Joint Compensation of IQ imbalance and Frequency Offset in OFDM systems.

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Abstract—**Zero-IF receivers are gaining interest because of their potential to enable low-cost OFDM terminals. However, Zero-IF receivers introduce IQ imbalance which may have a huge impact on the performance. Rather than increasing design time or component cost to decrease the IQ imbalance, an alternative is to tolerate the IQ imbalance and compensate it digitally. Current solutions require extra analog hardware or converge too slowly for bursty communication. Moreover, the tremendous impact of a frequency offset on the IQ estimation/compensation problem is not considered. In this paper, we analyze the joint IQ imbalance-frequency offset estimation and propose a low-cost, highly effective, all-digital compensation scheme. For large IQ imbalance ($\epsilon = 10\%$, $\Delta\phi = 10^o$) and large frequency offsets, our solution results in an average implementation loss below 0.5 dB. It therefore enables the design of low-cost, low-complexity OFDM receivers.**

I. Introduction

OFDM is a widely recognized and standardized modulation technique [1], [2]. Unfortunately, OFDM is also sensitive to non-idealities in the receiver front-end [3]. This sensitivity leads either to heavy front-end specifications and thus an expensive front-end or to large performance degradations. IQ imbalance has been identified as a key front-end effect for OFDM systems.

However, OFDM is also very sensitive to frequency offsets [4]. Therefore, we investigate the joint IQ imbalance and frequency offset estimation and compensation and introduce a low-complexity compensation scheme to combat the IQ imbalance and Carrier Frequency Offset (CFO). To the best of our knowledge, no such solution has been proposed so far. [5], [6] propose a calibration solution which requires extra, carefully designed, analog hardware. [7] investigates only the impact of IQ imbalance on the CFO estimation. [8], [9] offer a IQ compensation with slow convergence and ignore the impact of the frequency offset. In [10] we proposed an IQ compensation with a fast convergence, but it only handles small frequency offsets.

In this paper, we propose an all-digital, low-complexity IQ compensation which converges within 1 OFDM training symbol and can handle large frequency offsets.

II. IQ imbalance

This section is a brief summary of [10] to introduce the IQ model and the frame work to compare other algorithms. More details can be found in that paper.

A. Effect/Model

IQ imbalance can be characterized by 2 parameters: the amplitude imbalance ϵ between the I and Q branch, and the phase orthogonality mismatch $\Delta\phi$. The complex baseband equation for the IQ imbalance effect on the ideal time domain signal $\mathbf{r}$ is given by [11] as

$$\begin{aligned}\mathbf{r_{iq}} &= (1+\epsilon)\cos\Delta\phi\Re\{\mathbf{r}\} - (1+\epsilon)\sin\Delta\phi\Im\{\mathbf{r}\} \\ &+ \jmath[(1-\epsilon)\cos\Delta\phi\Im\{\mathbf{r}\} - (1-\epsilon)\sin\Delta\phi\Re\{\mathbf{r}\}] \quad (1)\\ &= (\cos\Delta\phi + \jmath\epsilon\sin\Delta\phi)\cdot\mathbf{r} + (\epsilon\cos\Delta\phi - \jmath\sin\Delta\phi)\cdot\mathbf{r}^* \\ &= \alpha\cdot\mathbf{r} + \beta\cdot\mathbf{r}^* \quad (2)\end{aligned}$$

with $\mathbf{r_{iq}}$ the time domain signal with IQ imbalance, $\Re()$ denotes the real part, $\Im()$ the imaginary part and $()^*$ the complex conjugate and

$$\alpha = \cos\Delta\phi + \jmath\epsilon\sin\Delta\phi \quad (3)$$
$$\beta = \epsilon\cos\Delta\phi - \jmath\sin\Delta\phi \quad (4)$$

Frequency domain signals are underscored, while time domain signals are not. Signals are indicated in bold and scalar parameters in normal font.

Throughout the rest of the paper, the term IQ parameters refers to α and β for calculations and estimations ; to indicate physical parameters, however, we use the more direct ϵ and $\Delta\phi$.

