Multi-channel Wiener filtering for binaural noise reduction

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Abstract

This report presents an extension of the speech distortion weighted multi-channel Wiener filter (SDW-MWF) for noise reduction in binaural hearing aids, taking into account binaural localisation cues. By adding a term related to the interaural time difference (ITD) and the interaural level difference (ILD) of the noise component to the cost function of the SDW-MWF, the ITD and the ILD cue of both the speech and the noise component can be preserved, without compromising the noise reduction performance. In addition, it is also possible to extend the linearly constrained minimum variance (LCMV) beamformer using the acoustic transfer function (TF) ratio with binaural localisation cues.

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1 Introduction

Noise reduction algorithms in hearing aids are crucial for hearing impaired persons to improve speech intelligibility in background noise. Multi-microphone systems are able to exploit spatial in addition to spectral information and are hence preferred to single-microphone systems. Commonly used multi-microphone noise reduction techniques for - monaural and binaural - hearing aids are based on fixed beamforming [1, 2, 3, 4, 5, 6, 7, 8], adaptive beamforming [9, 10, 11, 12, 13, 14, 15, 16], or multi-channel Wiener filtering [17, 18, 19, 20, 21, 22, 23, 24].

In a binaural hearing aid system, output signals for both ears are generated, either by using both hearing aids independently or by sharing information between the hearing aids. In addition to reducing background noise and limiting speech distortion, another important objective of a binaural algorithm is to preserve the listener's impression of the auditory environment in order to exploit the natural binaural hearing advantage. This can be achieved by preserving the binaural cues, i.e. the interaural time and level difference (ITD, ILD), of the speech and the noise components.

In [5], fixed beamforming techniques have been proposed that combine noise reduction with the preservation of binaural cues. A first, simple solution consists of the combination of two identical monaural-output endfire arrays or two hardware directional microphones, one at each ear. Assuming that both arrays have the same directivity pattern, the ITD and ILD cues are preserved. However, this method only exploits half of the available microphone signals. Better performance is expected if all microphone signals would be exploited. In a second solution, all microphone signals of a single broadside array are combined to compute a left and a right output. The filter weights are optimized such that they maximize the directivity index while restricting the ITD error below some threshold. Interaural level differences are not preserved. The proposed broadside microphone array consists of a 4-microphone system that is either eye-glass or headband mounted. In [8], a binaural superdirective beamformer has been presented using head related transfer functions.

In contrast to fixed beamformers, adaptive beamforming techniques make use of data-dependent filter coefficients that can be adapted to time-varying scenarios. As a result, they generally result in a better performance than fixed beamforming techniques. Binaural adaptive beamforming techniques, based on the Generalised Sidelobe Canceller (GSC), have been proposed in [10, 12, 15]. The algorithm in [10] takes a microphone signal from each ear as inputs. Binaural hearing is provided by dividing the frequency spectrum into a low-pass and a high-pass portion. The low frequencies of the left and the right signal are passed through unaltered in order to preserve the ITD cues, whereas the high frequencies are adaptively processed using the GSC and added to the low frequencies. A major drawback of this approach is that not only the speech but also the noise in the low-frequency portion is passed through, significantly comprimising the noise reduction performance. As the cut-off frequency increases, the preservation of the ITD cues improves at the expense of noise reduction. In [12, 15], the preservation of the ITD and the ILD cues is restricted to an angular region around the front, while at other angles the background noise is reduced. In [12], the preservation of the interaural time differences is imposed by means of a Frost beamformer [25] with multiple constraints. In [15], an alternative

optimization criterion based on weighted least squares has been proposed. This criterion allows for a trade-off between noise reduction and restoration of the binaural cues around the frontal directions. Both algorithms assume the desired signal to be located in front and require a priori knowledge of the microphone positions and the microphone characteristics.
In [23], a binaural multi-channel Wiener filter, providing an enhanced output signal at both ears, has been discussed. In addition to significantly suppressing the background noise, it has been shown that this algorithm preserves the ITD cues of the speech component. On the contrary, the binaural cues of the noise component may be distorted. In addition to reducing the noise level, it is also important to (partially) preserve these binaural noise cues in order to exploit the binaural hearing advantage of normal hearing and hearing impaired persons [26, 27, 28, 29, 30, 31, 32, 33, 34] or in order to further process the binaural output signals with a speech enhancement procedure based on a difference in binaural speech and noise cues [35, 36, 37, 38, 39]. An extension of the MWF that partially preserves these binaural noise cues has been proposed in [24]. However, this technique results in a considerable reduction of the noise reduction performance. This paper describes a novel extension of the MWF, adding a term related to the binaural cues of the noise component to the cost function of the MWF. Experimental results show that the ITD and ILD cues of both the speech and the noise component can be preserved without compromising the noise reduction performance.

2 Configuration and notation

Consider the binaural hearing instrument configuration depicted in Fig. 1, where the left and the right hearing instrument have a microphone array consisting of M_0 and M_1 microphones. The mth microphone signal in the left hearing instrument $Y_{0,m}(\omega)$ can be decomposed as

$$Y_{0,m}(\omega) = X_{0,m}(\omega) + V_{0,m}(\omega), \quad m = 0 \dots M_0 - 1, \tag{2.1}$$

where $X_{0,m}(\omega)$ represents the speech component and $V_{0,m}(\omega)$ represents the noise component. Assuming that one desired signal source is present, the speech component $X_{0,m}(\omega)$ is equal to

$$X_{0,m}(\omega) = A_{0,m}(\omega)S(\omega)\ , \tag{2.2}$$

with $A_{0,m}(\omega)$ the acoustical transfer function (TF) between the speaker and the mth microphone in the left hearing instrument and $S(\omega)$ the speech signal. Similarly, the mth microphone signal in the right hearing instrument $Y_{1,m}(\omega)$ can be written as

$$Y_{1,m}(\omega) = X_{1,m}(\omega) + V_{1,m}(\omega) = A_{1,m}(\omega)S(\omega) + V_{1,m}(\omega)\ . \tag{2.3}$$

Define the M-dimensional signal vector $\mathbf{Y}(\omega)$, with $M = M_0 + M_1$, as[1]

$$\mathbf{Y}(\omega) = \left[\ Y_{0,0}(\omega) \quad \dots \quad Y_{0,M_0-1}(\omega) \quad Y_{1,0}(\omega) \quad \dots \quad Y_{1,M_1-1}(\omega)\ \right]^T\ . \tag{2.4}$$

[1]We assume that all microphone signals are simultaneously available at both the left and the right hearing instrument, requiring some sort of communication between the hearing instruments (e.g. cable, wireless link).

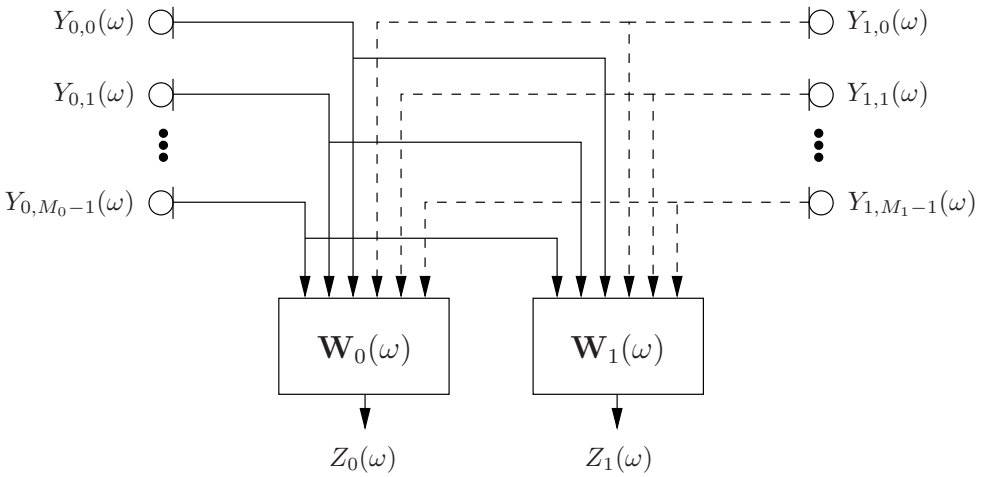


Figure 1: Binaural configuration

The signal vector can be written as

$$\mathbf{Y}(\omega) = \mathbf{X}(\omega) + \mathbf{V}(\omega) = \mathbf{A}(\omega)S(\omega) + \mathbf{V}(\omega) \, , \tag{2.5}$$

with $\mathbf{X}(\omega)$ and $\mathbf{V}(\omega)$ defined similarly as in (2.4) and the TF vector $\mathbf{A}(\omega)$ equal to

$$\mathbf{A}(\omega) = \left[\begin{array}{cccccc} A_{0,0}(\omega) & \dots & A_{0,M_0-1}(\omega) & A_{1,0}(\omega) & \dots & A_{1,M_1-1}(\omega) \end{array} \right]^T \, . \tag{2.6}$$

The output signals at the left and the right hearing instrument $Z_0(\omega)$ and $Z_1(\omega)$ are equal to

$$Z_0(\omega) = \mathbf{W}_0^H(\omega)\mathbf{Y}(\omega), \quad Z_1(\omega) = \mathbf{W}_1^H(\omega)\mathbf{Y}(\omega) \, , \tag{2.7}$$

where $\mathbf{W}_0(\omega)$ and $\mathbf{W}_1(\omega)$ are M-dimensional complex weight vectors[2]. The output signal at the left hearing instrument can be written as

$$Z_0(\omega) = Z_{x0}(\omega) + Z_{v0}(\omega) = \mathbf{W}_0^H(\omega)\mathbf{X}(\omega) + \mathbf{W}_0^H(\omega)\mathbf{V}(\omega) \, , \tag{2.8}$$

where $Z_{x0}(\omega)$ represents the speech component and $Z_{v0}(\omega)$ represents the noise component. Similarly, the output signal at the right hearing instrument can be written as $Z_1(\omega) = Z_{x1}(\omega) + Z_{v1}(\omega)$.
We define the $2M$-dimensional complex stacked weight vector $\mathbf{W}(\omega)$ as

$$\mathbf{W}(\omega) = \left[\begin{array}{c} \mathbf{W}_0(\omega) \\ \mathbf{W}_1(\omega) \end{array} \right] \, . \tag{2.9}$$

Hence, the filters $\mathbf{W}_0(\omega)$ and $\mathbf{W}_1(\omega)$ are equal to

$$\mathbf{W}_0(\omega) = \left[\begin{array}{cc} \mathbf{I}_M & \mathbf{0}_M \end{array} \right] \mathbf{W}(\omega), \quad \mathbf{W}_1(\omega) = \left[\begin{array}{cc} \mathbf{0}_M & \mathbf{I}_M \end{array} \right] \mathbf{W}(\omega) \, , \tag{2.10}$$

[2]Instead of using all available microphone signals for computing the binaural output signals, it is of course also possible just to use a subset of the microphone signals, e.g. compute $Z_0(\omega)$ using only the microphone signals in the left hearing instrument and compute $Z_1(\omega)$ using only the microphone signals in the right hearing instrument.

with $\mathbf{I}_M$ the $M \times M$-dimensional identity matrix.
The real and the imaginary part of $\mathbf{W}(\omega)$ are denoted by $\mathbf{W}_R(\omega)$ and $\mathbf{W}_I(\omega)$. We define the $4M$-dimensional real weight vector $\widetilde{\mathbf{W}}(\omega)$ as

$$\widetilde{\mathbf{W}}(\omega) = \begin{bmatrix} \mathbf{W}_R(\omega) \\ \mathbf{W}_I(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{W}_{0R}(\omega) \\ \mathbf{W}_{1R}(\omega) \\ \mathbf{W}_{0I}(\omega) \\ \mathbf{W}_{1I}(\omega) \end{bmatrix} . \tag{2.11}$$

For conciseness, we will omit the frequency-domain variable ω in the remainder of the text.

3 Speech-distortion weighted multi-channel Wiener filter

3.1 Definition of cost function

The *multi-channel Wiener filter (MWF)* produces a minimum mean-square error (MMSE) estimate of the speech component in one of the microphone signals, hence simultaneously reducing residual noise and limiting speech distortion [17, 18, 23, 40, 41, 42]. The MSE cost function for the filter $\mathbf{W}_0$ estimating the speech component X_{0,r_0} in the r_0th microphone signal[3] of the left hearing instrument is equal to

$$\begin{aligned} J_{MSE,0}(\mathbf{W}_0) &= \mathcal{E}\{|X_{0,r_0} - Z_0|^2\} = \mathcal{E}\{|X_{0,r_0} - Z_{x0}|^2\} + \mathcal{E}\{|Z_{v0}|^2\} && (3.1) \\ &= \mathcal{E}\{|X_{0,r_0} - \mathbf{W}_0^H\mathbf{X}|^2\} + \mathcal{E}\{|\mathbf{W}_0^H\mathbf{V}|^2\} , && (3.2) \end{aligned}$$

assuming independence between the speech and the noise component. Similarly, the MSE cost function for the filter $\mathbf{W}_1$ estimating the speech component X_{1,r_1} in the r_1th microphone signal of the right hearing instrument is equal to

$$J_{MSE,1}(\mathbf{W}_1) = \mathcal{E}\{|X_{1,r_1} - Z_1|^2\} = \mathcal{E}\{|X_{1,r_1} - \mathbf{W}_1^H\mathbf{X}|^2\} + \mathcal{E}\{|\mathbf{W}_1^H\mathbf{V}|^2\} . \tag{3.3}$$

The total MSE cost function can now be written as

$$J_{MSE}(\mathbf{W}) = J_{MSE,0}(\mathbf{W}_0) + \alpha\, J_{MSE,1}(\mathbf{W}_1) , \tag{3.4}$$

where the factor α trades off the MSE cost functions at the left and the right hearing instrument. However, since both terms in $J_{MSE}(\mathbf{W})$ are independent of each other, this factor has no influence on the computation of the optimal filter $\mathbf{W}_{MSE}$.
In order to provide a trade-off between speech distortion and noise reduction, the *speech distortion weighted multi-channel Wiener filter (SDW-MWF)* minimises the weighted sum of the residual noise energy and the speech distortion energy [17, 18, 22, 41, 43]. The SDW cost function for the left hearing instrument becomes

$$\begin{aligned} J_{SDW,0}(\mathbf{W}_0) &= \mathcal{E}\{|X_{0,r_0} - Z_{x0}|^2\} + \mu_0\mathcal{E}\{|Z_{v0}|^2\} && (3.5) \\ &= \mathcal{E}\{|X_{0,r_0} - \mathbf{W}_0^H\mathbf{X}|^2\} + \mu_0\mathcal{E}\{|\mathbf{W}_0^H\mathbf{V}|^2\} , && (3.6) \end{aligned}$$

[3] Typically, the reference microphone is the first microphone, i.e. $r_0 = r_1 = 0$.

where μ_0 provides a trade-off between noise reduction and speech distortion at the left hearing instrument. Similarly, the SDW cost function for the right hearing instrument becomes

$$J_{SDW,1}(\mathbf{W}_1) = \mathcal{E}\{|X_{1,r_1} - \mathbf{W}_1^H\mathbf{X}|^2\} + \mu_1\mathcal{E}\{|\mathbf{W}_1^H\mathbf{V}|^2\}\ , \tag{3.7}$$

where μ_1 provides a trade-off between noise reduction and speech distortion at the right hearing instrument. When $\mu_0 = \mu_1 = 1$, the SDW cost functions reduce to the MSE cost functions. The total SDW cost function can now be written as

$$\boxed{J_{SDW}(\mathbf{W}) = J_{SDW,0}(\mathbf{W}_0) + \alpha\, J_{SDW,1}(\mathbf{W}_1)} \tag{3.8}$$

Again, the factor α has no influence on the computation of the optimal filter $\mathbf{W}_{SDW}$, and the filters $\mathbf{W}_0$ and $\mathbf{W}_1$ can be computed independently of each other.

