (or a trellis). The number of states in the state diagram or a trellis in this case is M^L (usually $L \ll N$).

A. SC Transmission

For SC transmission, the time-domain symbols $x[n]$ are drawn from the input sequence finite alphabet $\mathcal{X}$. Define $c^{(i,j)}[n]$ as the output symbol of the transition from state i to state j at time-index (recursion) n. Define the sequence $\mathcal{X}^{(i,j)}$ as the length $L+1$ sequence of symbols that characterizes the transition from state i to state j for $i,j = 0,\ldots,M^L-1$, $\mathcal{X}^{(i,j)} = \{x_0^{(i,j)},\ldots,x_L^{(i,j)}\}$, the output symbol $c^{(i,j)}[n]$ can be written as

$$c^{(i,j)}[n] = \sum_{l=0}^{L} g[n;l] x_{L-l}^{(i,j)}. \tag{6}$$

Using the Euclidean distance, the branch metric for the transition from state i to state j at time index n is given by

$$b^{(i,j)}[n] = \left| y[n] - c^{(i,j)}[n] \right|^2. \tag{7}$$

For time-invariant channels the outputs $c^{(i,j)}[n]$ are time-invariant, which means that in order to perform VA, one state diagram (transition table) need to be constructed and used for the equalization process. For time-varying channels, the output symbols $c^{(i,j)}[n]$ are also time-varying as they depend on the channel state information (CSI) at every time instant n. Therefore, in order to perform VA, the transition table need to be recomputed for each recursion. Note that, the complexity of this step can be reduced by using adaptive techniques [14], [15]. However, these techniques, have difficulty tracking rapidly time-varying channels.

Using the BEM to approximate the doubly selective channel, we can write the output symbol $c^{(i,j)}[n]$ in (6) as

$$\begin{aligned} c^{(i,j)}[n] &= \sum_{-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} x_{L-l}^{(i,j)} e^{j2\pi qn/K} \\ &= \sum_{q=-Q/2}^{Q/2} c_q^{(i,j)} e^{j2\pi qn/K}. \end{aligned} \tag{8}$$

Therefore, the branch metric for the transition from state i to state j at time index n is given by

$$b^{(i,j)}[n] = \big| y[n] - \sum_{q=-Q/2}^{Q/2} c_q^{(i,j)} e^{j2\pi qn/K} \big|^2. \tag{9}$$

Using the BEM as shown in (8), in order to perform VA, we require to construct $Q+1$ time-invariant transition tables. Each transition table corresponds to a branch of the BEM channel. The output of the transition from state i to state j at the qth branch $c_q^{(i,j)}$ is given by

$$c_q^{(i,j)} = \sum_{l=0}^{L} h_{q,l} x_{L-l}^{(i,j)}, \tag{10}$$

which is time-invariant over a block of N symbols (as long as the BEM coeffiecients $h_{q,l}$ are fixed). The time-varying outputs at each recursion is updated as in (8). This means that we only require to store $Q+1$ time-invariant transition tables. However, the computational complexity and memory requirement is discussed later on in this section.

In order to perform the add-compare select (ACS) step of the VA, we define $m^{(j)}[n]$[†] as the path metric of state j at time index n, and the set $\mathcal{I}$ as the set of states that have transitions to state j. The branch metric of state j at time index is then chosen as

$$m^{(j)}[n] = \min_{i \in \mathcal{I}} \big(m^{(j)}[n-1] + b^{(i,j)}[n] \big).$$

In a nutshell, instead of recomputing the trellis outputs for each time instant taking into account the channel status at that time instant, the BEM allows for an easy and computationally efficient mechanism to perform VA for time-varying channels. In this sense, we require to compute and store $Q+1$ time-invariant transition tables, and at each time instant the branch metrics are updated using (8). Note that in most cases Q is taken as $Q = 2, 4$.