We also analyze the effect of the IQ imbalance in the frequency domain. If $\underline{\mathbf{d}}$ is the incoming OFDM symbol (thus $\underline{\mathbf{d}} = \underline{\mathbf{d}}_\mathbf{t}\cdot\underline{\mathbf{c}} + \underline{\mathbf{n}}$ with $\underline{\mathbf{d}}_\mathbf{t}$ the transmitted OFDM symbol, $\underline{\mathbf{c}}$ the channel and $\underline{\mathbf{n}}$ the noise and $\cdot$ an element-wise multiplication), then $IFFT(\underline{\mathbf{d}})$ is the corresponding time domain signal. Applying the IQ imbalance (2) and taking the FFT to convert back to the frequency domain leads to

$$\begin{aligned}\underline{\mathbf{d}}_{\mathbf{iq}} &= FFT\{\alpha\cdot IFFT(\underline{\mathbf{d}}) + \beta\cdot[IFFT(\underline{\mathbf{d}})]^*\} \\ &= \alpha\cdot\underline{\mathbf{d}} + \beta\cdot\underline{\mathbf{d}}^*_\mathbf{m} \quad (5)\end{aligned}$$

where $\underline{\mathbf{d}}_{\mathbf{iq}}$ is the OFDM symbol with IQ imbalance and $\underline{\mathbf{d}}_\mathbf{m}$ the incoming OFDM symbol, mirrored over the carriers: $(\underline{\mathbf{d}})_m(i) = (\underline{\mathbf{d}})(mod(N_{sc} - i + 2, N_{sc}))$, with N_{sc} the number of subcarriers in the OFDM symbol, $1 \le i \le N$ and mod the modulo operation. Carrier 1 is the DC carrier.

In this paper we focus on IQ compensation for bursty communication, for which channel estimation is performed on the basis of a known training symbol. Both IEEE802.11a [1] and HIPERLAN-II [2] provide such a Long Training Symbol (LTS) (BPSK symbol $\underline{\mathbf{t}}$). The effect of IQ imbalance on channel estimation can be calculated based on (5) (and ignoring the noise for the moment ; its effect is taken into account for the performance analysis)

$$\begin{aligned}\underline{\mathbf{h}} &= \underline{\mathbf{t}}\cdot[\alpha(\underline{\mathbf{t}}\cdot\underline{\mathbf{c}}) + \beta(\underline{\mathbf{t}}\cdot\underline{\mathbf{c}})^*_m] \\ &= \alpha\cdot\underline{\mathbf{c}} + \beta\cdot\underline{\mathbf{t}}'\cdot\underline{\mathbf{c}}^*_\mathbf{m} \quad (6)\end{aligned}$$

where $\underline{\mathbf{h}}$ is the channel estimate calculated from the LTS, $\underline{\mathbf{c}}$ is the exact channel vector and $\underline{\mathbf{t}}' = \underline{\mathbf{t}}\cdot\underline{\mathbf{t}}_\mathbf{m}$.

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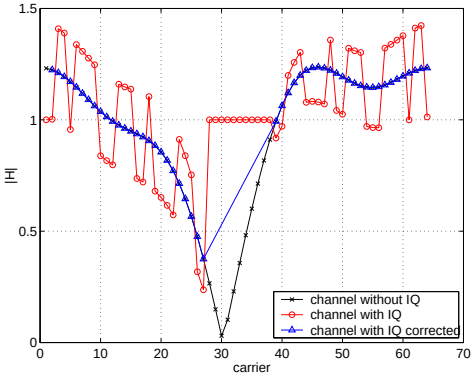


Fig. 1. The effect of IQ imbalance and correction on channel estimation.