3.2 Computation of cost function and derivatives

The SDW cost function for the left hearing instrument in (3.6) is equal to

$$J_{SDW,0}(\mathbf{W}_0) = P_0 + \mathbf{W}_0^H(\mathbf{R}_x + \mu_0\mathbf{R}_v)\mathbf{W}_0 - \mathbf{W}_0^H\mathbf{r}_{x0} - \mathbf{r}_{x0}^H\mathbf{W}_0\ , \tag{3.9}$$

with

$$\mathbf{R}_x = \mathcal{E}\{\mathbf{X}\mathbf{X}^H\} \qquad \mathbf{R}_v = \mathcal{E}\{\mathbf{V}\mathbf{V}^H\} \tag{3.10}$$

$$\mathbf{r}_{x0} = \mathcal{E}\{\mathbf{X}X_{0,r_0}^*\} \qquad P_0 = \mathcal{E}\{|X_{0,r_0}|^2\}\ . \tag{3.11}$$

Note that the $M \times M$-dimensional complex matrices $\mathbf{R}_x$ and $\mathbf{R}_v$ are Hermitian, i.e. $\mathbf{R}_x^H = \mathbf{R}_x$ and $\mathbf{R}_v^H = \mathbf{R}_v$, and positive definite. Similarly, the SDW cost function for the right hearing instrument in (3.7) is equal to

$$J_{SDW,1}(\mathbf{W}_1) = P_1 + \mathbf{W}_1^H(\mathbf{R}_x + \mu_1\mathbf{R}_v)\mathbf{W}_1 - \mathbf{W}_1^H\mathbf{r}_{x1} - \mathbf{r}_{x1}^H\mathbf{W}_1\ , \tag{3.12}$$

with

$$\mathbf{r}_{x1} = \mathcal{E}\{\mathbf{X}X_{1,r_1}^*\} \qquad P_1 = \mathcal{E}\{|X_{1,r_1}|^2\}\ . \tag{3.13}$$

Using (2.9), the total SDW cost function in (3.8) can be written as

$$\boxed{J_{SDW}(\mathbf{W}) = P + \mathbf{W}^H\mathbf{R}\mathbf{W} - \mathbf{W}^H\mathbf{r} - \mathbf{r}^H\mathbf{W}} \tag{3.14}$$

with

$$P = P_0 + \alpha P_1 \tag{3.15}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_x + \mu_0\mathbf{R}_v & \mathbf{0}_M \\ \mathbf{0}_M & \alpha\,(\mathbf{R}_x + \mu_1\mathbf{R}_v) \end{bmatrix} \tag{3.16}$$

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_{x0} \\ \alpha\,\mathbf{r}_{x1} \end{bmatrix}\ . \tag{3.17}$$

Note that the $2M \times 2M$-dimensional complex matrix $\mathbf{R}$ is Hermitian and positive definite. The gradient and the Hessian of $J_{SDW}(\mathbf{W})$ are equal to

$$\frac{\partial J_{SDW}(\mathbf{W})}{\partial \mathbf{W}} = 2\mathbf{R}\mathbf{W} - 2\mathbf{r}, \quad \frac{\partial^2 J_{SDW}(\mathbf{W})}{\partial^2 \mathbf{W}} = 2\mathbf{R}\ . \tag{3.18}$$

By setting the gradient equal to $\mathbf{0}$, the optimal filter minimising $J_{SDW}(\mathbf{W})$ can be computed as

$$\boxed{\mathbf{W}_{SDW} = \mathbf{R}^{-1}\mathbf{r}} \tag{3.19}$$

such that the optimal filters for the left and right hearing instruments are equal to

$$\mathbf{W}_{SDW,0} = (\mathbf{R}_x + \mu_0\mathbf{R}_v)^{-1}\mathbf{r}_{x0}, \quad \mathbf{W}_{SDW,1} = (\mathbf{R}_x + \mu_1\mathbf{R}_v)^{-1}\mathbf{r}_{x1}\ . \tag{3.20}$$

Using (2.11), the cost function in (3.14) can be written as

$$\boxed{J_{SDW}(\widetilde{\mathbf{W}}) = P + \widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}\widetilde{\mathbf{W}} - 2\widetilde{\mathbf{W}}^T\tilde{\mathbf{r}}} \tag{3.21}$$

with

$$\widetilde{\mathbf{R}} = \begin{bmatrix} \mathbf{R}_R & -\mathbf{R}_I \\ \mathbf{R}_I & \mathbf{R}_R \end{bmatrix} \tag{3.22}$$

$$\tilde{\mathbf{r}} = \begin{bmatrix} \mathbf{r}_R \\ \mathbf{r}_I \end{bmatrix}\ . \tag{3.23}$$

Note that the $4M \times 4M$-dimensional real matrix $\widetilde{\mathbf{R}}$ is symmetric and positive definite. The gradient and the Hessian of $J_{SDW}(\widetilde{\mathbf{W}})$ are equal to

$$\frac{\partial J_{SDW}(\widetilde{\mathbf{W}})}{\partial \widetilde{\mathbf{W}}} = 2\widetilde{\mathbf{R}}\widetilde{\mathbf{W}} - 2\tilde{\mathbf{r}}, \quad \frac{\partial^2 J_{SDW}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} = 2\widetilde{\mathbf{R}}\ . \tag{3.24}$$

Since the Hessian is positive definite, the cost function $J_{SDW}(\widetilde{\mathbf{W}})$ is a convex function.

4 Linearly constrained minimum variance beamformer using transfer function ratio (TF-LCMV)

4.1 Definition of constrained optimisation problem

In [25], a linearly constrained minimum variance (LCMV) beamforming algorithm was derived under the assumption that the transfer function (TF) between the signal source and each microphone consists of only gain and delay values, i.e. no reverberation present. In [44] the LCMV beamformer was extended for arbitrary TFs in a reverberant environment.

The objective of the LCMV beamformer is to minimise the total output energy under the constraint that the speech component in the output signal is equal to a filtered version (usually a delayed version) of the speech signal S. Hence, the filter $\mathbf{W}_0$ generating the output signal Z_0 at the left hearing aid can be computed by minimising the minimum variance cost funcion

$$J_{MV,0}(\mathbf{W}_0) = \mathcal{E}\{|Z_0|^2\} = \mathbf{W}_0^H\mathbf{R}_y\mathbf{W}_0\ , \tag{4.1}$$

subject to the constraint

$$Z_{x0} = \mathbf{W}_0^H\mathbf{X} = F_0^*S\ , \tag{4.2}$$

with F_0 a prespecified filter. Using (2.2), this is equivalent to the linear constraint

$$\mathbf{W}_0^H \mathbf{A} = F_0^* \ . \tag{4.3}$$

In order to solve this constrained optimisation problem, the TF vector $\mathbf{A}$ needs to be known. Accurately estimating the acoustic transfer functions is quite a difficult task, especially when background noise is present [45, 46]. However, in [44] a procedure has been presented for estimating the acoustic transfer function ratio vector

$$\mathbf{H}_0 = \frac{\mathbf{A}}{A_{0,r_0}} \ , \tag{4.4}$$

exploiting the non-stationarity of the speech signal, and assuming that both the TFs and the noise signal are stationary during some analysis interval. When the speech component in the output signal is now constrained to be equal to a filtered version of the speech component $X_{0,r_0} = A_{0,r_0} S$ in the reference microphone signal (instead of the speech signal S), the constrained optimisation problem (TF-LCMV) becomes

$$\min_{\mathbf{W}_0} J_{MV,0}(\mathbf{W}_0) = \mathbf{W}_0^H \mathbf{R}_y \mathbf{W}_0, \quad \text{subject to } \mathbf{W}_0^H \mathbf{H}_0 = F_0^* \ . \tag{4.5}$$

Similarly, the filter $\mathbf{W}_1$ generating the output signal Z_1 at the right hearing aid is the solution of the constrained optimisation problem

$$\min_{\mathbf{W}_1} J_{MV,1}(\mathbf{W}_1) = \mathbf{W}_1^H \mathbf{R}_y \mathbf{W}_1, \quad \text{subject to } \mathbf{W}_1^H \mathbf{H}_1 = F_1^* \ , \tag{4.6}$$

with the TF ratio vector for the right hearing instrument equal to

$$\mathbf{H}_1 = \frac{\mathbf{A}}{A_{1,r_1}} \ . \tag{4.7}$$

Hence, the total constrained optimisation problem comes down to minimising

$$\boxed{J_{MV}(\mathbf{W}) = J_{MV,0}(\mathbf{W}_0) + \alpha J_{MV,1}(\mathbf{W}_1)} \tag{4.8}$$

subject to the linear constraints

$$\boxed{\mathbf{W}_0^H \mathbf{H}_0 = F_0^* \text{ and } \mathbf{W}_1^H \mathbf{H}_1 = F_1^*} \tag{4.9}$$

where α trades off the MV cost functions in the left and the right hearing aid. However, since both terms in $J_{MV}(\mathbf{W})$ are independent of each other, this factor has no influence on the computation of the optimal filter $\mathbf{W}_{MV}$.

4.2 Computation of total cost function and solution

Using (2.9), the total cost function $J_{MV}(\mathbf{W})$ in (4.8) can be written as

$$\boxed{J_{MV}(\mathbf{W}) = \mathbf{W}^H \mathbf{R}_t \mathbf{W}} \tag{4.10}$$

with the $2M \times 2M$-dimensional complex matrix $\mathbf{R}_t$ equal to

$$\mathbf{R}_t = \begin{bmatrix} \mathbf{R}_y & \mathbf{0}_M \\ \mathbf{0}_M & \alpha \mathbf{R}_y \end{bmatrix} . \tag{4.11}$$

Using (2.9), the two linear constraints in (4.9) can be written as

$$\boxed{\mathbf{W}^H \mathbf{H} = \mathbf{F}^H} \tag{4.12}$$

with the $2M \times 2$-dimensional matrix $\mathbf{H}$ equal to

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{0}_{M\times 1} \\ \mathbf{0}_{M\times 1} & \mathbf{H}_1 \end{bmatrix} , \tag{4.13}$$

and the 2-dimensional vector $\mathbf{F}$ equal to

$$\mathbf{F} = \begin{bmatrix} F_0 \\ F_1 \end{bmatrix} . \tag{4.14}$$

It can be easily shown that the solution of the constrained optimisation problem (4.10) and (4.12) is equal to

$$\boxed{\mathbf{W}_{MV} = \mathbf{R}_t^{-1} \mathbf{H} \left[\mathbf{H}^H \mathbf{R}_t^{-1} \mathbf{H} \right]^{-1} \mathbf{F}} \tag{4.15}$$

such that

$$\mathbf{W}_{MV,0} = \frac{\mathbf{R}_y^{-1} \mathbf{H}_0 F_0}{\mathbf{H}_0^H \mathbf{R}_y^{-1} \mathbf{H}_0} \qquad \mathbf{W}_{MV,1} = \frac{\mathbf{R}_y^{-1} \mathbf{H}_1 F_1}{\mathbf{H}_1^H \mathbf{R}_y^{-1} \mathbf{H}_1} . \tag{4.16}$$

4.3 Relationship between SDW-MWF and TF-LCMV

Assuming that one desired signal source is present, i.e. $\mathbf{X} = \mathbf{A}S$, the correlation matrix $\mathbf{R}_x$ is equal to

$$\mathbf{R}_x = P_s \mathbf{A} \mathbf{A}^H , \tag{4.17}$$

with $P_s = \mathcal{E}\{|S|^2\}$. It can be easily shown, using the matrix inversion lemma, that

$$(\mathbf{R}_x + \mu_0 \mathbf{R}_v)^{-1} = \frac{1}{\mu_0} \left[\mathbf{R}_v^{-1} - \frac{P_s \mathbf{R}_v^{-1} \mathbf{A} \mathbf{A}^H \mathbf{R}_v^{-1}}{\mu_0 + P_s \mathbf{A}^H \mathbf{R}_v^{-1} \mathbf{A}} \right] , \tag{4.18}$$

and

$$(\mathbf{R}_x + \mu_0 \mathbf{R}_v)^{-1} \mathbf{A} = \frac{\mathbf{R}_v^{-1} \mathbf{A}}{\mu_0 + P_s \mathbf{A}^H \mathbf{R}_v^{-1} \mathbf{A}} . \tag{4.19}$$

Hence, the filter $\mathbf{W}_{SDW,0}$ in (3.20) is equal to

$$\mathbf{W}_{SDW,0} = (\mathbf{R}_x + \mu_0 \mathbf{R}_v)^{-1} \mathbf{r}_{x0} = (\mathbf{R}_x + \mu_0 \mathbf{R}_v)^{-1} P_s \mathbf{A} A_{0,r_0}^* \tag{4.20}$$

$$= \frac{P_s \mathbf{R}_v^{-1} \mathbf{A} A_{0,r_0}^*}{\mu_0 + P_s \mathbf{A}^H \mathbf{R}_v^{-1} \mathbf{A}} , \tag{4.21}$$

and the filter $\mathbf{W}_{MV,0}$ in (4.16), assuming $F_0 = 1$, is equal to

$$\mathbf{W}_{MV,0} = \frac{\mathbf{R}_y^{-1}\mathbf{H}_0}{\mathbf{H}_0^H\mathbf{R}_y^{-1}\mathbf{H}_0} = \frac{\mathbf{R}_y^{-1}\mathbf{A}A_{0,r_0}^*}{\mathbf{A}^H\mathbf{R}_y^{-1}\mathbf{A}} \tag{4.22}$$

$$= \frac{\mathbf{R}_v^{-1}\mathbf{A}A_{0,r_0}^*}{\mathbf{A}^H\mathbf{R}_v^{-1}\mathbf{A}} \ . \tag{4.23}$$

Using (4.21) and (4.23), it can be easily shown that

$$\boxed{\mathbf{W}_{SDW,0} = \underbrace{\frac{P_s}{P_s + \mu_0\left[\mathbf{A}^H\mathbf{R}_v^{-1}\mathbf{A}\right]^{-1}}}_{G_{post}}\mathbf{W}_{MV,0}} \tag{4.24}$$

such that the SDW-MWF is equivalent with the TF-LCMV combined with a single-channel postfilter G_{post}, which is dependent on the signal-to-noise ratio, the spatial separation between the speech and the noise sources and the factor μ_0. This is a similar result as already derived in [40].
Note that when μ_0 approaches 0, both filters apparently become equal. However, this seems to contradict the fact that for $\mu_0 = 0$, the filter $\mathbf{W}_{SDW,0}$ is a vector consisting of zeros, with the r_0th element equal to 1, hence not being equal to $\mathbf{W}_{MV,0}$?? The similarity between both filters is also higher for high SNR and when the spatial separation between the speech and the noise sources is good, i.e. high $\mathbf{A}^H\mathbf{R}_v^{-1}\mathbf{A}$.

