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Fullband Error Adaptation of Subband Adaptive Filters [1]

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Fullband Error Adaptation of Subband Adaptive Filters

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Abstract

For many years now, subband and frequency–domain adaptive filtering techniques have been proposed for the identification of high–order FIR systems. Standard fullband algorithms are less attractive as the implementation cost is higher and their convergence behaviour is typically worse.
Subband processing has many desirable properties. However, when used to implement adaptive filters, various side effects occur which reduce performance. On the other hand, frequency–domain adaptive filters, such as the PBFDAF, do not suffer from these problems despite being (nearly) equivalent to subband adaptive filters, be it with "poor" filter banks.
In this paper an alternative fullband error based adaptation scheme for subband adaptive systems will be proposed. It will be shown that the weight updating mechanism of the so–called unconstrained PBFDAF is closely related to the proposed fullband error adaptation algorithm.

1 Introduction

Subband and frequency–domain adaptive schemes have been a topic of interest for many years now. They are employed to identify high–order FIR systems and are a promising alternative for standard fullband adaptation algorithms such as LMS. Still, with the available multirate techniques, it is difficult to meet all the requirements.
Frequency–domain techniques are well understood [6] and hence their performance is very tractable. On the other hand, *subband adaptive filters* —at first sight— may have a lower complexity and a better performance. Unfortunately, this picture of the subband approach is certainly too optimistic.
In this paper we will focus on an alternative fullband error adaptation algorithm for subband adaptive systems and show that the unconstrained PBFDAF uses this fullband error adaptation scheme to update its filter weights. This is an attempt to generalise and extend the frequency–domain aliasing compensation techniques to subband adaptive systems.

2 Subband Adaptive Filtering

2.1 General setup

The general setup for a subband adaptive system is shown in figure 1. The loudspeaker and microphone are

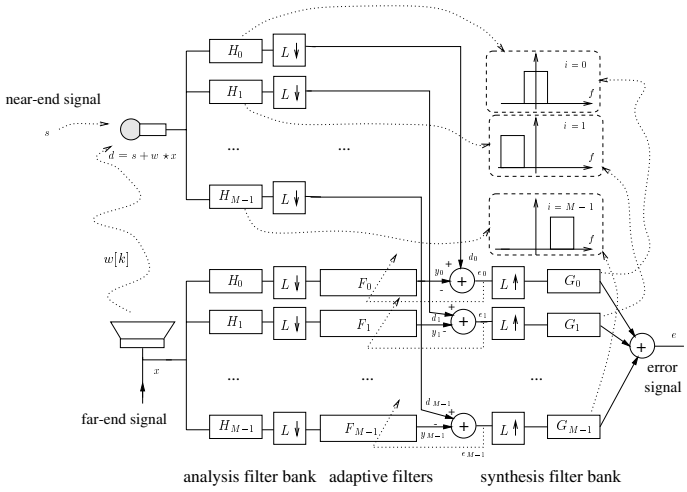


Figure 1: Subband adaptive filter with ideal filter banks: echo cancellation setup

added for convenience, indicating how the adaptive filter structure may be employed in an acoustic echo cancellation setup. While acoustic echo cancellation has been a driving application for many researchers in this field, all results obviously apply to other applications too.
The input signals x and d are fed into identical M–band analysis filter banks. After subsampling with a factor L, (mostly LMS–based) adaptive filtering is done in

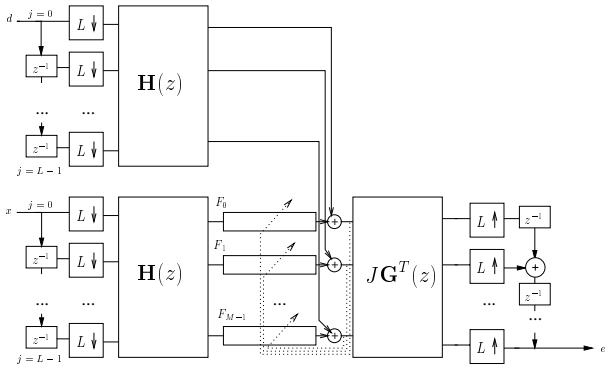


Figure 2: Subband adaptive filter : polyphase implementation

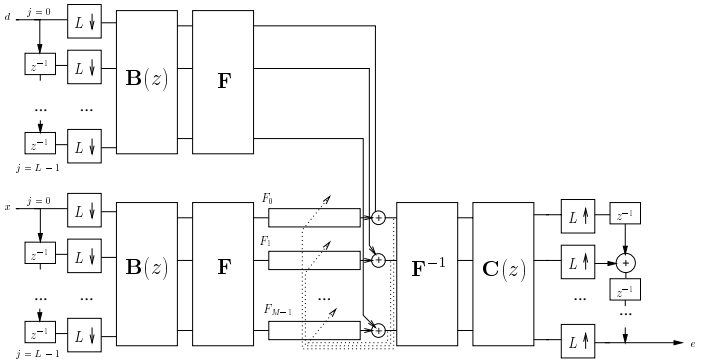


Figure 3: DFT modulated subband adaptive filter

each subband. The outputs of the subband adaptive filters are recombined in the synthesis filter bank and fed to the output. The ideal characteristics of the analysis bank filters H_i and synthesis bank filters G_i are shown (ideal bandpass filters). Due to aliasing effects, this setup will only work for $M \geqslant L$.