B. Estimation/Compensation

From (6) the corrected channel response is

$$\underline{\hat{\mathbf{c}}} = \frac{\alpha^* \underline{\mathbf{h}} - \beta \underline{\mathbf{t}'} \cdot \underline{\mathbf{h}}^*_{\mathbf{m}}}{|\alpha|^2 - |\beta|^2} \tag{7}$$

The estimation of α and β is based on the information that the corrected channel response should have a smooth channel characteristic: since the coherence bandwidth of the channel is (a lot) larger than the inter-carrier-spacing in a WLAN system, the channel response does not change substantially between successive frequency taps (the x-line in figure 1). With IQ imbalance, sharp transitions occur in the measured channel response $\underline{\mathbf{h}}$ due to the β degradation term (the o-line in figure 1). Thus, correcting the IQ imbalance means making the channel response 'smooth' again. Therefore, we select the set of IQ parameters (α, β) which renders the corrected channel response $\hat{c}$ as smooth as possible ; in other words we minimize the Mean Square Error (MSE) between consecutive channel coefficients

$$MSE = \sum_l |\underline{\hat{\mathbf{c}}}_{l+1} - \underline{\hat{\mathbf{c}}}_l|^2 \tag{8}$$

The derivation can be found in [10]. The results are

$$\hat{\beta} = \frac{\sum_l (\underline{\mathbf{h}}_{l+1} - \underline{\mathbf{h}}_l)(\underline{\mathbf{t}'}_{-l-1}\underline{\mathbf{h}}_{-l-1} - \underline{\mathbf{t}'}_{-l}\underline{\mathbf{h}}_{-l})}{\sum_l |\underline{\mathbf{t}'}_{-l-1}\underline{\mathbf{h}}_{-l-1} - \underline{\mathbf{t}'}_{-l}\underline{\mathbf{h}}_{-l}|^2} \tag{9}$$

$$\hat{\alpha} = \sqrt{1 - \Im^2\{\hat{\beta}\}} - \jmath \frac{\Re\{\hat{\beta}\}\Im\{\hat{\beta}\}}{\sqrt{1 - \Im^2\{\hat{\beta}\}}} \tag{10}$$

Figure 1 shows that we can correct the influence of the IQ imbalance on the channel estimate extremely well: the corrected channel response (the Δ-line) coincides (almost) perfectly with the exact channel response (the x-line). Note that carriers 28 to 38 are zero carriers, which means no channel estimate is needed on those carriers.

This algorithm provides us with a corrected channel response and an estimate of the IQ parameters α and β (or equivalently ϵ and $\Delta\phi$). Since ϵ and $\Delta\phi$ and thus α and β are typically static over many symbols, we can use their estimates from the channel correction also for the correction of the IQ imbalance on the data. As we already have an estimate of the IQ parameters before the data symbols arrive we can compensate the effect in the time domain, i.e. before the FFT: this allows us to correct the IQ imbalance as early as possible. The setup is shown in figure 2.

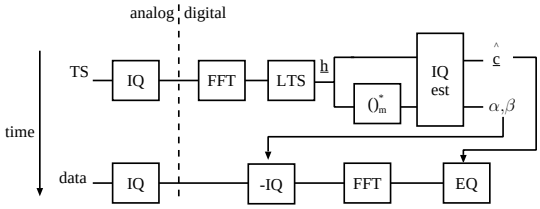


Fig. 2. Overview of the IQ estimation/compensation.

To obtain the corrected signal $\hat{\mathbf{r}}$ from the observed time domain signal $\mathbf{r_{iq}}$, we need to solve (2) for $\mathbf{r}$, i.e.

$$\hat{\mathbf{r}} = \frac{\alpha^* \cdot \mathbf{r_{iq}} - \beta \cdot \mathbf{r}^*_{\mathbf{iq}}}{|\alpha|^2 - |\beta|^2} \tag{11}$$

Since this algorithm is based on a frequency-domain estimation of the IQ parameters, throughout the rest of the paper we refer to it as the 'IQ-Frequency Domain' or IQ-FD algorithm.