4.4 From constrained to unconstrained optimisation problem

It is well known that the constrained optimisation problem (4.10) and (4.12) can be transformed into an unconstrained optimisation problem [44, 47]. The filters $\mathbf{W}_0$ and $\mathbf{W}_1$ can be parametrised as

$$\mathbf{W}_0 = \mathbf{H}_0 V_0 - \mathbf{H}_{a0}\mathbf{W}_{a0} \tag{4.25}$$

$$\mathbf{W}_1 = \mathbf{H}_1 V_1 - \mathbf{H}_{a1}\mathbf{W}_{a1} \ , \tag{4.26}$$

with the *blocking matrices* $\mathbf{H}_{a0}$ and $\mathbf{H}_{a1}$ equal to the $M \times (M-1)$-dimensional null-spaces of $\mathbf{H}_0$ and $\mathbf{H}_1$, and $\mathbf{W}_{a0}$ and $\mathbf{W}_{a1}$ $(M-1)$-dimensional filter vectors. Assuming that $r_0 = 0$, a possible choice for the blocking matrix $\mathbf{H}_{a0}$ is [44]

$$\mathbf{H}_{a0} = \begin{bmatrix} -\frac{A_1^*}{A_0^*} & -\frac{A_2^*}{A_0^*} & \dots & -\frac{A_{M-1}^*}{A_0^*} \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} . \tag{4.27}$$

By applying the constraints (4.9) and using the fact that $\mathbf{H}_{a0}^H\mathbf{H}_0 = \mathbf{0}$ and $\mathbf{H}_{a1}^H\mathbf{H}_1 = \mathbf{0}$, we find that

$$\mathbf{V}_0^*\mathbf{H}_0^H\mathbf{H}_0 = F_0^*, \quad \mathbf{V}_1^*\mathbf{H}_1^H\mathbf{H}_1 = F_1^* \ , \tag{4.28}$$

such that

$$\mathbf{W}_0 = \mathbf{W}_{q0} - \mathbf{H}_{a0}\mathbf{W}_{a0} \tag{4.29}$$

$$\mathbf{W}_1 = \mathbf{W}_{q1} - \mathbf{H}_{a1}\mathbf{W}_{a1} \ , \tag{4.30}$$

with the *fixed beamformers* (quiescent responses) $\mathbf{W}_{q0}$ and $\mathbf{W}_{q1}$ equal to

$$\mathbf{W}_{q0} = \frac{\mathbf{H}_0 \, F_0}{\mathbf{H}_0^H \mathbf{H}_0}, \quad \mathbf{W}_{q1} = \frac{\mathbf{H}_1 \, F_1}{\mathbf{H}_1^H \mathbf{H}_1} \ . \tag{4.31}$$

The binaural TF-LCMV beamformer structure, using this parametrisation is depicted in Fig. 2. The constrained optimisation of the M-dimensional filters $\mathbf{W}_0$ and $\mathbf{W}_1$ now has been transformed into the unconstrained optimisation of the $(M-1)$-dimensional filters $\mathbf{W}_{a0}$ and $\mathbf{W}_{a1}$. The microphone signals filtered by the fixed beamformers

$$U_0 = \mathbf{W}_{q0}^H \mathbf{Y}, \quad U_1 = \mathbf{W}_{q1}^H \mathbf{Y} \ , \tag{4.32}$$

will be referred to as *speech references*, whereas the signals filtered by the blocking matrices

$$\mathbf{U}_{a0} = \mathbf{H}_{a0}^H \mathbf{Y}, \quad \mathbf{U}_{a1} = \mathbf{H}_{a1}^H \mathbf{Y} \ , \tag{4.33}$$

will be referred to as *noise references*.
Using the filter parametrisation in (4.29) and (4.30), the filter $\mathbf{W}$ can be written as

$$\boxed{\mathbf{W} = \mathbf{W}_q - \mathbf{H}_a \mathbf{W}_a} \tag{4.34}$$

with the $2M$-dimensional vector $\mathbf{W}_q$ equal to

$$\mathbf{W}_q = \begin{bmatrix} \mathbf{W}_{q0} \\ \mathbf{W}_{q1} \end{bmatrix} \ , \tag{4.35}$$

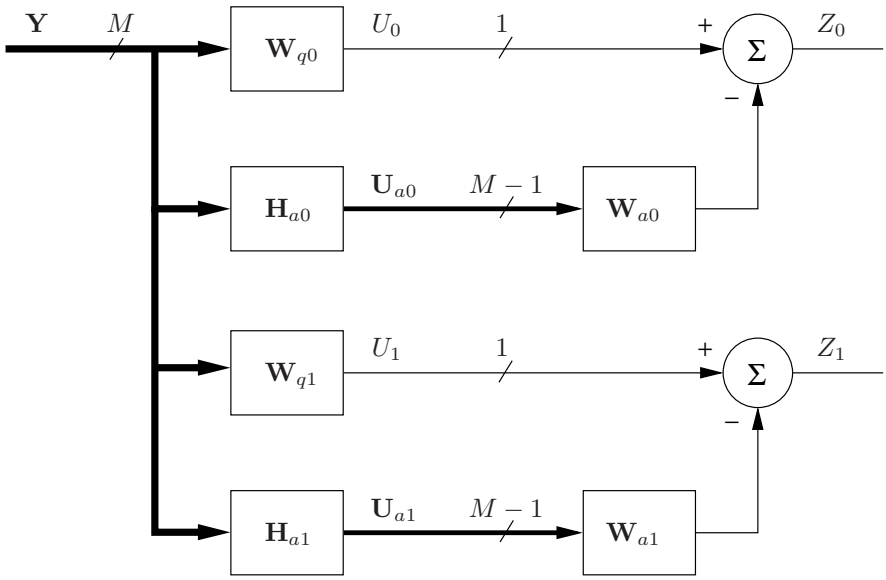


Figure 2: Binaural TF-LCMV beamformer structure

the $2(M-1)$-dimensional filter $\mathbf{W}_a$ equal to

$$\mathbf{W}_a = \begin{bmatrix} \mathbf{W}_{a0} \\ \mathbf{W}_{a1} \end{bmatrix} , \tag{4.36}$$

and the $2M \times 2(M-1)$-dimensional blocking matrix $\mathbf{H}_a$ equal to

$$\mathbf{H}_a = \begin{bmatrix} \mathbf{H}_{a0} & \mathbf{0}_{M\times(M-1)} \\ \mathbf{0}_{M\times(M-1)} & \mathbf{H}_{a1} \end{bmatrix} . \tag{4.37}$$

The unconstrained optimisation problem for the filter $\mathbf{W}_a$ then is equal to

$$\boxed{J_{MV}(\mathbf{W}_a) = (\mathbf{W}_q - \mathbf{H}_a\mathbf{W}_a)^H\mathbf{R}_t(\mathbf{W}_q - \mathbf{H}_a\mathbf{W}_a)} \tag{4.38}$$

such that the filter minimising $J_{MV}(\mathbf{W}_a)$ is equal to

$$\boxed{\mathbf{W}_{MV,a} = (\mathbf{H}_a^H\mathbf{R}_t\mathbf{H}_a)^{-1}\mathbf{H}_a^H\mathbf{R}_t\mathbf{W}_q} \tag{4.39}$$

and

$$\mathbf{W}_{MV,a0} = (\mathbf{H}_{a0}^H\mathbf{R}_y\mathbf{H}_{a0})^{-1}\mathbf{H}_{a0}^H\mathbf{R}_y\mathbf{W}_{q0} \tag{4.40}$$
$$\mathbf{W}_{MV,a1} = (\mathbf{H}_{a1}^H\mathbf{R}_y\mathbf{H}_{a1})^{-1}\mathbf{H}_{a1}^H\mathbf{R}_y\mathbf{W}_{q1} . \tag{4.41}$$

Note that these filters obviously also minimise the unconstrained cost function

$$J_{MV}(\mathbf{W}_{a0}, \mathbf{W}_{a1}) = \mathcal{E}\{|U_0 - \mathbf{W}_{a0}^H\mathbf{U}_{a0}|^2\} + \alpha\mathcal{E}\{|U_1 - \mathbf{W}_{a1}^H\mathbf{U}_{a1}|^2\} , \tag{4.42}$$

and the filters $\mathbf{W}_{MV,a0}$ and $\mathbf{W}_{MV,a1}$ can also be written as

$$\mathbf{W}_{MV,a0} = \mathcal{E}\{\mathbf{U}_{a0}\mathbf{U}_{a0}^H\}^{-1}\mathcal{E}\{\mathbf{U}_{a0}^H U_0^*\} \tag{4.43}$$
$$\mathbf{W}_{MV,a1} = \mathcal{E}\{\mathbf{U}_{a1}\mathbf{U}_{a1}^H\}^{-1}\mathcal{E}\{\mathbf{U}_{a1}^H U_1^*\} . \tag{4.44}$$

Assuming that one desired signal source is present, as in Section 4.3, it can be easily shown that

$$\mathbf{H}_{a0}^H\mathbf{R}_y = \mathbf{H}_{a0}^H(P_s|A_{0,r_0}|^2\mathbf{H}_0\mathbf{H}_0^H + \mathbf{R}_v) = \mathbf{H}_{a0}^H\mathbf{R}_v , \tag{4.45}$$

and similarly, $\mathbf{H}_{a1}^H\mathbf{R}_y = \mathbf{H}_{a1}^H\mathbf{R}_v$. In other words, the blocking matrices $\mathbf{H}_{a0}$ and $\mathbf{H}_{a1}$ cancel all speech components, such that the noise references only contain noise components. Hence, the optimal filters can also be written as

$$\mathbf{W}_{MV,a0} = (\mathbf{H}_{a0}^H\mathbf{R}_v\mathbf{H}_{a0})^{-1}\mathbf{H}_{a0}^H\mathbf{R}_v\mathbf{W}_{q0} \tag{4.46}$$
$$\mathbf{W}_{MV,a1} = (\mathbf{H}_{a1}^H\mathbf{R}_v\mathbf{H}_{a1})^{-1}\mathbf{H}_{a1}^H\mathbf{R}_v\mathbf{W}_{q1} . \tag{4.47}$$

5 Preservation of binaural cues

Since the SDW-MWF produces an optimal estimate of the speech component in the reference microphones for both hearing instruments, the binaural cues, i.e. the interaural time difference (ITD) and the interaural level difference (ILD), of the

speech are generally well preserved [23]. On the contrary, the binaural cues of the noise are generally not preserved. In addition to reducing the noise level, it is also important to (partially) preserve these binaural noise cues in order to exploit the binaural hearing advantage of normal hearing and hearing impaired persons [26, 27, 28, 29, 30, 31, 32, 33, 34] or in order to further process the binaural output signals with a speech enhancement procedure based on a difference in binaural speech and noise cues [36, 38, 39].

5.1 Partial estimation of the noise component

An extension of the MWF that partly preserves the binaural cues of the noise component has been proposed in [24], where the objective of the MWF is to produce an MMSE estimate of the speech component and part of the noise component in one of the microphone signals, i.e. the cost function for the left hearing instrument becomes

$$\begin{aligned} \bar{J}_{MSE,0}(\mathbf{W}_0) &= \mathcal{E}\{|(X_{0,r_0} + \lambda_0 V_{0,r_0}) - \mathbf{W}_0^H \mathbf{Y}|^2\} && (5.1) \\ &= \mathcal{E}\{|X_{0,r_0} - \mathbf{W}_0^H \mathbf{X}|^2\} + \mathcal{E}\{|\lambda_0 V_{0,r_0} - \mathbf{W}_0^H \mathbf{V}|^2\} \ . && (5.2) \end{aligned}$$

with $0 \leq \lambda_0 \leq 1$. When $\lambda_0 = 0$, this cost function reduces to $J_{MSE,0}(\mathbf{W}_0)$ in (3.2). When $\lambda_0 = 1$, the optimal filter $\mathbf{W}_{MSE,0}$ is obviously equal to a vector consisting of zeros, with the r_0th element equal to 1. This MWF is in fact an extension of single-channel Wiener filters that have been presented in [42, 48].
Similarly to the SDW-MWF in (3.6), it is also possible to trade-off noise reduction and speech distortion, i.e.

$$\begin{aligned} \bar{J}_{SDW,0}(\mathbf{W}_0) &= \mathcal{E}\{|X_{0,r_0} - \mathbf{W}_0^H \mathbf{X}|^2\} + \mu_0 \mathcal{E}\{|\lambda_0 V_{0,r_0} - \mathbf{W}_0^H \mathbf{V}|^2\} && (5.3) \\ &= P_0 + \mathbf{W}_0^H (\mathbf{R}_x + \mu_0 \mathbf{R}_v) \mathbf{W}_0 - \mathbf{W}_0^H (\mathbf{r}_{x0} + \mu_0 \lambda_0 \mathbf{r}_{v0}) \\ &\quad -(\mathbf{r}_{x0} + \mu_0 \lambda_0 \mathbf{r}_{v0})^H \mathbf{W}_0 \ , && (5.4) \end{aligned}$$

such that the optimal filter is equal to

$$\mathbf{W}_{SDW,0} = (\mathbf{R}_x + \mu_0 \mathbf{R}_v)^{-1} (\mathbf{r}_{x0} + \mu_0 \lambda_0 \mathbf{r}_{v0}) \ . \tag{5.5}$$

Similar expressions can be derived for the right hearing instrument.

