A Column Space Based Approach to Solve Multiparameter Eigenvalue Problems

Christof Vermeersch and Bart De Moor, *Fellow, IEEE & SIAM*
Center for Dynamical Systems, Signal Processing, and Data Analytics (STADIUS),
Dept. of Electrical Engineering (ESAT), KU Leuven
{christof.vermeersch,bart.demoor}@esat.kuleuven.be

1 Introduction

Multiparameter eigenvalue problems (MEPs) emerge both in nature and science [1]. More particularly, they often arise in systems and control, e.g., the least-squares realization of linear time-invariant models [2] and the globally optimal identification of autoregressive moving-average models [3]. A multidimensional realization problem in the null space of the block Macaulay matrix that contains the MEP results in a standard eigenvalue problem, of which the eigenvalues and -vectors yield the solutions of the MEP. Since this null space based algorithm uses established numerical linear algebra tools, like the singular value and eigenvalue decomposition, it finds the solutions within machine precision. In this research, we propose a new, complementary approach to solve MEPs, which considers the column space of the block Macaulay matrix instead of its null space.

2 Research methodology

An MEP extends the typical structure of the well-known standard eigenvalue problem [1]:

$$\left(\sum_{\omega} A_{\omega}\omega\right) z = 0, \tag{1}$$

where $\omega(\lambda_1,\ldots,\lambda_n) = \lambda_1^{k_1}\cdots\lambda_n^{k_n}$ is a monomial function of the n eigenvalues $\lambda_1,\ldots,\lambda_n \in \mathbb{C}$ and the vector $z \in \mathbb{C}^q$ is the eigenvector. The matrices $A_\omega \in \mathbb{R}^{p\times q}$ contain the corresponding coefficients. Contrary to standard eigenvalue problems, the available literature about solving MEPs remains quite limited. Typically, numerical optimization methods are used to find (approximations of) the eigenvalues and -vectors. Algebraic approaches, on the other hand, confine themselves mostly to linear or quadratic two-parameter eigenvalue problems. Vermeersch and De Moor [3] have shown that a multidimensional realization problem in the null space of the block Macaulay matrix that contains this MEP results in a standard eigenvalue problem, which yields the solutions of the MEP. Instead of the null space, we consider the column space of the block Macaulay matrix and work on the data directly. The complementarity between both spaces yields an alternative approach to solve MEPs, without the need to construct a numerical basis of the null space.

3 Presentation outline

In our presentation, we will explain how we can find the solutions of an MEP via a multidimensional realization problem in the column space of the block Macaulay matrix. Furthermore, the presentation will elaborate on this block Macaulay matrix and the complementarity between its null space and column space.

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