C. Performance

To test the performance of the estimation/compensation scheme, we ran simulations for coded 64QAM in a multi-path environment. The multi-path channel are obtained through ray-tracing simulations [12]. The impact of severe IQ imbalance ($\epsilon = 10\%, \Delta\phi = 10^o$) on coded ($R = 3/4$ from the IEEE or HIPERLAN standard) 64QAM transmission is quite dramatic: it renders the system useless, causing a error floor of 5.10^{-1}. Our combined channel and IQ estimation/compensation scheme gets the degradation at a BER of 10^{-5} down to 0.25 dB (figure 4). The compensation scheme with the exact knowledge of the IQ parameters fully eliminates the IQ imbalance.

III. IQ Compensation in the presence of CFO

A. Effect/Model

In the time domain a Carrier Frequency Offset (CFO) causes an additional phase rotation proportional with time $\mathbf{t}$ on an incoming signal $\mathbf{r}$

$$\mathbf{r_{cfo}} = \mathbf{r} \cdot e^{\jmath 2\pi CFO \cdot \mathbf{t}} \tag{12}$$

In the frequency domain, a frequency offset causes leakage [13]. This means an OFDM symbol $\underline{\mathbf{d}}$ is received under a frequency offset as

$$(\underline{\mathbf{d}}_{\mathbf{cfo}})_i = \sum_{j=0}^{N_{sc}-1} \underline{\mathbf{d}}_j \cdot \gamma_{i-j} \tag{13}$$

where γ is the inter-carrier-interference caused by the frequency offset CFO

$$\gamma_n = e^{\jmath\pi(\frac{CFO}{\Delta f} - n)\frac{N_{sc}-1}{N_{sc}}} \frac{\sin \pi(\frac{CFO}{\Delta f} - n)}{\sin \frac{\pi}{N_{sc}}(\frac{CFO}{\Delta f} - n)} \tag{14}$$

with Δf is the inter-carrier-spacing and N_{sc} the total number of carriers in the system.

Mathematical analysis of the RF signals shows that the effect of the IQ imbalance and CFO can be modeled at baseband by first applying the frequency offset CFO, followed by all IQ imbalance contributions (from RF, LO and baseband) combined in (α,β). The resulting signal $\mathbf{r_{cfo-iq}}$ is thus

$$\mathbf{r_{cfo-iq}} = \alpha \cdot \mathbf{r} \cdot e^{\jmath 2\pi CFO \cdot \mathbf{t}} + \beta \cdot \mathbf{r}^* \cdot e^{-\jmath 2\pi CFO \cdot \mathbf{t}} \quad (15)$$

where $\mathbf{r}$ is the input signal.

B. Estimation/Compensation

(13) shows that if no CFO is present, the symbol transmitted on carrier i will be received at carrier i. However, in the presence of CFO, each carrier is spread out over all other carriers according to (13)-(14), the so-called inter-carrier-interference or leakage. Because of this leakage, the assumptions used to derive existing IQ estimation/compensation algorithms are no longer valid and thus their performance will degrade. For example, the degradation for IQ-FD algorithm at a BER of 10^{-5} for coded 64QAM already exceeds 1 dB at a CFO of 20 kHz and exceeds 5 dB at 40 kHz. In practice, the CFO can easily exceed 40 kHz. This clearly illustrates the need for an algorithm to take both IQ imbalance and frequency offset into account.

We derive an IQ compensation scheme according to the same principles as for the IQ-FD algorithm (as in [10]), but now taking the frequency offset into account.

We start from the time domain signal (15) and assume we can accurately estimate the frequency offset ; later we comment on this assumption. If we compensate for the frequency offset we obtain

$$\mathbf{r_1} = \alpha \cdot \mathbf{r} + \beta \cdot \mathbf{r}^* \cdot e^{-2\jmath 2\pi CFO \cdot \mathbf{t}} \quad (16)$$

We observe two contributions in the resulting signal: the first one contains a scaled version of the desired signal ; the second contribution contains a scaled version of the complex conjugate of the signal, rotated over $-2 \cdot CFO$. In the frequency domain, the former gives rise to the desired signal, while the latter yields a mirror image (because of the complex conjugate), subject to leakage (caused by the $-2 \cdot CFO$).