5.2 Extension of SDW-MWF with binaural cues

In this report we present a different way to preserve the binaural noise cues by adding a term to the SDW cost function that is related to the ITD cue and the ILD cue of the noise component, linking the computation of the filters $\mathbf{W}_0$ and $\mathbf{W}_1$. The total cost function can then be expressed as

$$J_{tot}(\mathbf{W}) = J_{SDW}(\mathbf{W}) + \beta \underbrace{|ITD_{out}(\mathbf{W}) - ITD_{in}|^2}_{J_{ITD}(\mathbf{W})} + \gamma \underbrace{|ILD_{out}(\mathbf{W}) - ILD_{in}|^2}_{J_{ILD}(\mathbf{W})} \tag{5.6}$$

where β and γ are weighting factors[4]. The main challenge is to come up with a perceptually relevant mathematical expression for these binaural cues.

5.2.1 Interaural time difference (ITD)

This section discusses the cost function related to the ITD of the noise, i.e.

$$J_{ITD}(\mathbf{W}) = |ITD_{out}(\mathbf{W}) - ITD_{in}|^2 \ . \tag{5.7}$$

We will assume that the ITD can be expressed using (the phase of) the cross-correlation between two signals. The input cross-correlation between the noise components in the reference microphones is equal to

$$s = \mathcal{E}\{V_{0,r_0} V_{1,r_1}^*\} = \mathbf{R}_v(r_0, r_1) \ . \tag{5.8}$$

We assume that the input cross-correlation of the noise components is known, e.g. through measurement during noise-only periods. Similarly, the output cross-correlation between the noise components in the output signals is equal to

$$\mathcal{E}\{Z_{v0} Z_{v1}^*\} = \mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1 \ . \tag{5.9}$$

Two possibilities now arise for expressing the cost function in (5.7):

1. Using the difference between the input and the output cross-correlation, the cost function is equal to

$$\boxed{J_{ITD,1}(\mathbf{W}) = |\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1 - s|^2} \tag{5.10}$$

2. Using the difference between the tangent of the phase of the input and the output cross-correlation, the cost function is equal to

$$\boxed{J_{ITD,2}(\mathbf{W}) = \left[\frac{(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_I}{(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_R} - \frac{s_I}{s_R}\right]^2 = \frac{\left[(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_I - \frac{s_I}{s_R}(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_R\right]^2}{(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_R^2}} \tag{5.11}$$

   Note that this cost function is scale-independent, i.e. $J_{ITD,2}(\lambda_0 \mathbf{W}_0, \lambda_1 \mathbf{W}_1) = J_{ITD,2}(\mathbf{W}_0, \mathbf{W}_1)$, $\lambda_0, \lambda_1 \in \mathbb{R}$.

3. For mathematical convenience (calculation of gradient and Hessian, convex cost function), we can also use only the denominator of (5.11) as cost function, i.e.

$$\boxed{J_{ITD,3}(\mathbf{W}) = \left[(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_I - \frac{s_I}{s_R}(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_1)_R\right]^2} \tag{5.12}$$

[4]These weighting factors should probably be frequency-dependent, since it is well known that e.g for sound localisation the ITD cue is more important for low frequencies, whereas the ILD cue is more important for high frequencies [49, 50, 51].

4. However, when using the tangent of the phase of the cross-correlation, a phase difference of 180° between the input and the output cross-correlation also minimises $J_{ITD,2}(\mathbf{W})$ and $J_{ITD,3}(\mathbf{W})$, which is actually not desired. A better cost function can be constructed using the cosine of the phase difference $\phi(\mathbf{W})$ between the input and the output correlation, i.e.

$$\boxed{J_{ITD,4}(\mathbf{W}) = 1 - \cos\big(\phi(\mathbf{W})\big) = 1 - \frac{s_R\,(\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1)_R + s_I\,(\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1)_I}{\sqrt{s_R^2+s_I^2}\sqrt{(\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1)_R^2 + (\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1)_I^2}}} \tag{5.13}$$

Note: Instead of using the input cross-correlation in (5.8) as the desired output cross-correlation, it is also possible to use other values. If the output noise component should be perceived as coming from the direction θ, where $\theta = 0°$ represents the direction in front of the head, then the desired output cross-correlation (for free-field conditions) is equal to

$$s(\omega) = e^{-j\tau(\theta)\omega}\,, \tag{5.14}$$

with

$$\tau(\theta) = \frac{d\sin\theta}{c} f_s\,, \tag{5.15}$$

with d the distance between the two ears, $c \approx 340\,\mathrm{m/s}$ the speed of sound, and f_s the sampling frequency. In order to improve the perception, it would actually be better to use the head-related transfer functions (HRTF), i.e.

$$s(\omega) = HRTF_0(\omega,\theta)\,HRTF_1^*(\omega,\theta)\,. \tag{5.16}$$

Another possibility is to use

$$s = 0\,, \tag{5.17}$$

corresponding to uncorrelated output noise components.

Using (2.9), the output cross-correlation in (5.9) is equal to

$$\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1 = \mathbf{W}^H\bar{\mathbf{R}}_v^{01}\mathbf{W}\,, \tag{5.18}$$

with

$$\bar{\mathbf{R}}_v^{01} = \begin{bmatrix} \mathbf{0}_M & \mathbf{R}_v \\ \mathbf{0}_M & \mathbf{0}_M \end{bmatrix}\,. \tag{5.19}$$

Note that the $2M \times 2M$-dimensional complex matrix $\bar{\mathbf{R}}_v^{01}$ is neither Hermitian nor positive definite. Using (2.11), the real and the imaginary part of the output cross-correlation can be written as

$$(\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1)_R = \widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}} \tag{5.20}$$

$$(\mathbf{W}_0^H\mathbf{R}_v\mathbf{W}_1)_I = \widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}}\,, \tag{5.21}$$

with

$$\widetilde{\mathbf{R}}_{v1} = \begin{bmatrix} \bar{\mathbf{R}}_{v,R}^{01} & -\bar{\mathbf{R}}_{v,I}^{01} \\ \bar{\mathbf{R}}_{v,I}^{01} & \bar{\mathbf{R}}_{v,R}^{01} \end{bmatrix}, \quad \widetilde{\mathbf{R}}_{v2} = \begin{bmatrix} \bar{\mathbf{R}}_{v,I}^{01} & \bar{\mathbf{R}}_{v,R}^{01} \\ -\bar{\mathbf{R}}_{v,R}^{01} & \bar{\mathbf{R}}_{v,I}^{01} \end{bmatrix}\,. \tag{5.22}$$

Note that the $4M \times 4M$-dimensional real matrices $\widetilde{\mathbf{R}}_{v1}$ and $\widetilde{\mathbf{R}}_{v2}$ are neither symmetric nor positive definite.

We will now discuss the different ITD cost functions:

1. Using (5.20) and (5.21), the cost function $J_{ITD,1}$ in (5.10) is equal to

$$\boxed{\begin{array}{rcl} J_{ITD,1}(\widetilde{\mathbf{W}}) & = & \Big[\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}} - s_R\Big]^2 + \Big[\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}} - s_I\Big]^2 \\ & = & (s_R^2 + s_I^2) - 2\widetilde{\mathbf{W}}^T\big(s_R\widetilde{\mathbf{R}}_{v1} + s_I\widetilde{\mathbf{R}}_{v2}\big)\widetilde{\mathbf{W}} + \\ & & \big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}}\big)^2 \end{array}} \tag{5.23}$$

   The gradient and the Hessian of $J_{ITD,1}$ are equal to

$$\begin{aligned} \frac{\partial J_{ITD,1}(\widetilde{\mathbf{W}})}{\partial \widetilde{\mathbf{W}}} &= 2\Big[\big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}} - s_R\big)\big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big)\widetilde{\mathbf{W}} \\ &\quad + \big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}} - s_I\big)\big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big)\widetilde{\mathbf{W}}\Big] \qquad (5.24) \\ \frac{\partial^2 J_{ITD,1}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} &= 2\Big[\big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}} - s_R\big)\big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big) \\ &\quad + \big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}} - s_I\big)\big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big) \\ &\quad + \big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T\big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big) \\ &\quad + \big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T\big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big)\Big] \ . \qquad (5.25) \end{aligned}$$

   Since

$$\begin{aligned} \widetilde{\mathbf{W}}^T\frac{\partial^2 J_{ITD,1}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}}\widetilde{\mathbf{W}} &= 12\big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}}\big)^2 + 12\big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}}\big)^2 \\ &\quad -4\big(s_R\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}} + s_I\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v2}\widetilde{\mathbf{W}}\big) \qquad (5.26) \end{aligned}$$

   can not be guaranteed to be positive for all $\widetilde{\mathbf{W}}$, the cost function $J_{ITD,1}$ is not convex, such that algorithms for minimising this cost function may suffer from local minima.

2. Using (5.20) and (5.21), the cost function $J_{ITD,2}$ in (5.11) is equal to

$$\boxed{J_{ITD,2}(\widetilde{\mathbf{W}}) = \frac{\big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}}\big)^2}{\big(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}}\big)^2}} \tag{5.27}$$

   with

$$\widetilde{\mathbf{R}}_{vd} = \widetilde{\mathbf{R}}_{v2} - \frac{s_I}{s_R}\widetilde{\mathbf{R}}_{v1} = \begin{bmatrix} \bar{\mathbf{R}}_{v,I}^{01} - \frac{s_I}{s_R}\bar{\mathbf{R}}_{v,R}^{01} & \bar{\mathbf{R}}_{v,R}^{01} + \frac{s_I}{s_R}\bar{\mathbf{R}}_{v,I}^{01} \\ -\bar{\mathbf{R}}_{v,R}^{01} - \frac{s_I}{s_R}\bar{\mathbf{R}}_{v,I}^{01} & \bar{\mathbf{R}}_{v,I}^{01} - \frac{s_I}{s_R}\bar{\mathbf{R}}_{v,R}^{01} \end{bmatrix} . \tag{5.28}$$

Note that the $4M \times 4M$-dimensional real matrix $\widetilde{\mathbf{R}}_{vd}$ is neither symmetric nor positive definite. The gradient and the Hessian of $J_{ITD,2}$ are equal to

$$\begin{aligned}\frac{\partial J_{ITD,2}(\widetilde{\mathbf{W}})}{\partial \widetilde{\mathbf{W}}} &= 2\frac{(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})}{(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}})^3}\Big[(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}})(\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T)\widetilde{\mathbf{W}} \\ &\quad - (\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})(\widetilde{\mathbf{R}}_{v1}+\widetilde{\mathbf{R}}_{v1}^T)\widetilde{\mathbf{W}}\Big] \qquad (5.29)\\ \frac{\partial^2 J_{ITD,2}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} &= \frac{2}{(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}})^4}\Big\{(\widetilde{\mathbf{R}}_{H,2}\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{H,2}) \\ &\quad +(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}})^2(\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T) \\ &\quad -(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}})(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})^2(\widetilde{\mathbf{R}}_{v1}+\widetilde{\mathbf{R}}_{v1}^T) \\ &\quad -(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})^2(\widetilde{\mathbf{R}}_{v1}+\widetilde{\mathbf{R}}_{v1}^T)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T(\widetilde{\mathbf{R}}_{v1}+\widetilde{\mathbf{R}}_{v1}^T)\Big\}\,, \qquad (5.30)\end{aligned}$$

with

$$\widetilde{\mathbf{R}}_{H,2} = (\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{v1}\widetilde{\mathbf{W}})(\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T) - 2(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})(\widetilde{\mathbf{R}}_{v1}+\widetilde{\mathbf{R}}_{v1}^T)\,. \qquad (5.31)$$

It can be easily shown that for every $\widetilde{\mathbf{W}}$,

$$\widetilde{\mathbf{W}}^T\frac{\partial^2 J_{ITD,2}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}}\widetilde{\mathbf{W}} = 0\,. \qquad (5.32)$$

3. The cost function $J_{ITD,3}$ in (5.12) is equal to

$$\boxed{J_{ITD,3}(\widetilde{\mathbf{W}}) = (\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})^2} \qquad (5.33)$$

The gradient of $J_{ITD,3}$ is equal to

$$\frac{\partial J_{ITD,3}(\widetilde{\mathbf{W}})}{\partial \widetilde{\mathbf{W}}} = 2(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})(\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T)\widetilde{\mathbf{W}}\,. \qquad (5.34)$$

Hence all (local) minima $\widetilde{\mathbf{W}}_{ITD,3}$ of $J_{ITD,3}$ either satisfy

$$\widetilde{\mathbf{W}}_{ITD,3}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}}_{ITD,3} = 0 \quad \text{or} \quad (\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T)\widetilde{\mathbf{W}}_{ITD,3} = \mathbf{0}\,, \qquad (5.35)$$

such that it can be easily show that for all minima

$$J_{ITD,3}(\widetilde{\mathbf{W}}_{ITD,3}) = 0\,, \qquad (5.36)$$

i.e. all local minima are also global minima. The Hessian of $J_{ITD,3}$ is equal to

$$\begin{aligned}\frac{\partial^2 J_{ITD,3}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} &= 2\Big[(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{R}}_{vd}\widetilde{\mathbf{W}})(\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T)+ \\ &\quad (\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T(\widetilde{\mathbf{R}}_{vd}+\widetilde{\mathbf{R}}_{vd}^T)\Big]\,. \qquad (5.37)\end{aligned}$$

Since

$$\widetilde{\mathbf{W}}^T \frac{\partial^2 J_{ITD,3}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} \widetilde{\mathbf{W}} = 12\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{vd} \widetilde{\mathbf{W}}\big)^2 = 12 J_{ITD,3}(\widetilde{\mathbf{W}}) \tag{5.38}$$

is positive for all $\widetilde{\mathbf{W}}$ (and equal to zero for all minima), the cost function $J_{ITD,3}$ is convex.

