Linear Multiparameter Eigenvalue Problems

Abstract for the 28th ERNSI Workshop on System Identification

Christof Vermeersch[ab] and Bart De Moor[ac]

August 23, 2018



Type of contribution: poster

Abstract

Much has been written about standard eigenvalue problems and their properties. However, linear multiparameter eigenvalue problems (linear MEPs) remain largely uncharted territory. Linear MEPs contain multiple eigenvalues and, therefore, extend the typical structure of the well-known standard eigenvalue problem [1]:

$$(A_0 + A_1\lambda_1 + A_2\lambda_2 + \cdots + A_n\lambda_n)\, z = 0.$$

They emerge naturally in many physical applications, for example when solving partial differential equations via a separation of variables [3]: Helmholtz, Laplace, and Schrödinger equations typically lead to linear MEPs. In systems and control, MEPs arise when identifying or reducing models of linear time-invariant systems [2, 4].

Despite their fundamental linear algebra nature, the available literature about solving (linear) MEPs remains quite limited. Typically, numerical optimization methods are used to find (approximations of) the eigenvalues and -vectors. Algebraic approaches, on the other hand, confine themselves mostly to two-parameter problems.

We approach this problem from a different perspective and incorporate the linear MEP in the so-called block Macaulay matrix, which we iteratively extend until its null space has a special block multi-shift-invariant structure. Via a multidimensional realization problem in that null space, we obtain a standard eigenvalue problem that yields all the solutions of the original linear MEP.

Acknowledgments

Christof Vermeersch is an FWO Strategic Basic Research fellow (application number 1SA1319N). This research receives support from FWO under EOS project 30478160 (SeLMA) and research project I013218N (Alamire), from IOF under fellowship 13-0260, from the EU under H2020-SC1-2016-2017 Gran Agreement No.727721 (MIDAS), from IWT and VLAIO through PhD grants, from VLAIO under the industrial project HBC.2018.0405, and from KU Leuven Internal Funds: C16/15/059 and C32/16/013.

References

[1] F. V. Atkinson. *Multiparameter Eigenvalue Problems*. Mathematics in Science and Engineering. Academic Press, New York, 1972.

[2] B. De Moor. Least-squares realization of LTI models is an eigenvalue problem. Technical report, KU Leuven, 2018. Internal Report 18-140, KU Leuven, Leuven, 2018. Accepted for presentation at and publication in the proceedings of ECC 2019.

[3] B. Plestenjak, C. I. Gheorghiu, and M. E. Hochstenbach. Spectral collocation for multiparameter eigenvalue problems arising from separable boundary value problems. *Journal of Computational Physics*, 298(1):585–601, 2015.

[4] C. Vermeersch and B. De Moor. Globally optimal ARMA model identification is an eigenvalue problem. *IEEE Control Systems Letters*, 3(4):xx–xx, 2019.

[a] KU Leuven, Department of Electrical Engineering (ESAT), Center for Dynamical Systems, Signal Processing, and Data Analytics (STADIUS). Kasteelpark Arenberg 10, 3001 Leuven, Belgium

[b] christof.vermeersch@esat.kuleuven.be

[c] bart.demoor@esat.kuleuven.be