On the other hand, if we compensate (15) by $+CFO$, we obtain

$$\begin{aligned} \mathbf{r_2} &= \alpha \cdot \mathbf{r} \cdot e^{2\jmath 2\pi CFO \cdot \mathbf{t}} + \beta \cdot \mathbf{r}^* \\ \Rightarrow \mathbf{r_2^*} &= \beta^* \cdot \mathbf{r} + \alpha^* \cdot \mathbf{r}^* \cdot e^{-2\jmath 2\pi CFO \cdot \mathbf{t}} \end{aligned} \quad (17)$$

As in (16), we see the same two contributions: the desired signal and the complex conjugate of the signal rotated over $-2 \cdot CFO$, but with different weighting factors. Considering (16) and (17) a set of equations in the variables $\mathbf{r}$ and $\mathbf{r}^* \cdot e^{-2\jmath 2\pi CFO \cdot \mathbf{t}}$ and solving for $\mathbf{r}$ leads to

$$\hat{\mathbf{r}} = \frac{\alpha^* \cdot \mathbf{r_1} - \beta \cdot \mathbf{r_2^*}}{|\alpha|^2 - |\beta|^2} \quad (18)$$

(18) yields the corrected time domain signal based on $\mathbf{r_1}$ and $\mathbf{r_2}$. Equivalently, we write the equation for the channel correction

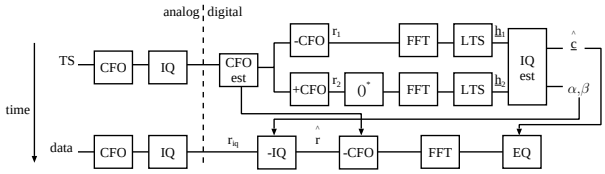


Fig. 3. Overview of the joint IQ-CFO estimation/compensation.

based on $\underline{\mathbf{h_1}} = fft(\mathbf{r_1}) \cdot \underline{\mathbf{t}}$ and $\underline{\mathbf{h_2}} = fft(\mathbf{r_2^*}) \cdot \underline{\mathbf{t}}$ (as indicated in figure 3) as follows

$$\hat{\underline{\mathbf{c}}} = \frac{\alpha^* \cdot \underline{\mathbf{h_1}} - \beta \cdot \underline{\mathbf{h_2}}}{|\alpha|^2 - |\beta|^2} \quad (19)$$

(19) provides the corrected channel response if we know (α,β). As in the previous section, we put the constraint that the corrected channel response is as smooth as possible. In other words, we minimize the MSE from (8).

This yields

$$\hat{\beta} = \frac{\sum_l (\underline{\mathbf{h}}_{1,l+1} - \underline{\mathbf{h}}_{1,l})(\underline{\mathbf{h}}^*_{2,l+1} - \underline{\mathbf{h}}^*_{2,l})}{\sum_l |\underline{\mathbf{h}}_{2,l+1} - \underline{\mathbf{h}}_{2,l}|^2} \quad (20)$$

$$\hat{\alpha} = \sqrt{1 - \Im^2\{\hat{\beta}\}} - \jmath \frac{\Re\{\hat{\beta}\}\Im\{\hat{\beta}\}}{\sqrt{1 - \Im^2\{\hat{\beta}\}}} \quad (21)$$

Once (α,β) are known, (11) can be used to correct the IQ imbalance on the received data signal, followed by a frequency offset correction.

The resulting block diagram is depicted in figure 3.

If no frequency offset is present, then the joint IQ-CFO compensation (figure 3) reduces to the IQ compensation (figure 2) after rearranging some blocks and (20-21-19) reduce to (9-10-7).

Since the estimation is done in the frequency domain, it can be easily redesigned to incorporate frequency-dependent IQ. In that case also the compensation is best handled in the frequency domain through a correction similar to (19).

C. Performance

We illustrate the performance of this joint IQ-CFO compensation scheme for a WLAN case study with coded ($R = 3/4$ from the IEEE or HIPERLAN standard) 64QAM transmission.