4. The cost function $J_{ITD,4}$ in (5.13) is equal to

$$\boxed{J_{ITD,4}(\widetilde{\mathbf{W}}) = 1 - \frac{\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{vs} \widetilde{\mathbf{W}}}{\sqrt{\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2}}} \tag{5.39}$$

with

$$\widetilde{\mathbf{R}}_{vs} = \frac{s_R \widetilde{\mathbf{R}}_{v1} + s_I \widetilde{\mathbf{R}}_{v2}}{\sqrt{s_R^2 + s_I^2}} = \frac{1}{\sqrt{s_R^2 + s_I^2}} \begin{bmatrix} s_R \bar{\mathbf{R}}^{01}_{v,R} + s_I \bar{\mathbf{R}}^{01}_{v,I} & -s_R \bar{\mathbf{R}}^{01}_{v,I} + s_I \bar{\mathbf{R}}^{01}_{v,R} \\ s_R \bar{\mathbf{R}}^{01}_{v,I} - s_I \bar{\mathbf{R}}^{01}_{v,R} & s_R \bar{\mathbf{R}}^{01}_{v,R} + s_I \bar{\mathbf{R}}^{01}_{v,I} \end{bmatrix} . \tag{5.40}$$

The gradient of $J_{ITD,4}$ is equal to

$$\begin{aligned} \frac{\partial J_{ITD,4}(\widetilde{\mathbf{W}})}{\partial \widetilde{\mathbf{W}}} = & -\frac{\big(\widetilde{\mathbf{R}}_{vs} + \widetilde{\mathbf{R}}_{vs}^T\big)\widetilde{\mathbf{W}}}{\sqrt{\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2}} \\ & + \frac{\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{vs} \widetilde{\mathbf{W}}\big)}{\Big[\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2\Big]^{\frac{3}{2}}} \widetilde{\mathbf{R}}_{H,4} \widetilde{\mathbf{W}} \, , \end{aligned} \tag{5.41}$$

with

$$\widetilde{\mathbf{R}}_{H,4} = \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)\big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big) + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)\big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big) \, . \tag{5.42}$$

The Hessian of $J_{ITD,4}$ is equal to

$$\begin{aligned} \frac{\partial^2 J_{ITD,4}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} = & -\frac{\widetilde{\mathbf{R}}_{vs} + \widetilde{\mathbf{R}}_{vs}^T}{\sqrt{\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2}} \\ & + \frac{\big(\widetilde{\mathbf{R}}_{vs} + \widetilde{\mathbf{R}}_{vs}^T\big)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{H,4} + \widetilde{\mathbf{R}}_{H,4} \widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T \big(\widetilde{\mathbf{R}}_{vs} + \widetilde{\mathbf{R}}_{vs}^T\big)}{\Big[\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2\Big]^{\frac{3}{2}}} \\ & + \frac{\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{vs} \widetilde{\mathbf{W}}\big)}{\Big[\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2\Big]^{\frac{3}{2}}} \cdot \\ & \Big[\widetilde{\mathbf{R}}_{H,4} + \big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T \big(\widetilde{\mathbf{R}}_{v1} + \widetilde{\mathbf{R}}_{v1}^T\big) \\ & + \big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big)\widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T \big(\widetilde{\mathbf{R}}_{v2} + \widetilde{\mathbf{R}}_{v2}^T\big)\Big] \\ & - 3\frac{\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{vs} \widetilde{\mathbf{W}}\big)\widetilde{\mathbf{R}}_{H,4} \widetilde{\mathbf{W}}\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{H,4}}{\Big[\big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\big)^2 + \big(\widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_{v2} \widetilde{\mathbf{W}}\big)^2\Big]^{\frac{5}{2}}} \, . \end{aligned} \tag{5.43}$$

It can be easily shown that for every $\widetilde{\mathbf{W}}$,

$$\widetilde{\mathbf{W}}^T \frac{\partial^2 J_{ITD,4}(\widetilde{\mathbf{W}})}{\partial^2 \widetilde{\mathbf{W}}} \widetilde{\mathbf{W}} = 0 \; . \tag{5.44}$$

5.2.2 Interaural level difference (ILD)

This section discusses the cost function related to the ILD of the noise, i.e.

$$J_{ILD}(\mathbf{W}) = |ILD_{out}(\mathbf{W}) - ILD_{in}|^2 \; . \tag{5.45}$$

We will assume that the ILD can be expressed as the power ratio of two signals. The input power ratio of the noise components in the reference microphones is equal to

$$\frac{\mathcal{E}\{|V_{0,r_0}|^2\}}{\mathcal{E}\{|V_{1,r_1}|^2\}} = \frac{\mathbf{R}_v(r_0, r_0)}{\mathbf{R}_v(r_1, r_1)} = \frac{P_{v0}}{P_{v1}} \; . \tag{5.46}$$

We assume that the input power ratio of the noise components is known, e.g. through measurement during noise-only periods. Similarly, the output power ratio of the noise components in the output signals is equal to

$$\frac{\mathcal{E}\{|Z_{v0}|^2\}}{\mathcal{E}\{|Z_{v1}|^2\}} = \frac{\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_0}{\mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1} \; . \tag{5.47}$$

The cost function in (5.45) can then be expressed as

$$\boxed{J_{ILD,1}(\mathbf{W}) = \left[\frac{\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_0}{\mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1} - \frac{P_{v0}}{P_{v1}} \right]^2 = \frac{\left[(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_0) - \frac{P_{v0}}{P_{v1}} (\mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1) \right]^2}{(\mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1)^2}} \tag{5.48}$$

which is quite similar to the cost function $J_{ITD,2}(\mathbf{W})$ in (5.11). For mathematical convenience, we can also use only the denominator of (5.48) as cost function, i.e.

$$\boxed{J_{ILD,2}(\mathbf{W}) = \left[(\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_0) - \frac{P_{v0}}{P_{v1}} (\mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1) \right]^2} \tag{5.49}$$

Note: Similarly to (5.16), if the output noise component should be perceived as coming from the direction θ, we can also use

$$\frac{|HRTF_0(\omega, \theta)|^2}{|HRTF_1(\omega, \theta)|^2} \tag{5.50}$$

as the desired output power ratio.

Using (2.9), the output noise powers can be written as

$$\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_0 = \mathbf{W}^H \bar{\mathbf{R}}_v^{00} \mathbf{W}, \quad \mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1 = \mathbf{W}^H \bar{\mathbf{R}}_v^{11} \mathbf{W} \; , \tag{5.51}$$

with

$$\bar{\mathbf{R}}_v^{00} = \begin{bmatrix} \mathbf{R}_v & \mathbf{0}_M \\ \mathbf{0}_M & \mathbf{0}_M \end{bmatrix}, \quad \bar{\mathbf{R}}_v^{11} = \begin{bmatrix} \mathbf{0}_M & \mathbf{0}_M \\ \mathbf{0}_M & \mathbf{R}_v \end{bmatrix} \; . \tag{5.52}$$

Note that the $2M \times 2M$-dimensional complex matrices $\bar{\mathbf{R}}_v^{00}$ and $\bar{\mathbf{R}}_v^{11}$ are Hermitian and positive definite. Using (2.11), the output noise powers can be written as

$$\mathbf{W}_0^H \mathbf{R}_v \mathbf{W}_0 = \widetilde{\mathbf{W}}^T \widehat{\mathbf{R}}_{v0} \widetilde{\mathbf{W}}, \quad \mathbf{W}_1^H \mathbf{R}_v \mathbf{W}_1 = \widetilde{\mathbf{W}}^H \widehat{\mathbf{R}}_{v1} \widetilde{\mathbf{W}} \,, \tag{5.53}$$

with

$$\widehat{\mathbf{R}}_{v0} = \begin{bmatrix} \bar{\mathbf{R}}_{v,R}^{00} & -\bar{\mathbf{R}}_{v,I}^{00} \\ \bar{\mathbf{R}}_{v,I}^{00} & \bar{\mathbf{R}}_{v,R}^{00} \end{bmatrix}, \quad \widehat{\mathbf{R}}_{v1} = \begin{bmatrix} \bar{\mathbf{R}}_{v,R}^{11} & -\bar{\mathbf{R}}_{v,I}^{11} \\ \bar{\mathbf{R}}_{v,I}^{11} & \bar{\mathbf{R}}_{v,R}^{11} \end{bmatrix} . \tag{5.54}$$

Note that the $4M \times 4M$-dimensional real matrices $\widehat{\mathbf{R}}_{v0}$ and $\widehat{\mathbf{R}}_{v1}$ are symmetric and positive definite.

We will now discuss the different ILD cost functions:

1. The cost function $J_{ILD,1}$ in (5.48) is equal to

$$\boxed{J_{ILD,1}(\widetilde{\mathbf{W}}) = \frac{\left(\widetilde{\mathbf{W}}^T \widehat{\mathbf{R}}_{vd} \widetilde{\mathbf{W}}\right)^2}{\left(\widetilde{\mathbf{W}}^T \widehat{\mathbf{R}}_{v1} \widetilde{\mathbf{W}}\right)^2}} \tag{5.55}$$

   with

$$\widehat{\mathbf{R}}_{vd} = \widehat{\mathbf{R}}_{v0} - \frac{P_{v0}}{P_{v1}} \widehat{\mathbf{R}}_{v1} = \begin{bmatrix} \mathbf{R}_{v,R} & \mathbf{0}_M & -\mathbf{R}_{v,I} & \mathbf{0}_M \\ \mathbf{0}_M & -\frac{P_{v0}}{P_{v1}} \mathbf{R}_{v,R} & \mathbf{0}_M & \frac{P_{v0}}{P_{v1}} \mathbf{R}_{v,I} \\ \mathbf{R}_{v,I} & \mathbf{0}_M & \mathbf{R}_{v,R} & \mathbf{0}_M \\ \mathbf{0}_M & -\frac{P_{v0}}{P_{v1}} \mathbf{R}_{v,I} & \mathbf{0}_M & -\frac{P_{v0}}{P_{v1}} \mathbf{R}_{v,R} \end{bmatrix} . \tag{5.56}$$

2. The cost function $J_{ILD,2}$ in (5.49) is equal to

$$\boxed{J_{ILD,2}(\widetilde{\mathbf{W}}) = \left(\widetilde{\mathbf{W}}^T \widehat{\mathbf{R}}_{vd} \widetilde{\mathbf{W}}\right)^2} \tag{5.57}$$

The gradient and the Hessian of both cost functions can be computed similarly as in Section 5.2.1, by replacing $\widetilde{\mathbf{R}}_{vd}$ by $\widehat{\mathbf{R}}_{vd}$ and $\widetilde{\mathbf{R}}_{v1}$ by $\widehat{\mathbf{R}}_{v1}$.

5.2.3 Total cost function

As already presented in (5.6), the total cost function is equal to

$$J_{tot}(\widetilde{\mathbf{W}}) = J_{SDW}(\widetilde{\mathbf{W}}) + \beta \, J_{ITD}(\widetilde{\mathbf{W}}) + \gamma \, J_{ILD}(\widetilde{\mathbf{W}}) \,, \tag{5.58}$$

using (3.21) for J_{SDW}, and either using (5.23), (5.27) or (5.33) for J_{ITD}, and either using (5.55) or (5.57) for J_{ILD}. None of the presented cost functions for J_{ITD} and J_{ILD} lead to a closed-form expression for the filter minimising the total cost function J_{tot}. Hence, iterative optimisation techniques need to be used for minimising this cost function. Many of these optimisation techniques are able to take advantage of the analytical expressions for the gradient and the Hessian that have been derived in sections 5.2.1 and 5.2.2. When using $J_{ITD,3}$ and $J_{ILD,2}$, the total cost function J_{tot} is a convex function.

Note that it might be good idea to make the weighting parameters β and γ dependent on how well the optimal filter for the SDW-MWF cost function $\mathbf{W}_{SDW}$ in (3.19) already fits the ITD and ILD cost functions, i.e.

$$\bar{J}_{tot}(\widetilde{\mathbf{W}}) = J_{SDW}(\widetilde{\mathbf{W}}) + \bar{\beta} J_{ITD}(\widetilde{\mathbf{W}}_{SDW})\, J_{ITD}(\widetilde{\mathbf{W}}) + \bar{\gamma} J_{ILD}(\widetilde{\mathbf{W}}_{SDW})\, J_{ILD}(\widetilde{\mathbf{W}}) \; . \tag{5.59}$$

5.3 Extension of TF-LCMV beamformer with binaural cues

5.3.1 Constrained optimisation problem

Using (2.11), the MV cost function in (4.10) can be written as

$$J_{MV}(\widetilde{\mathbf{W}}) = \widetilde{\mathbf{W}}^T \widetilde{\mathbf{R}}_t \widetilde{\mathbf{W}} \; , \tag{5.60}$$

with

$$\widetilde{\mathbf{R}}_t = \begin{bmatrix} \mathbf{R}_{t,R} & -\mathbf{R}_{t,I} \\ \mathbf{R}_{t,I} & \mathbf{R}_{t,R} \end{bmatrix} \; , \tag{5.61}$$

and the linear constraints in (4.12) can be written as

$$\widetilde{\mathbf{W}}^T \widetilde{\mathbf{H}} = \widetilde{\mathbf{F}}^T \; , \tag{5.62}$$

with the $4M \times 4$-dimensional matrix $\widetilde{\mathbf{H}}$ and the 4-dimensional vector $\widetilde{\mathbf{F}}$ equal to

$$\widetilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{0,R} & -\mathbf{H}_{0,I} \\ \mathbf{H}_{0,I} & \mathbf{H}_{0,R} \end{bmatrix}, \quad \widetilde{\mathbf{F}} = \begin{bmatrix} \mathbf{F}_R \\ \mathbf{F}_I \end{bmatrix} \; . \tag{5.63}$$

Hence, when extending the TF-LCMV beamformer discussed with the binaural cues of the noise component, the total cost function is equal to

$$J_{tot}(\widetilde{\mathbf{W}}) = J_{MV}(\widetilde{\mathbf{W}}) + \beta\, J_{ITD}(\widetilde{\mathbf{W}}) + \gamma\, J_{ILD}(\widetilde{\mathbf{W}}) \; , \tag{5.64}$$

subject to the linear constraints

$$\widetilde{\mathbf{W}}^T \widetilde{\mathbf{H}} = \widetilde{\mathbf{F}}^T \; . \tag{5.65}$$

Since no closed-form expression is available for the filter solving this constrained optimisation problem, we will resort to iterative constrained optimisation techniques.

5.3.2 Unconstrained optimisation problem

Using the parametrisation in (4.34), the total cost function can be written as

$$J_{tot}(\mathbf{W}_a) = J_{MV}(\mathbf{W}_a) + \beta\, J_{ITD}(\mathbf{W}_a) + \gamma\, J_{ILD}(\mathbf{W}_a) \; , \tag{5.66}$$

where $J_{MV}(\mathbf{W}_a)$ is defined in (4.38), and where the ITD and ILD cost functions need to be redefined in function of the filter vector $\mathbf{W}_a$.