[†]We have used the notation of [19] to the extent that serves our purpose

B. OFDM

For OFDM transmission, the information-bearing symbols are parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed to the time-domain by the inverse discrete Fourier transform (IDFT). A cyclic prefix (CP) of length $\nu \geq L$ is added to the head of each block. The time-domain blocks are then serially transmitted over the time-varying channel. Assuming S_k is the transmitted symbol on the kth sub-carrier of the OFDM block, $x[n]$ can be written as:

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j2\pi(n-\nu)k/N}.$$

Note that this description includes the transmission of a CP of length ν. The received sequence after removing the CP, can be written as

$$\mathbf{y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \mathbf{D}_q \bar{\mathbf{Z}}_l \mathbf{F}^H \mathbf{S} + \mathbf{v}, \tag{11}$$

where $\mathbf{F}$ is the $N \times N$ unitary DFT matrix, $\bar{\mathbf{Z}}_l$ is an $N \times N$ circulant matrix with $\mathbf{e}_l$ as its first column. The frequency-domain transmitted vector $\mathbf{S} = [S_0, \ldots, S_{N-1}]^T$, where the frequency-domain symbols here are drawn from the finite input symbols alphabet set defines as $\mathcal{S}$[‡]. Define $\mathbf{Y} = [Y_0, \ldots, Y_{N-1}]^T$ as the received vector in frequency-domain, $\mathbf{Y}$ can be written as

$$\mathbf{Y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \mathbf{F} \mathbf{D}_q \bar{\mathbf{Z}}_l \mathbf{F}^H \mathbf{S} + \mathbf{V}. \tag{12}$$

For $K = N$, i.e. the BEM basis functions are taken on a DFT grid of size N, we can use the properties $\mathbf{D}_q = \mathbf{F}^H \bar{\mathbf{Z}}_q \mathbf{F}$ and $\bar{\mathbf{Z}}_l = \mathbf{F} \mathbf{D}_{-l} \mathbf{F}^H$. Hence, (12) can be written as

$$\mathbf{Y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} h_{q,l} \bar{\mathbf{Z}}_q \mathbf{D}_{-l} \mathbf{S} + \mathbf{V}. \tag{13}$$

Using the property $\bar{\mathbf{Z}}_q \mathbf{D}_{-l} = e^{-j2\pi ql/N} \mathbf{D}_{-l} \bar{\mathbf{Z}}_q$, we arrive at

$$\mathbf{Y} = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^{L} \tilde{h}_{q,l} \mathbf{D}_{-l} \bar{\mathbf{Z}}_q \mathbf{S} + \mathbf{V}, \tag{14}$$

where $\tilde{h}_{q,l} = e^{-j2\pi ql/N} h_{q,l}$. Note the similarity between (14) and (4). From (14), using a BEM resolution $K = N$, OFDM transmission corresponds to SC transmission over doubly selective channel, where the doubly selective channel is now of order Q (instead of L for SC transmission), and the time-variation is captured by means of $L+1$ (instead of $Q+1$ for SC) time-varying complex exponential basis functions. The channel BEM coefficients are characterized by $\tilde{h}_{q,l}$. Hence, we can depict a figure similar to Figure 1, but with input S_k, and output Y_k. Therefore, we can perform VA on the new channel model. Define the sequence $\mathcal{S}^{(i,j)}$ as the length $Q+1$ sequence

[‡]Note that we choose to define the frequency-domain input alphabet set different than the time-domain input alphabet set for SC transmission $\mathcal{X}$. Actually both sets can be the same, but we choose different names to show the analogy between SC transmission and OFDM transmission in this context.

of symbols that characterizes the transition from state i to state j, for $i, j = 0, \ldots, M^Q - 1$, $\mathcal{S}^{(i,j)} = \{S_0^{(i,j)}, \ldots, S_Q^{(i,j)}\}$, the output symbol $C^{(i,j)}[k]$ of the transition from state i to state j at recursion k (recursion refers here to the sub-carrier index) can be written as

$$C^{(i,j)}[k] = \sum_{l=0}^{L} C_l^{(i,j)} e^{-j2\pi lk/N}, \tag{15}$$

where $C_l^{(i,j)}$ is given by:

$$C_l^{(i,j)} = \sum_{q=-Q/2}^{Q/2} \tilde{h}_{q,l} S_{Q/2-q}^{(i,j)}. \tag{16}$$

Using the Euclidean distance, the branch metric for the transition from state i to state j at recursion k is given by

$$B^{(i,j)}[k] = \left|Y_k - C^{(i,j)}[k]\right|^2. \tag{17}$$

Again we can perform the ASC operation of the VA as follows. Define $M^{(j)}[k]$ as the path metric of state j at sub-carrier k, and the set $\mathcal{I}$ as the set of states that have transitions to state j. The branch metric of state j at recursion k is then chosen as

$$M^{(j)}[k] = \min_{i \in \mathcal{I}} \left(M^{(j)}[k-1] + B^{(i,j)}[k]\right).$$

For a BEM resolution $K > N$, the ICI is unlimited and covers the whole OFDM block, which prevents from developing a proper VA for practical applications. However, the main part of ICI comes from the neighboring Q sub-carriers.