In the absence of CFO, this scheme is equivalent to the IQ-FD algorithm and thus has the same performance: for large IQ imbalance ($\epsilon = 10\%, \Delta\phi = 10^o$) it reduces the degradation at a BER of 10^{-5} down to 0.25 dB, as shown in figure 4 and 5.

Figure 5 illustrates the IQ compensation works for large frequency offsets as well: the degradation doesn't exceed 0.5 dB at a BER of 10^{-5} coded 64QAM even for frequency offsets exceeding 100 kHz.

The CFO is estimated by means of the average phase rotation between the 2 Long Training Symbols

$$\hat{CFO} = \frac{angle(E[\mathbf{x_2} \cdot \mathbf{x_1^*}])}{2\pi N_{sc} T_s} \quad (22)$$

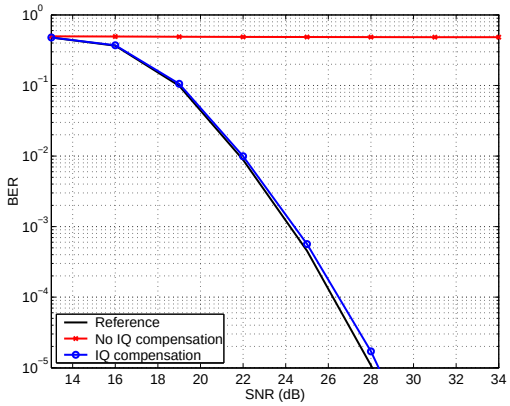


Fig. 4. The performance of the IQ compensation scheme for coded (R=3/4) 64QAM and ($\epsilon = 10\%$,$\Delta\phi = 10^o$) in the absence of a frequency offset.

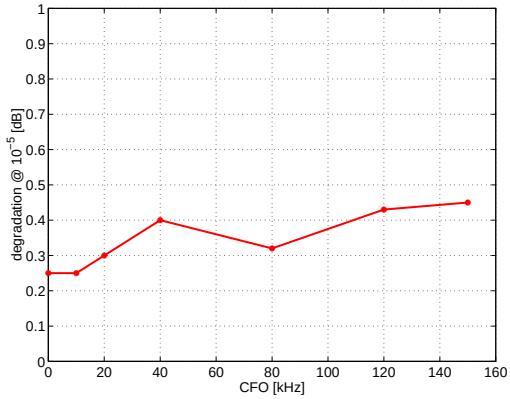


Fig. 5. Remaining degradation of the IQ compensation @ 10^{-5} coded 64QAM as a function of the frequency offset.

with $\mathbf{x_1}$ and $\mathbf{x_2}$ the received training sequences, $N_{sc}T_s$ the time between 2 corresponding samples of the training sequences.

This CFO estimation performs well in the presence of IQ imbalance: the simulation results from figure 5 are based on the CFO estimates and show the estimates are accurate enough to produce a small residual degradation in the IQ imbalance estimation/compensation.

The IQ estimation/compensation algorithm requires no extra analog hardware and a very small additional digital complexity. The IQ imbalance may occur anywhere in the receiver, because the RF, LO and baseband contributions are jointly estimated and compensated.

Since the IQ estimation only requires a known Training Symbol, this scheme is applicable to any system which uses a Training Symbol to estimate the channel (e.g. Wireless LAN [2], [1] and Broadband Fixed Wireless Access [14]). Moreover, the scheme is also independent of the data that follows the Training Symbol. This means it is applicable to any constellation size and to OFDM as well as Single-Carrier with Frequency-Domain Processing.

IV. Conclusions

In this paper, we introduced a low-complexity estimation/compensation scheme that tackles the IQ imbalance caused by Zero-IF receivers. The scheme converges on one OFDM training symbol. It removes large IQ imbalance ($\epsilon = 10\%, \Delta\phi = 10^o$) and reduces the average remaining degradation down to 0.5 dB, even in the presence of large frequency offsets. This allows to relax the IQ mismatch specifications significantly. In turn, this allows to use cheaper components and to shorten the design time. Therefore, our estimation/compensation scheme enables low-cost, low-complexity OFDM receivers.

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