6 Controlling the ratio of the output noise components

6.1 Definition of cost function

Instead of controlling the output noise cross-correlation (cf. Section 5.2.1) and the output noise power ratio (cf. Section 5.2.2), another possibility is to control the ratio between the output noise components

$$\frac{Z_{v0}}{Z_{v1}} = \frac{\mathbf{W}_0^H \mathbf{V}}{\mathbf{W}_1^H \mathbf{V}} \tag{6.1}$$

to be equal to some predefined TF ratio H_d, e.g. the ratio between the HRTFs

$$H_d(\omega) = \frac{HRTF_0(\omega, \theta)}{HRTF_1(\omega, \theta)} \, . \tag{6.2}$$

The cost function to be minimised then is equal to[5]

$$J_{d1}(\mathbf{W}_0, \mathbf{W}_1) = \mathcal{E}\Big\{ \Big| \frac{\mathbf{W}_0^H \mathbf{V}}{\mathbf{W}_1^H \mathbf{V}} - H_d \Big|^2 \Big\} \, . \tag{6.4}$$

However, it is not possible to write this expression using the noise correlation matrix $\mathbf{R}_v$. For mathematical convenience, we now define a similar cost function

$$\begin{aligned} J_{d2}(\mathbf{W}_0, \mathbf{W}_1) &= \mathcal{E}\big\{ \big| \mathbf{W}_0^H \mathbf{V} - H_d \, \mathbf{W}_1^H \mathbf{V} \big|^2 \big\} & (6.5) \\ &= \mathcal{E}\Big\{ \Big| \mathbf{W}^H \begin{bmatrix} \mathbf{V} \\ -H_d \, \mathbf{V} \end{bmatrix} \Big|^2 \Big\} & (6.6) \\ &= \mathbf{W}^H \begin{bmatrix} \mathbf{R}_v & -H_d^* \, \mathbf{R}_v \\ -H_d \, \mathbf{R}_v & |H_d|^2 \mathbf{R}_v \end{bmatrix} \mathbf{W} \, . & (6.7) \end{aligned}$$

Since the cost function $J_{d2}(\mathbf{W}_0, \mathbf{W}_1)$ depends on the power of the noise component, whereas the original cost function $J_{d1}(\mathbf{W}_0, \mathbf{W}_1)$ is independent of the amplitude of the noise component, we will perform a normalisation with the power of the noise component, i.e.

$$\boxed{J_d(\mathbf{W}) = \mathbf{W}^H \mathbf{R}_{vt} \mathbf{W}} \tag{6.8}$$

with

$$\boxed{\mathbf{R}_{vt} = \frac{M}{\mathrm{diag}(\mathbf{R}_v)} \begin{bmatrix} \mathbf{R}_v & -H_d^* \, \mathbf{R}_v \\ -H_d \, \mathbf{R}_v & |H_d|^2 \mathbf{R}_v \end{bmatrix}} \tag{6.9}$$

[5] If we denote the short time Fourier transform (STFT) of the noise component in the left hearing aid at time t and frequency ω as $\mathbf{V}(t, \omega)$, the cost function to be minimised is equal to

$$\sum_{t=0}^{T} \Big| \frac{\mathbf{W}_0(\omega)^H \mathbf{V}(t, \omega)}{\mathbf{W}_1(\omega)^H \mathbf{V}(t, \omega)} - H_d(\omega) \Big|^2 \, . \tag{6.3}$$

Note: since the original cost function $J_{d1}(\mathbf{W}_0, \mathbf{W}_1)$ is also independent of the size of the filter coefficients, i.e. $J_{d1}(\kappa\mathbf{W}_0, \kappa\mathbf{W}_1) = J_{d1}(\mathbf{W}_0, \mathbf{W}_1)$, we could in addition normalise (6.8) with the norm of the filter,

$$J_{dn}(\mathbf{W}) = \frac{\mathbf{W}^H\mathbf{R}_{vt}\mathbf{W}}{\mathbf{W}^H\mathbf{W}} \ , \tag{6.10}$$

such that $J_{dn}(\kappa\mathbf{W}) = J_{dn}(\mathbf{W})$.

6.2 Extension of SDW-MWF

The total cost function is defined as the weighted sum of the cost functions $J_{SDW}(\mathbf{W})$, defined in (3.14), and $J_d(\mathbf{W})$, defined in (6.8), i.e.

$$\begin{aligned} J_{tot}(\mathbf{W}) &= J_{SDW}(\mathbf{W}) + \delta J_d(\mathbf{W}) & (6.11)\\ &= P + \mathbf{W}^H\mathbf{R}\mathbf{W} - \mathbf{W}^H\mathbf{r} - \mathbf{r}^H\mathbf{W} + \delta\mathbf{W}^H\mathbf{R}_{vt}\mathbf{W} \ , & (6.12) \end{aligned}$$

where δ provides a trade-off between both cost functions. The filter minimising $J_{tot}(\mathbf{W})$ is equal to

$$\boxed{\mathbf{W}_{tot} = (\mathbf{R} + \delta\mathbf{R}_{vt})^{-1}\mathbf{r}} \tag{6.13}$$

6.3 Extension of TF-LCMV beamformer

6.3.1 Constrained optimisation problem

The total cost function is defined as the weighted sum of the cost functions $J_{MV}(\mathbf{W})$, defined in (4.10), and $J_d(\mathbf{W})$, defined in (6.8), i.e.

$$\begin{aligned} J_{tot}(\mathbf{W}) &= J_{MV}(\mathbf{W}) + \delta J_d(\mathbf{W}) & (6.14)\\ &= \mathbf{W}^H\mathbf{R}_t\mathbf{W} + \delta\mathbf{W}^H\mathbf{R}_{vt}\mathbf{W} \ , & (6.15) \end{aligned}$$

where δ provides a trade-off between both cost functions, and subject to the linear constraints defined in (4.12),

$$\mathbf{W}^H\mathbf{H} = \mathbf{F}^H \ . \tag{6.16}$$

The filter minimising this constrained cost function is equal to

$$\boxed{\mathbf{W}_{opt} = (\mathbf{R}_t + \delta\mathbf{R}_{vt})^{-1}\mathbf{H}\left[\mathbf{H}^H(\mathbf{R}_t + \delta\mathbf{R}_{vt})^{-1}\mathbf{H}\right]^{-1}\mathbf{F}} \tag{6.17}$$

6.3.2 Unconstrained optimisation problem: adaptive solution

As indicated in Section 4.4, the constrained optimisation problem of the $2M$-dimensional filter $\mathbf{W}$ is equivalent to the unconstrained optimisation problem of the $2(M-1)$-dimensional filters $\mathbf{W}_a$, defined in (4.42), i.e.

$$J_{MV}(\mathbf{W}_a) = \mathcal{E}\left\{\left|U_0 - \mathbf{W}_a^H\begin{bmatrix}\mathbf{U}_{a0}\\ \mathbf{0}_{M-1}\end{bmatrix}\right|^2\right\} + \alpha\mathcal{E}\left\{\left|U_1 - \mathbf{W}_a^H\begin{bmatrix}\mathbf{0}_{M-1}\\ \mathbf{U}_{a1}\end{bmatrix}\right|^2\right\} . \tag{6.18}$$

Using (4.29) and (4.30), the cost function in (6.5) can be written as

$$J_{d2}(\mathbf{W}_a) = \mathcal{E}\{\left|(\mathbf{W}_{q0}^H - \mathbf{W}_{a0}^H\mathbf{H}_{a0}^H)\mathbf{V} - (\mathbf{W}_{q1}^H - \mathbf{W}_{a1}^H\mathbf{H}_{a1}^H)H_d\mathbf{V}\right|^2\} \tag{6.19}$$

$$= \mathcal{E}\left\{\left|(U_{v0} - H_d\,U_{v1}) - \mathbf{W}_a^H\begin{bmatrix}\mathbf{U}_{v,a0}\\ -H_d\mathbf{U}_{v,a1}\end{bmatrix}\right|^2\right\}, \tag{6.20}$$

with U_{v0} and U_{v1} the noise component of the speech references U_0 and U_1, and $\mathbf{U}_{v,a0}$ and $\mathbf{U}_{v,a1}$ the noise component of the noise references[6] $\mathbf{U}_{a0}$ and $\mathbf{U}_{a1}$.
The total cost function is defined as the weighted sum of the cost functions $J_{MV}(\mathbf{W}_a)$ and $J_{d2}(\mathbf{W}_a)$, i.e.

$$J_{tot}(\mathbf{W}_a) = J_{MV}(\mathbf{W}_a) + \delta J_{d2}(\mathbf{W}_a)\,, \tag{6.21}$$

where δ provides a trade-off between both cost functions and may include the normalisation with the power of the noise component, cf. (6.9). The gradient of $J_{tot}(\mathbf{W}_a)$ is equal to

$$\begin{aligned}\frac{\partial J_{tot}(\mathbf{W}_a)}{\partial \mathbf{W}_a} &= -2\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{a0}\\ \mathbf{0}_{M-1}\end{bmatrix}U_0^*\right\} + 2\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{a0}\\ \mathbf{0}_{M-1}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{a0}^H & \mathbf{0}_{M-1}^H\end{bmatrix}\right\}\mathbf{W}_a\\
&\quad -2\alpha\,\mathcal{E}\left\{\begin{bmatrix}\mathbf{0}_{M-1}\\ \mathbf{U}_{a1}\end{bmatrix}U_1^*\right\} + 2\alpha\,\mathcal{E}\left\{\begin{bmatrix}\mathbf{0}_{M-1}\\ \mathbf{U}_{a1}\end{bmatrix}\begin{bmatrix}\mathbf{0}_{M-1}^H & \mathbf{U}_{a1}^H\end{bmatrix}\right\}\mathbf{W}_a\\
&\quad -2\delta\,\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{v,a0}\\ -H_d\mathbf{U}_{v,a1}\end{bmatrix}(U_{v0} - H_d\,U_{v1})^*\right\}\\
&\quad +2\delta\,\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{v,a0}\\ -H_d\mathbf{U}_{v,a1}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{v,a0}^H & -H_d^*\mathbf{U}_{v,a1}^H\end{bmatrix}\right\}\mathbf{W}_a \qquad (6.22)\\
&= -2\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{a0}\\ \mathbf{0}_{M-1}\end{bmatrix}Z_0^*\right\} - 2\alpha\,\mathcal{E}\left\{\begin{bmatrix}\mathbf{0}_{M-1}\\ \mathbf{U}_{a1}\end{bmatrix}Z_1^*\right\}\\
&\quad -2\delta\,\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{v,a0}\\ -H_d\mathbf{U}_{v,a1}\end{bmatrix}(Z_{v0} - H_d\,Z_{v1})^*\right\}. \qquad (6.23)\end{aligned}$$

By setting the gradient equal to zero, we obtain the normal equations, i.e.

$$\underbrace{\left(\begin{bmatrix}\mathcal{E}\{\mathbf{U}_{a0}\mathbf{U}_{a0}^H\} & \mathbf{0}_{M-1}\\ \mathbf{0}_{M-1} & \alpha\mathcal{E}\{\mathbf{U}_{a1}\mathbf{U}_{a1}^H\}\end{bmatrix} + \delta\begin{bmatrix}\mathcal{E}\{\mathbf{U}_{v,a0}\mathbf{U}_{v,a0}^H\} & -H_d^*\mathcal{E}\{\mathbf{U}_{v,a0}\mathbf{U}_{v,a1}^H\}\\ -H_d\mathcal{E}\{\mathbf{U}_{v,a1}\mathbf{U}_{v,a0}^H\} & |H_d|^2\mathcal{E}\{\mathbf{U}_{v,a1}\mathbf{U}_{v,a1}^H\}\end{bmatrix}\right)}_{\mathbf{R}_a}\mathbf{W}_a$$

$$= \underbrace{\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{a0}\\ \mathbf{0}_{M-1}\end{bmatrix}U_0^*\right\} + \alpha\mathcal{E}\left\{\begin{bmatrix}\mathbf{0}_{M-1}\\ \mathbf{U}_{a1}\end{bmatrix}U_1^*\right\} + \delta\mathcal{E}\left\{\begin{bmatrix}\mathbf{U}_{v,a0}\\ -H_d\mathbf{U}_{v,a1}\end{bmatrix}(U_{v0} - H_d\,U_{v1})^*\right\}}_{\mathbf{r}_a}.$$

such that the optimal filter is equal to

$$\mathbf{W}_{a,opt} = \mathbf{R}_a^{-1}\mathbf{r}_a\,. \tag{6.24}$$

[6]Theoretically no speech component is present in the noise references, i.e. $\mathbf{U}_{a0} = \mathbf{U}_{v,a0}$ and $\mathbf{U}_{a1} = \mathbf{U}_{v,a1}$.