C. Complexity Diversity Trade-off

Performing VA in time-domain and using perfect channel state information (CSI) requires $N(L+1)M^L$ multiply-add (MA) operations, in addition to M^L memory locations to store the transition tables (which required to be recomputed for each time instant (or recursion)). Performing VA in time-domain and using the BEM requires $(L+1)(Q+1)M^L + N(Q+1)$ MA operations, in addition to $(L+1)M^Q$ memory locations to store the $L+1$ transition tables that characterize the state diagrams.

Performing VA in frequency-domain requires $(L+1)(Q+1)M^Q + N(L+1)$ MA operations, in addition to $(L+1)M^Q$ memory locations to store the $L+1$ transition tables that characterize the state diagrams.

In time-domain, VA is capable of exploiting the multipath diversity (diversity order is equal to $L+1$), whereas in frequency-domain it is capable of exploiting the Doppler diversity (using the BEM, the diversity order is equal to $Q+1$). In this sense we can tradeoff diversity and complexity. In one hand for $L > Q$, performing VA in frequency-domain is less complex than performing it in time-domain, but on the cost of less diversity order and vice versa. Exploiting full diversity (multi-path as well as Doppler) is possible by using maximum diversity techniques [18] in conjunction with the MLSE.

IV. Simulations

In this section we present some of the simulation results of the proposed equalization technique for SC as well as for OFDM transmission over doubly selective channels. In the simulations below, uncoded Quadrature Phase Shift Keying (QPSK) modulation is used.

• *SC Transmission:* For SC transmission the doubly selective channel is assumed to be of order $L = 3$. The channel taps are simulated as i.i.d random variables, correlated in time according to Jakes' model with correlation function $r_h[n] = J_0(2\pi n f_{max} T)$, where J_0 is the zeroth-order Bessel function of the first kind, $f_{\max} T = 0.001$ is the maximum normalized Doppler spread. We consider a window of size $N = 800$. The BEM resolution is chosen to be $K = N$ and $K = 2N$. For $K = N$ the number of basis functions is $Q = 2$, while for $K = 2N$ the number of basis functions is $Q = 4$. Alongside with MLSE, we examine the linear MMSE and the DFE, the feed forward and feedback filters are implemented as time-varying FIR filters designed using the BEM [10], [9]. The feed forward filter is designed to have order $L' = 20$, and number of basis functions $Q' = 20$, the feedback filter is designed to have order $L'' = 3$, and the number of basis functions is $Q'' = 4$. We measure the performance in terms of bit error rate (BER) vs. signal-to-noise ratio (SNR). The SNR is defined as $(L+1)E_s/\sigma_n^2$, where E_s is the transmitted symbol energy, and σ_n^2 is the additive white Gaussian noise variance. The simulation results are shown in Figure 2. As shown in Figure 2, for $K = N$, the MLSE, the MMSE and the DFE suffer from an early floor at a $BER = 3 \times 10^{-2}$. The MLSE slightly outperforms the MMSE and DFE equalizers, but not significantly so. For $K = 2N$, the MLSE significantly outperforms the MMSE and DFE equalizers, where an SNR gain of $2\ dB$ is observed for the MLSE over the DFE, and an SNR gain of $4\ dB$ over the MMSE equalizer. We also simulate the performance of the MLSE over TI channels. the channel is of order $L = 3$. As clearly seen from the Figure, the MLSE over the time-varying channel coincides with the MLSE over TI channels. This confirms the fact that the MLSE (VA) in time-domain is capable of exploiting the multi-path diversity. The matched filter bound (MFB) for time-invariant channels [20] is also included in the plot. An SNR loss of about $1\ dB$ for the MLSE implemented using VA compared to the MFB. Clearly the BER slope of the MLSE is the same of that of the MFB, which confirms the fact that VA in time-domain is capable of exploiting the multipath diversity.