2.2 Polyphase implementation

The outputs of the analysis bank filters are immediately downsampled. Hence it is cheaper to do all filter operations at the downsampled rate. By re–arranging figure 1 we obtain figure 2. $\mathbf{H}(z)$ and $\mathbf{G}(z)$ are respectively called the analysis and synthesis polyphase matrix. J is the anti–diagonal matrix. Element (i, j) of $\mathbf{H}(z)$ is

$$[\mathbf{H}(z)]_{ij} = H_{i_{j:L}}(z) \qquad \begin{cases} i = 0 \to M-1 \\ j = 0 \to L-1 \end{cases} \tag{1}$$

$H_{i_{j:L}}(z)$ is the j–th out of L polyphase component of the i–th subband filter $h_i[k]$, in other words the z–transform of $h_i[j + Lk]$. Similarly,

$$[\mathbf{G}(z)]_{ij} = G_{i_{j:L}}(z) \qquad \begin{cases} i = 0 \to M-1 \\ j = 0 \to L-1 \end{cases} \tag{2}$$

2.3 DFT modulated subband adaptive filters

Subband adaptive systems are often based on DFT modulated filter banks. M subband filters are derived from a single prototype filter $h_0[k]$:

$$\begin{aligned} h_i[k] &= h_0[k] e^{-j\frac{2\pi ki}{M}}, \ i = 0 \to M-1 \\ &\Leftrightarrow \\ H_i(z) &= H_0(e^{j\frac{2\pi i}{M}} z) \end{aligned} \tag{3}$$

It appears that the analysis filters are frequency shifted versions of each other and each filter covers a part of the frequency spectrum.

DFT modulated filter banks may be implemented efficiently using polyphase decomposition and fast signal transforms. In [1] a general framework for DFT modulated subband systems was proposed. A DFT modulated filter bank with L–fold downsampling can be implemented as a tapped delay line of size L followed by a structured $M \times L$ matrix $\mathbf{B}(z)$, containing polyphase components of the prototype h_0, and an $M \times M$ DFT matrix $\mathbf{F}$. For DFT modulated filter banks figure 2 can be redrawn resulting in figure 3. The synthesis bank is constructed in a similar fashion with an $L \times M$ matrix $\mathbf{C}(z)$.

By splitting signals into subbands and subsequent subsampling faster (initial) convergence and better tracking properties are hoped for. As the adaptive computations as well as the filter bank convolutions can be done at a reduced sampling rate, the subband approach is supposed to give a better performance at a lower cost. Unfortunately, this picture of the subband approach is certainly too optimistic [2].

3 Frequency–domain Adaptive Filters

As a cheaper alternative to LMS, the frequency–domain adaptive filter (FDAF) was introduced, which is a direct translation of Block LMS in frequency domain [6]. The implementation cost for the FDAF is low, but the input–output delay introduced by the algorithm is typically too high.

The FDAF can be extended by splitting the acoustic impulse response in equal parts. In this way a kind of mixed time– and frequency–domain adaptive filter is obtained, called the Partitioned Block Frequency–Domain Adaptive Filter (PBFDAF) [5][7]. Here block lengths can be adjusted, resulting in a cheap adaptive filter with acceptable processing delay.

The L_{FB}–taps fullband adaptive filter $w^{(n)}[k]$ at block index n is partitioned in $\frac{L_{FB}}{P}$ equal parts $\mathbf{w}_{p_n}$ of length

P each :[1]

$$\mathbf{w}_{p_n} \stackrel{\forall p}{=} \begin{bmatrix} w^{(n)}[pP] \\ \vdots \\ w^{(n)}[(p+1)P-1] \end{bmatrix}, \; p = 0 : \frac{L_{FB}}{P} - 1$$

$$\mathbf{W}_{p_n} \stackrel{\forall p}{=} \mathbf{F} \begin{bmatrix} \mathbf{w}_{p_n} \\ \mathbf{0} \end{bmatrix} \begin{matrix} \updownarrow P \\ \updownarrow L-1 \end{matrix} \tag{4}$$