The gradient descent approach for minimising $J_{tot}(\mathbf{W}_a)$ is

$$\mathbf{W}_a(j+1) = \mathbf{W}_a(j) - \frac{\rho}{2}\left[\frac{\partial J_{tot}(\mathbf{W}_a)}{\partial \mathbf{W}_a}\right]_{\mathbf{W}_a=\mathbf{W}_a(j)} , \tag{6.25}$$

where j denotes the iteration index and ρ is the stepsize parameter. A stochastich gradient algorithm for updating $\mathbf{W}_a$ is obtained by replacing the iteration index j by the time index k and leaving out the expectation values, i.e.

$$\begin{aligned}\mathbf{W}_a(k+1) &= \mathbf{W}_a(k) + \rho\Bigg\{ \begin{bmatrix} \mathbf{U}_{a0}(k) \\ \mathbf{0}_{M-1} \end{bmatrix} Z_0^*(k) + \alpha \begin{bmatrix} \mathbf{0}_{M-1} \\ \mathbf{U}_{a1}(k) \end{bmatrix} Z_1^*(k) \\ &\quad + \delta \begin{bmatrix} \mathbf{U}_{v,a0}(k) \\ -H_d \mathbf{U}_{v,a1}(k) \end{bmatrix} \big(Z_{v0}(k) - H_d\, Z_{v1}(k)\big)^* \Bigg\} . \end{aligned} \tag{6.26}$$

It can be easily shown that

$$\mathcal{E}\{\mathbf{W}_a(k+1) - \mathbf{W}_{a,opt}\} = \big[\mathbf{I}_{2(M-1)} - \rho \mathbf{R}_a\big]^{k+1} \mathcal{E}\{\mathbf{W}_a(0) - \mathbf{W}_{a,opt}\} , \tag{6.27}$$

such that the adaptive algorithm in (6.26) is convergent in the mean if the step size ρ is smaller than $\frac{2}{\lambda_{max}}$, with λ_{max} the maximum eigenvalue of $\mathbf{R}_a$. The similarity with standard LMS lets us presume that setting

$$\rho < \frac{2}{\mathcal{E}\{\mathbf{U}_{a0}^H \mathbf{U}_{a0}\} + \alpha \mathcal{E}\{\mathbf{U}_{a1}^H \mathbf{U}_{a1}\} + \delta(\mathcal{E}\{\mathbf{U}_{v,a0}^H \mathbf{U}_{v,a0}\} + |H_d|^2 \mathcal{E}\{\mathbf{U}_{v,a1}^H \mathbf{U}_{v,a1}\})} \tag{6.28}$$

guarantees convergence. The adaptive NLMS-based algorithm for updating the filters $\mathbf{W}_{a0}(k)$ and $\mathbf{W}_{a1}(k)$ **during noise-only periods** hence becomes[7]

$$\boxed{\begin{aligned} Z_0(k) &= U_0(k) - \mathbf{W}_{a0}^H(k)\mathbf{U}_{a0}(k) \\ Z_1(k) &= U_1(k) - \mathbf{W}_{a1}^H(k)\mathbf{U}_{a1}(k) \\ Z_d(k) &= Z_0(k) - H_d\, Z_1(k) \\ P_{a0}(k) &= \lambda P_{a0}(k-1) + (1-\lambda)\mathbf{U}_{a0}^H(k)\mathbf{U}_{a0}(k) \\ P_{a1}(k) &= \lambda P_{a1}(k-1) + (1-\lambda)\mathbf{U}_{a1}^H(k)\mathbf{U}_{a1}(k) \\ P(k) &= (1+\delta)P_{a0}(k) + (\alpha + \delta|H_d|^2)P_{a1}(k) \\ \mathbf{W}_{a0}(k+1) &= \mathbf{W}_{a0}(k) + \frac{\rho'}{P(k)}\mathbf{U}_{a0}\big(Z_0(k) + \delta\, Z_d(k)\big)^* \\ \mathbf{W}_{a1}(k+1) &= \mathbf{W}_{a1}(k) + \frac{\rho'}{P(k)}\mathbf{U}_{a1}\big(\alpha\, Z_1(k) - \delta H_d^*\, Z_d(k)\big)^* \end{aligned}} \tag{6.29}$$

with λ a forgetting factor for updating the noise energy. This algorithm is similar to the TF-GSC implementation described in [44], where for the left hearing aid $Z_0(k)$ is replaced by $Z_0(k) + \delta\, Z_d(k)$, and for the right hearing aid $Z_1(k)$ is replaced by $\alpha Z_1(k) - \delta H_d\, Z_d(k)$. The extended TF-GSC structure controlling the TF ratio of the noise component is depicted in Fig. 3.

[7] Note: frequency-domain implementation similar to book chapter possible ??

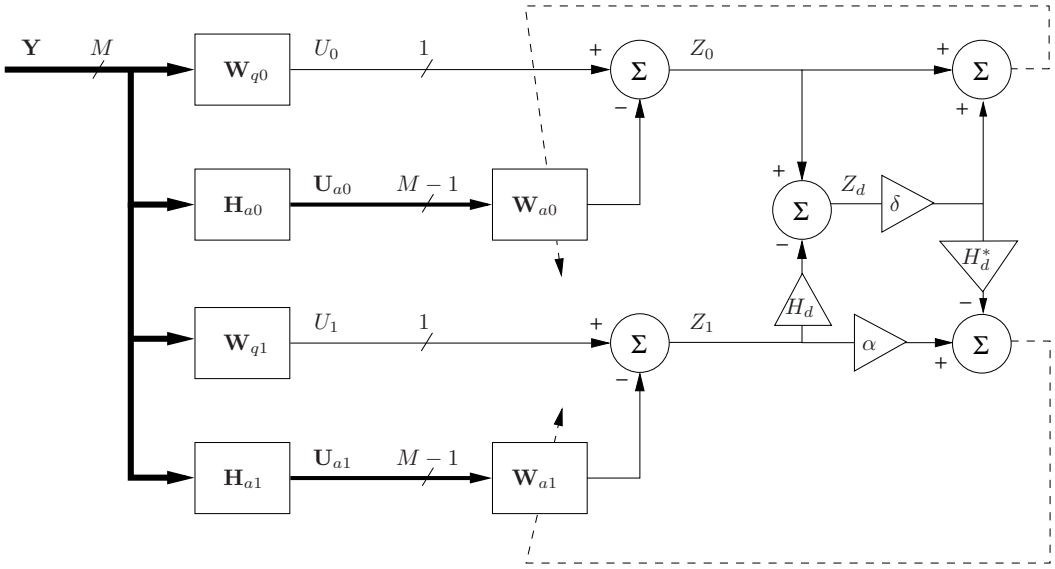


Figure 3: Binaural TF-GSC structure controlling the TF ratio of the noise component

7 Simulations

7.1 Set-up and performance measures

The recordings used in the simulations were made in a seminar room with dimensions 11'×11'×8'6", having a relatively low reverberation time ($T_{60} \approx 150\,\text{ms}$). Two Knowles FG microphones were placed horizontally inside both ears of a KEMAR mannequin ($M_0 = M_1 = 2$), with a microphone spacing of 1 cm. The desired speech source is positioned in front of the head (0°) and consists of English sentences. The noise scenario consists of a multi-talker babble source positioned at 45°. All recordings were performed at a sampling frequency of 16 kHz. For evaluation purposes, the speech and the noise signal were recorded separately. The unbiased broadband SNR of the reference microphone signals at the left and the right hearing aid ($r_0 = r_1 = 0$) is 0 dB and −3.2 dB.

The FFT-size used for frequency-domain processing is $N = 256$. As already mentioned in Section **??**, the noise correlation matrices $\mathbf{R}_v^n, n = 0 \ldots N-1$, are estimated during noise-only periods, the matrices $\mathbf{R}_y^n$ are estimated during speech-and-noise periods, and the speech correlation matrices are computed as $\mathbf{R}_x^n = \mathbf{R}_y^n - \mathbf{R}_v^n$. For all simulations we used $\mu_0 = \mu_1 = 1$.

As performance measures we use the *SNR improvement* between the input and the output signal at the left and the right hearing aid, and the *ITD cost function* for the noise and the speech component. The SNR improvement for the left hearing aid is defined as the mean of the SNR improvement in dB over all frequencies, i.e.

$$\Delta\text{SNR}_0 = \frac{10}{N} \sum_{n=0}^{N-1} \log_{10} \frac{\mathbf{W}_0^{n,H} \mathbf{R}_x^n \mathbf{W}_0^n}{\mathbf{W}_0^{n,H} \mathbf{R}_v^n \mathbf{W}_0^n} - \log_{10} \frac{\mathbf{R}_x^n(r_0, r_0)}{\mathbf{R}_v^n(r_0, r_0)} \,.$$

The SNR improvement for the right hearing aid is defined similarly. The ITD cost function for the noise component is defined as the mean of the cost function $J_{ITD}(\mathbf{W}^n)$ in (5.13) over all frequencies. The ITD cost function for the speech component is defined similarly, by replacing $\mathbf{R}_v$ with $\mathbf{R}_x$ in (5.8) and (5.13).

8 Conclusion

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Appendix A: Derivative of $\mathbf{w}^H\mathbf{A}\mathbf{w}$

Consider a real M-dimensional vector $\mathbf{w}$ and a real $M \times M$-dimensional matrix $\mathbf{A}$. The derivative of the cost function $J = \mathbf{w}^T\mathbf{A}\mathbf{w}$ is equal to

$$\frac{\partial J}{\partial \mathbf{w}} = (\mathbf{A} + \mathbf{A}^T)\mathbf{w} \;, \tag{A.1}$$

such that only for symmetric $\mathbf{A}$ the derivative becomes $\frac{\partial J}{\partial \mathbf{w}} = 2\mathbf{A}\mathbf{w}$.

Now consider a complex M-dimensional vector $\mathbf{w}$ and a complex $M \times M$-dimensional matrix $\mathbf{A}$. The cost function $J = \mathbf{w}^H\mathbf{A}\mathbf{w}$ can be written as

$$\begin{aligned} J &= (\mathbf{w}_R^T - j\mathbf{w}_I^T)(\mathbf{A}_R + j\mathbf{A}_I)(\mathbf{w}_R + j\mathbf{w}_I) && \text{(A.2)} \\ &= (\mathbf{w}_R^T\mathbf{A}_R\mathbf{w}_R + \mathbf{w}_I^T\mathbf{A}_I\mathbf{w}_R - \mathbf{w}_R^T\mathbf{A}_I\mathbf{w}_I + \mathbf{w}_I^T\mathbf{A}_R\mathbf{w}_I) && \text{(A.3)} \\ &\quad + j(\mathbf{w}_R^T\mathbf{A}_I\mathbf{w}_R - \mathbf{w}_I^T\mathbf{A}_R\mathbf{w}_R + \mathbf{w}_R^T\mathbf{A}_R\mathbf{w}_I + \mathbf{w}_I^T\mathbf{A}_I\mathbf{w}_I) \;. && \text{(A.4)} \end{aligned}$$

The derivative $\frac{\partial J}{\partial \mathbf{w}}$ can be computed as

$$\frac{\partial J}{\partial \mathbf{w}} = \frac{\partial J}{\partial \mathbf{w}_R} + j\frac{\partial J}{\partial \mathbf{w}_I} \;. \tag{A.5}$$

Using (A.1), the derivatives $\frac{\partial J}{\partial \mathbf{w}_R}$ and $\frac{\partial J}{\partial \mathbf{w}_I}$ can be written as

$$\begin{aligned} \frac{\partial J}{\partial \mathbf{w}_R} &= (\mathbf{A}_R + \mathbf{A}_R^T)\mathbf{w}_R + \mathbf{A}_I^T\mathbf{w}_I - \mathbf{A}_I\mathbf{w}_I + j\left[(\mathbf{A}_I + \mathbf{A}_I^T)\mathbf{w}_R - \mathbf{A}_R^T\mathbf{w}_I + \mathbf{A}_R\mathbf{w}_I\right] \\ \frac{\partial J}{\partial \mathbf{w}_I} &= (\mathbf{A}_R + \mathbf{A}_R^T)\mathbf{w}_I - \mathbf{A}_I^T\mathbf{w}_R + \mathbf{A}_I\mathbf{w}_R + j\left[(\mathbf{A}_I + \mathbf{A}_I^T)\mathbf{w}_I + \mathbf{A}_R^T\mathbf{w}_R - \mathbf{A}_R\mathbf{w}_R\right] \;, \end{aligned}$$

such that $\frac{\partial J}{\partial \mathbf{w}}$ is equal to

$$\frac{\partial J}{\partial \mathbf{w}} = 2(\mathbf{A}_R + j\mathbf{A}_I)\mathbf{w}_R + 2j(\mathbf{A}_R + j\mathbf{A}_I)\mathbf{w}_I = 2\mathbf{A}\mathbf{w} \;. \tag{A.6}$$

Note that this result also holds for non-Hermitian matrices $\mathbf{A}$.

Appendix B: Jacobian of $f(\mathbf{w})\mathbf{g}(\mathbf{w})$

Consider a real M-dimensional vector $\mathbf{w}$ and a real N-dimensional vector $\mathbf{h}(\mathbf{w})$ equal to

$$\mathbf{h}(\mathbf{w}) = f(\mathbf{w})\mathbf{g}(\mathbf{w}) \;, \tag{B.1}$$

with $f(\mathbf{w})$ a real scalar and $\mathbf{g}(\mathbf{w})$ a real N-dimensional vector. The $N \times M$-dimensional Jacobian of $\mathbf{h}(\mathbf{w})$ is equal to

$$\boxed{\frac{\partial \mathbf{h}(\mathbf{w})}{\partial \mathbf{w}} = \mathbf{g}(\mathbf{w})\left[\frac{\partial f(\mathbf{w})}{\partial \mathbf{w}}\right]^T + f(\mathbf{w})\frac{\partial \mathbf{g}(\mathbf{w})}{\partial \mathbf{w}}} \tag{B.2}$$

with $\frac{\partial \mathbf{g}(\mathbf{w})}{\partial \mathbf{w}}$ the $N \times M$-dimensional Jacobian of $\mathbf{g}(\mathbf{w})$.

Proof: The nth element of $\mathbf{h}(\mathbf{w})$ is equal to

$$h_n(\mathbf{w}) = f(\mathbf{w})g_n(\mathbf{w}) \,, \tag{B.3}$$

such that

$$\frac{\partial h_n(\mathbf{w})}{\partial w_m} = \frac{\partial f(\mathbf{w})}{\partial w_m} g_n(\mathbf{w}) + f(\mathbf{w}) \frac{\partial g_n(\mathbf{w})}{\partial w_m} \,. \tag{B.4}$$

Hence the Jacobian of $\mathbf{h}(\mathbf{w})$ is equal to

$$\begin{bmatrix} \frac{\partial h_1(\mathbf{w})}{\partial w_1} & \frac{\partial h_1(\mathbf{w})}{\partial w_2} & \cdots & \frac{\partial h_1(\mathbf{w})}{\partial w_M} \\ \frac{\partial h_2(\mathbf{w})}{\partial w_1} & \frac{\partial h_2(\mathbf{w})}{\partial w_2} & \cdots & \frac{\partial h_2(\mathbf{w})}{\partial w_M} \\ \vdots & \vdots & & \vdots \\ \frac{\partial h_N(\mathbf{w})}{\partial w_1} & \frac{\partial h_N(\mathbf{w})}{\partial w_2} & \cdots & \frac{\partial h_N(\mathbf{w})}{\partial w_M} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{\partial f(\mathbf{w})}{\partial w_1} g_1(\mathbf{w}) & \frac{\partial f(\mathbf{w})}{\partial w_2} g_1(\mathbf{w}) & \cdots & \frac{\partial f(\mathbf{w})}{\partial w_M} g_1(\mathbf{w}) \\ \frac{\partial f(\mathbf{w})}{\partial w_1} g_2(\mathbf{w}) & \frac{\partial f(\mathbf{w})}{\partial w_2} g_2(\mathbf{w}) & \cdots & \frac{\partial f(\mathbf{w})}{\partial w_M} g_2(\mathbf{w}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial f(\mathbf{w})}{\partial w_1} g_N(\mathbf{w}) & \frac{\partial f(\mathbf{w})}{\partial w_2} g_N(\mathbf{w}) & \cdots & \frac{\partial f(\mathbf{w})}{\partial w_M} g_N(\mathbf{w}) \end{bmatrix} +$$

$$f(\mathbf{w}) \begin{bmatrix} \frac{\partial g_1(\mathbf{w})}{\partial w_1} & \frac{\partial g_1(\mathbf{w})}{\partial w_2} & \cdots & \frac{\partial g_1(\mathbf{w})}{\partial w_M} \\ \frac{\partial g_2(\mathbf{w})}{\partial w_1} & \frac{\partial g_2(\mathbf{w})}{\partial w_2} & \cdots & \frac{\partial g_2(\mathbf{w})}{\partial w_M} \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_N(\mathbf{w})}{\partial w_1} & \frac{\partial g_N(\mathbf{w})}{\partial w_2} & \cdots & \frac{\partial g_N(\mathbf{w})}{\partial w_M} \end{bmatrix} \,, \tag{B.5}$$

which can be written as

$$\begin{bmatrix} \frac{\partial f(\mathbf{w})}{\partial w_1} \mathbf{g}(\mathbf{w}) & \frac{\partial f(\mathbf{w})}{\partial w_2} \mathbf{g}(\mathbf{w}) & \cdots & \frac{\partial f(\mathbf{w})}{\partial w_M} \mathbf{g}(\mathbf{w}) \end{bmatrix} + f(\mathbf{w}) \frac{\partial \mathbf{g}(\mathbf{w})}{\partial \mathbf{w}} \tag{B.6}$$

$$= \mathbf{g}(\mathbf{w}) \left[\frac{\partial f(\mathbf{w})}{\partial \mathbf{w}} \right]^T + f(\mathbf{w}) \frac{\partial \mathbf{g}(\mathbf{w})}{\partial \mathbf{w}} \tag{B.7}$$