• *OFDM Transmission:* For OFDM transmission, we consider a doubly selective channel that is modeled using the BEM with order $L = 6$, and the number of basis functions $Q = 2$. The BEM coefficients are simulated as complex Gaussian random variables. Along side the MLSE, the time-varying per-tone equalizer (PTEQ) is also examined [21], [12]. For the PTEQ, we examine the case when the the number of basis functions $Q' = 8$, as well as the number of basis functions $Q' = 14$. As shown in Figure 3, the MLSE significantly outperforms the MMSE PTEQ for both cases $Q' = 8$ and $Q' = 14$. The SNR gain of the MLSE over the MMSE PTEQ with $Q' = 8$ is $4.2\ dB$, while it is $3.2\ dB$ over the MMSE PTEQ with $Q' = 14$. Again, an SNR loss of $1\ dB$

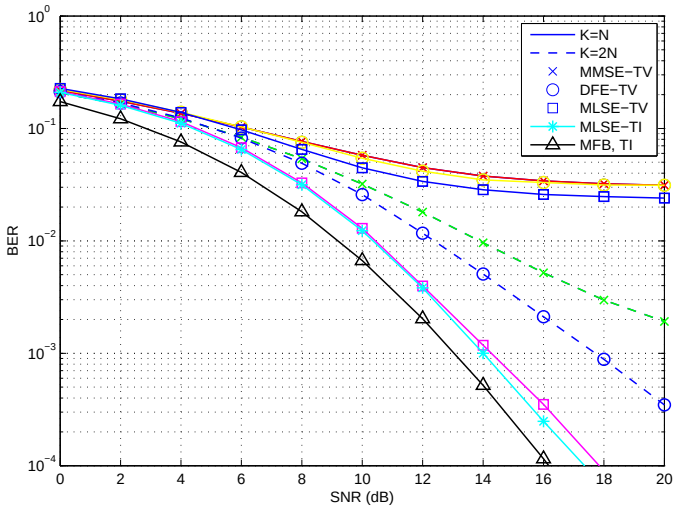


Fig. 2. MLSE for SC transmission.

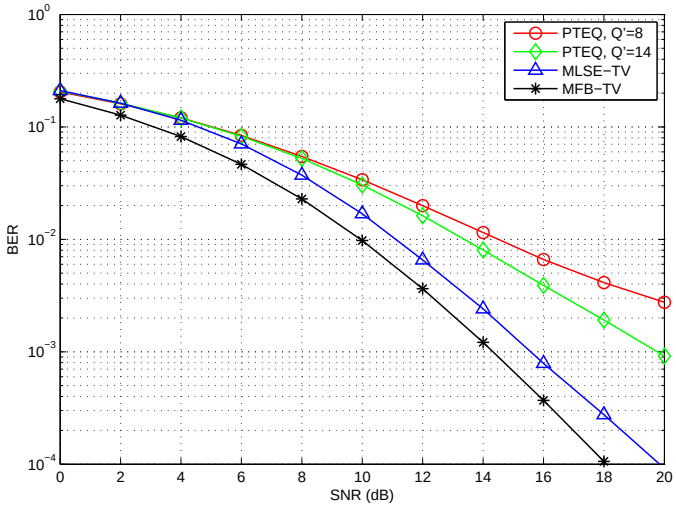


Fig. 3. MLSE for OFDM transmission.

for the MLSE over the MFB of the time-varying channel [22]. The BER slope of the MLSE is clearly the same as that of the MFB, which confirms the fact that VA in frequency-domain is capable of exploiting the Doppler diversity.

V. Conclusions

In this paper we propose an MLSE for doubly selective channels, implemented using VA. The doubly selective channel can be viewed as time-varying FIR filter. Using the BEM to approximate the doubly selective channel, it can be viewed as a bank of time-invariant FIR filters, where the time-variation of each filter is captured by means of a time-varying complex exponential basis function. By doing so, we show that, the branch metrics required for the VA can be computed and updated at a much lower complexity. This however, on the cost of extra complexity. This idea can be equally applied to SC transmission as well as to MC transmission system implemented using OFDM. The latter is equivalent to performing the VA in frequency-domain. Through computer simulations, we show that the VA is capable of exploiting the multi-path diversity in time-domain and the Doppler diversity in frequency-domain. Performing VA in time-domain or in frequency-domain allows for complexity diversity (performance) trade-off.

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