The equations defining the PBFDAF are[2] :

$$\mathbf{X}_{p_n} \stackrel{\forall p}{=} \text{diag}\left\{ \mathbf{F} \begin{bmatrix} x[(n+1)L - pP - M + 1] \\ \vdots \\ x[(n+1)L - pP] \end{bmatrix} \right\} \tag{5}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{0}_{P-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_L \end{bmatrix} \mathbf{F}^{-1} \sum_{p=0}^{\frac{L_{FB}}{P}-1} \mathbf{X}_{p_n} \mathbf{W}_{p_n} \tag{6}$$

$$\mathbf{d} = \begin{bmatrix} \mathbf{0} \\ \mathbf{d}_n \end{bmatrix} \begin{matrix} \updownarrow P-1 \\ \updownarrow L \end{matrix}, \quad \mathbf{d}_n = \begin{bmatrix} d[nL+1] \\ \vdots \\ d[(n+1)L] \end{bmatrix} \tag{7}$$

$$\mathbf{e} = \mathbf{d} - \mathbf{y} \tag{8}$$

$$\mathbf{W}_{p_{n+1}} \stackrel{\forall p}{=} \mathbf{W}_{p_n} + \mathbf{F} \begin{bmatrix} \mathbf{I}_P & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{L-1} \end{bmatrix} \mathbf{F}^{-1} \Delta \mathbf{X}_{p_n}^H \mathbf{F} \mathbf{e} \tag{9}$$

In each iteration L new x–samples are taken in, and L new filter output samples are produced. L is called the block length, the corresponding input/output delay is $2L-1$. $\mathbf{F}$ is an $M \times M$ DFT matrix, $\Delta = 2\text{diag}(\mu_m)$ contains the subband dependent step sizes and $M = P + L - 1$. If P is divisible by L (which is typically the case), $\mathbf{X}_{p_n} = \mathbf{X}_{0_{n-pP/L}}$ and hence equation 5 requires only 1 DFT operation, which corresponds to $p = 0$. The other $\mathbf{X}_{p_n}$ can be recovered from previous iterations. Very often M is chosen equal to $M = P+L-1+s = 2^r$ with $s > 0$ and r an integer. In most practical applications $P = L$.

There exists two variants of this algorithm, called the constrained and the unconstrained PBFDAF. For the unconstrained version the weight update compensation is left out resulting in the following weight update equation :

$$\mathbf{W}_{p_{n+1}} \stackrel{\forall p}{=} \mathbf{W}_{p_n} + \Delta \mathbf{X}_{p_n}^H \mathbf{F} \mathbf{e} \tag{10}$$

The unconstrained updating requires 3 DFTs per iteration whereas the constrained PBFDAF is more expensive, having an extra $\frac{2L_{FB}}{P}$ DFTs to compute. The latter on the other hand has better convergence properties.

The PBFDAF turns out to be a special subband adaptive filter having interesting convergence properties at a low implementation cost [2]. Despite the low frequency–selective filter banks upon which the PBFDAF is based the algorithm doesn't seem to suffer from aliasing effects and hence the convergence properties are comparable to those of LMS.

[1] We assume that $\frac{L_{FB}}{P}$ is integer.

[2] For signal conventions : see figure 1

4 Alternative adaptation scheme

In an attempt to generalise and extend the frequency–domain error correction to subband adaptive systems, we now focus on an alternative adaptation scheme for subband adaptive systems, which adjusts the subband filters F_m (see figure 4) based on the fullband error e instead of using the subband error signals $\epsilon_m = d_m - y_m$ directly as is normally done in a standard subband scheme (see figure 1).

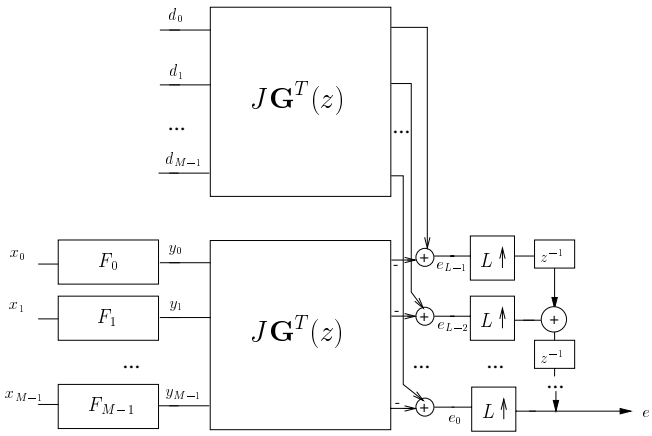


Figure 4: Oversampled subband adaptive filter : synthesis part

4.1 Fullband error adaptation scheme

Define :