□

Example 1: The Jacobian of $\mathbf{h}(\mathbf{w}) = (\mathbf{w}^T \mathbf{A} \mathbf{w}) \mathbf{B} \mathbf{w}$, with $\mathbf{A}$ and $\mathbf{B}$ real $M \times M$-dimensional matrices, is equal to

$$\frac{\partial \mathbf{h}(\mathbf{w})}{\partial \mathbf{w}} = \mathbf{B} \mathbf{w} \mathbf{w}^T (\mathbf{A} + \mathbf{A}^T) + (\mathbf{w}^T \mathbf{A} \mathbf{w}) \mathbf{B} \,. \tag{B.8}$$

Example 2: The Hessian of the function $J(\mathbf{w}) = f^n(\mathbf{w})$, with $n \geq 2$, is equal to

$$\frac{\partial^2 J(\mathbf{w})}{\partial^2 \mathbf{w}} = n f^{n-1}(\mathbf{w}) \frac{\partial^2 f(\mathbf{w})}{\partial^2 \mathbf{w}} + n(n-1) f^{n-2}(\mathbf{w}) \frac{\partial f(\mathbf{w})}{\partial \mathbf{w}} \left[\frac{\partial f(\mathbf{w})}{\partial \mathbf{w}} \right]^T \,. \tag{B.9}$$

References

[1] J. M. Kates, "Superdirective arrays for hearing aids," *Journal of the Acoustical Society of America*, vol. 94, no. 4, pp. 1930–1933, Oct. 1993.

[2] W. Soede, A. J. Berkhout, and F. A. Bilsen, "Development of a directional hearing instrument based on array technology," *Journal of the Acoustical Society of America*, vol. 94, no. 2, pp. 785–798, Aug. 1993.

[3] R. W. Stadler and W. M. Rabinowitz, "On the potential of fixed arrays for hearing aids," *Journal of the Acoustical Society of America*, vol. 94, no. 3, pp. 1332–1342, Sept. 1993.

[4] J. M. Kates and M. R. Weiss, "A comparison of hearing-aid array-processing techniques," *Journal of the Acoustical Society of America*, vol. 99, no. 5, pp. 3138–3148, May 1996.

[5] J.G. Desloge, W.M. Rabinowitz, and P.M. Zurek, "Microphone-array hearing aids with binaural output–Part I: Fixed-processing systems," *IEEE Trans. Speech and Audio Processing*, vol. 5, no. 6, pp. 529–542, Nov. 1997.

[6] I.L.D.M. Merks, M.M. Boone, and A.J. Berkhout, "Design of a broadside array for a binaural hearing aid," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, New Paltz NY, USA, Oct. 1997.

[7] V. Hamacher, "Comparison of advanced monaural and binaural noise reduction algorithms for hearing aids," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, Orlando FL, USA, May 2002, pp. 4008–4011.

[8] T. Lotter, *Single and multimicrophone speech enhancement for hearing aids*, Ph.D. thesis, RWTH Aachen, Germany, Aug. 2004.

[9] J. E. Greenberg and P. M. Zurek, "Evaluation of an adaptive beamforming method for hearing aids," *Journal of the Acoustical Society of America*, vol. 91, no. 3, pp. 1662–1676, Mar. 1992.

[10] D.P. Welker, J.E. Greenberg, J.G Desloge, and P.M. Zurek, "Microphone-array hearing aids with binaural output–Part II: A two-microphone adaptive system," *IEEE Trans. Speech and Audio Processing*, vol. 5, no. 6, pp. 543–551, Nov. 1997.

[11] J. Vanden Berghe and J. Wouters, "An adaptive noise canceller for hearing aids using two nearby microphones," *Journal of the Acoustical Society of America*, vol. 103, no. 6, pp. 3621–3626, June 1998.

[12] Y. Suzuki, S. Tsukui, F. Asano, R. Nishimura, and T. Sone, "New design method of a binaural microphone array using multiple constraints," *IEICE Trans. Fundamentals*, vol. E82-A, no. 4, pp. 588–596, Apr. 1999.

[13] J. E. Greenberg and P. M. Zurek, *Microphone-Array Hearing Aids*, chapter 11 in "Microphone Arrays: Signal Processing Techniques and Applications" (Brandstein, M. S. and Ward, D. B., Eds.), pp. 229–253, Springer-Verlag, May 2001.

[14] P.W. Shields and D.R. Campbell, "Improvements in intelligibility of noisy reverberant speech using a binaural subband adaptive noise-cancellation processing scheme," *Journal of the Acoustical Society of America*, vol. 110, no. 6, pp. 3232–3242, Dec. 2001.

[15] R. Nishimura, Y. Suzuki, and F. Asano, "A new adaptive binaural microphone array system using a weighted least squares algorithm," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, Orlando FL, USA, May 2002, pp. 1925–1928.

[16] M.E. Lockwood, D.L. Jones, R.C. Bilger, C.R. Lansing, W.D. O'Brien, B.C. Wheeler, and A.S. Feng, "Performance of time- and frequency-domain binaural beamformers based on recorded signals from real rooms," *Journal of the Acoustical Society of America*, vol. 115, no. 1, pp. 379–391, Jan. 2004.

[17] S. Doclo and M. Moonen, *GSVD-Based Optimal Filtering for Multi-Microphone Speech Enhancement*, chapter 6 in "Microphone Arrays: Signal Processing Techniques and Applications" (Brandstein, M. S. and Ward, D. B., Eds.), pp. 111–132, Springer-Verlag, May 2001.

[18] S. Doclo and M. Moonen, "GSVD-based optimal filtering for single and multimicrophone speech enhancement," *IEEE Trans. Signal Processing*, vol. 50, no. 9, pp. 2230–2244, Sept. 2002.

[19] J.-B. Maj, M. Moonen, and J. Wouters, "SVD-based optimal filtering technique for noise reduction in hearing aids using two microphones," *EURASIP Journal on Applied Signal Processing*, vol. 2002, no. 4, pp. 432–443, Apr. 2002.

[20] A. Spriet, M. Moonen, and J. Wouters, "A Multi-Channel Subband Generalized Singular Value Decomposition Approach to Speech Enhancement," *European Transactions on Telecommunications, special issue on Acoustic Echo and Noise Control*, vol. 13, no. 2, pp. 149–158, Mar.-Apr. 2002.

[21] A. Spriet, M. Moonen, and J. Wouters, "Spatially pre-processed speech distortion weighted multi-channel Wiener filtering for noise reduction," *Signal Processing*, vol. 84, no. 12, pp. 2367–2387, Dec. 2004.

[22] S. Doclo, A. Spriet, J. Wouters, and M. Moonen, *Speech Distortion Weighted Multichannel Wiener Filtering Techniques for Noise Reduction*, chapter 2 in "Speech Enhancement" (J. Benesty, J. Chen, S. Makino, Eds.), pp. 199–228, Springer-Verlag, 2005.

[23] T.J. Klasen, T. Van den Bogaert, M. Moonen, and J. Wouters, "Preservation of interaural time delay for binaural hearing aids through multi-channel Wiener filtering based noise reduction," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, Philadelphia PA, USA, Mar. 2005, pp. III 29–32.

[24] T.J. Klasen, T. Van den Bogaert, M. Moonen, and J. Wouters, "Binaural noise reduction algorithms for hearing aids that preserve interaural time delay cues," *Submitted to* IEEE Trans. Speech and Audio Processing, 2005.

[25] O. L. Frost III, "An Algorithm for Linearly Constrained Adaptive Array Processing," *Proc. IEEE*, vol. 60, pp. 926–935, Aug. 1972.

[26] H.N. Wright and R. Carhart, "The efficiency of binaural listening among the hearing impaired," *Arch. Otolaryngol.*, vol. 72, pp. 789–797, 1960.

[27] R. Carhart, "Monaural and binaural discrimination against competing sentences," *Int. Audiol.*, vol. 4, pp. 5–10, 1965.

[28] D.D. Dirks and R.H. Wilson, "The effect of spatially separated sound sources on speech intelligibility," *Journal of Speech and Hearing Research*, vol. 12, pp. 5–38, 1969.

[29] D.D. Dirks and R.H. Wilson, "Binaural hearing of speech for aided and unaided conditions," *Journal of Speech and Hearing Research*, vol. 12, pp. 650–664, 1969.

[30] N.W. MacKeith and R.R.A. Coles, "Binaural advantages in hearing of speech," *Journal of Laryngology and Otology*, vol. 85, pp. 213–232, 1971.

[31] R. Plomp and A.M. Mimpen, "Effect of the orientation of the speaker's head and the azimuth of a noise source on the speech reception threshold for sentences," *Acustica*, vol. 48, pp. 325–328, 1981.

[32] A.W. Bronkhorst and R. Plomp, "The effect of head-induced interaural time and level differences on speech intelligibility in noise," *Journal of the Acoustical Society of America*, vol. 83, no. 4, pp. 1508–1516, Apr. 1988.

[33] A.W. Bronkhorst and R. Plomp, "Binaural speech intelligibility in noise for hearing-impaired listeners," *Journal of the Acoustical Society of America*, vol. 86, no. 4, pp. 1374–1383, Oct. 1989.

[34] J. Peissig and B. Kollmeier, "Directivity of binaural noise reduction in spatial multiple noise-source arrangements for normal and impaired listeners," *Journal of the Acoustical Society of America*, vol. 101, no. 3, pp. 1660–1670, Mar. 1997.

[35] B. Kollmeier, J. Peissig, and V. Hohmann, "Real-time multiband dynamic compression and noise reduction for binaural hearing aids," *Journal of Rehabilitation Research and Development*, vol. 30, no. 1, pp. 82–94, 1993.

[36] B. Kollmeier and R. Koch, "Speech enhancement based on physiological and psychoacoustical modals of modulation perception and binaural interaction," *Journal of the Acoustical Society of America*, vol. 95, no. 3, pp. 1593–1602, Mar. 1994.

[37] T. Wittkop, *Two-channel noise reduction algorithms motivated by models of binaural interaction*, Ph.D. thesis, Universität Oldenburg, Germany, Mar. 2001.

[38] T. Wittkop and V. Hohmann, "Strategy-selective noise reduction for binaural digital hearing aids," *Speech Communication*, vol. 39, no. 1-2, pp. 111–138, Jan. 2003.

[39] R. Dong, J. Bondy, I. Bruce, and S. Haykin, "Dual-microphone speech enhancement using speech stream segregation," in *International Hearing Aid Research Conference*, Lake Tahoe CA, USA, Aug. 2004.

[40] K. U. Simmer, J. Bitzer, and C. Marro, *Post-Filtering Techniques*, chapter 3 in "Microphone Arrays: Signal Processing Techniques and Applications" (Brandstein, M. S. and Ward, D. B., Eds.), pp. 39–60, Springer-Verlag, May 2001.

[41] S. Doclo, *Multi-microphone noise reduction and dereverberation techniques for speech applications*, Ph.D. thesis, ESAT, Katholieke Universiteit Leuven, Belgium, May 2003.

[42] J. Benesty, J. Chen, A. Huang, and S. Doclo, *Study of the Wiener Filter for Noise Reduction*, chapter 2 in "Speech Enhancement" (J. Benesty, J. Chen, S. Makino, Eds.), pp. 9–42, Springer-Verlag, 2005.

[43] Y. Ephraim and H. L. Van Trees, "A Signal Subspace Approach for Speech Enhancement," *IEEE Trans. Speech and Audio Processing*, vol. 3, no. 4, pp. 251–266, July 1995.

[44] S. Gannot, D. Burshtein, and E. Weinstein, "Signal Enhancement Using Beamforming and Non-Stationarity with Applications to Speech," *IEEE Trans. Signal Processing*, vol. 49, no. 8, pp. 1614–1626, Aug. 2001.

[45] S. Doclo and M. Moonen, "Combined frequency-domain dereverberation and noise reduction technique for multi-microphone speech enhancement," in *Proc. Int. Workshop on Acoustic Echo and Noise Control (IWAENC)*, Darmstadt, Germany, Sept. 2001, pp. 31–34.

[46] S. Gannot and M. Moonen, "Subspace methods for multi-microphone speech dereverberation," in *Proc. Int. Workshop on Acoustic Echo and Noise Control (IWAENC)*, Darmstadt, Germany, Sept. 2001, pp. 47–50.

[47] L. J. Griffiths and C. W. Jim, "An alternative approach to linearly constrained adaptive beamforming," *IEEE Trans. Antennas Propagat.*, vol. 30, no. 1, pp. 27–34, Jan. 1982.

[48] B. de Vries and R. A. J. de Vries, "An integrated approach to hearing aid algorithm design for enhancement of audibility, intelligibility and comfort," in *Proc. of the IEEE Benelux Signal Processing Symposium (SPS2004)*, Hilvarenbeek, The Netherlands, Apr. 2004, pp. 65–68.

[49] J. Blauert, *Spatial Hearing: The Psychophysics of Human Sound Localisation*, MIT, Cambridge, MA, 1983.

[50] F. Wightman and D. Kistler, "The dominant role of low-frequency interaural time differences in sound localization," *Journal of the Acoustical Society of America*, vol. 91, no. 3, pp. 1648–1661, Mar. 1992.

[51] W. Hartmann, "How we localize sound," *Physics Today*, vol. 52, no. 11, pp. 24–29, Nov. 1999.