$$\underbrace{\begin{bmatrix} Y_0^k \\ \vdots \\ Y_{M-1}^k \end{bmatrix}}_{\mathbf{Y}_k} = \underbrace{\begin{bmatrix} X_0^k & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & X_{M-1}^k \end{bmatrix}}_{\mathbf{X}_k} \underbrace{\begin{bmatrix} F_0^k \\ \vdots \\ F_{M-1}^k \end{bmatrix}}_{\mathbf{w}_k}, \tag{11}$$

in which (see figure 4)

$$X_m^k = \begin{bmatrix} x_m[k - L_f + 1] & \dots & x_m[k - L_f - L_{SB} + 2] \\ \vdots & \ddots & \vdots \\ x_m[k] & \dots & x_m[k - L_{SB} + 1] \end{bmatrix} \tag{12}$$

$$F_m^k = \begin{bmatrix} f_m^k[0] \\ \vdots \\ f_m^k[L_{SB} - 1] \end{bmatrix}, \quad Y_m^k = \begin{bmatrix} y_m[k - L_f + 1] \\ \vdots \\ y_m[k] \end{bmatrix} \tag{13}$$

$\mathbf{w}_k$ is the adaptive filter vector. L_{SB} is the length[3] of the subband adaptive filters, L_f is the length of the synthesis bank polyphase filters. Further define

$$\underbrace{\begin{bmatrix} e_0[k] \\ \vdots \\ e_{L-1}[k] \end{bmatrix}}_{\mathbf{E}_k^{FB}} = \underbrace{\begin{bmatrix} S_{0_{0:L}} & \dots & S_{M-1_{0:L}} \\ \vdots & \ddots & \vdots \\ S_{0_{L-1:L}} & \dots & S_{M-1_{L-1:L}} \end{bmatrix}}_{\mathbf{S}^T} \underbrace{\begin{bmatrix} \epsilon_0^k \\ \vdots \\ \epsilon_{M-1}^k \end{bmatrix}}_{\mathbf{E}_k^{SB}} \tag{14}$$

in which

$$\underbrace{\begin{bmatrix} \epsilon_0^k \\ \vdots \\ \epsilon_{M-1}^k \end{bmatrix}}_{\mathbf{E}_k^{SB}} = \underbrace{\begin{bmatrix} D_0^k \\ \vdots \\ D_{M-1}^k \end{bmatrix}}_{\mathbf{D}_k} - \underbrace{\begin{bmatrix} Y_0^k \\ \vdots \\ Y_{M-1}^k \end{bmatrix}}_{\mathbf{Y}_k}, \tag{15}$$

$$D_m^k = \begin{bmatrix} d_m[k-L_f+1] \\ \vdots \\ d_m[k] \end{bmatrix}, \quad \epsilon_m^k = \begin{bmatrix} \epsilon_m[k-L_f+1] \\ \vdots \\ \epsilon_m[k] \end{bmatrix} \tag{16}$$

$$S_{m_{l:L}} = \begin{bmatrix} g_{m_{l:L}}[L_f-1] & \dots & g_{m_{l:L}}[0] \end{bmatrix} \tag{17}$$

$\mathbf{E}_k^{FB}$ is the fullband error vector, $\mathbf{E}_k^{SB}$ contains the subband errors, $\mathbf{S}$ represents the synthesis filter bank and $g_{m_{l:L}}[k] = g_m[kL+l]$ is the l–th out of L polyphase components of the m–th synthesis filter $g_m[k]$.
Optimal error suppression is obtained when Eq. 14 is as small as possible :

$$\min_{\mathbf{w}_k} \mathcal{E}\left\{||\mathbf{E}_k^{FB}||_2^2\right\} \tag{18}$$

The optimal $\mathbf{w}$ is

$$\mathbf{w}_{\text{opt}} = \arg\min_{\mathbf{w}_k} ||\mathbf{S}^T\mathbf{D}_k - \mathbf{S}^T\mathbf{X}_k\mathbf{w}_k||_2^2. \tag{19}$$

In practice the optimal subband filters are estimated adaptively using a steepest descent algorithm. The gradient with respect to $\mathbf{w}_k$ is

$$\nabla\mathbf{w}_k = 2\mathbf{X}_k^H\mathbf{S}^*\mathbf{S}^T(\mathbf{X}_k\mathbf{w}_k - \mathbf{D}_k). \tag{20}$$

Therefore, $\mathbf{w}_{\text{opt}}$ can be found in an iterative way using a fullband error block adaptation algorithm :

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 2\Delta\mathbf{X}_k^H\mathbf{S}^*\mathbf{S}^T(\mathbf{D}_k - \mathbf{X}_k\mathbf{w}_k) \tag{21}$$

$$= \mathbf{w}_k + 2\Delta\mathbf{X}_k^H\mathbf{S}^*\mathbf{S}^T\mathbf{E}_k^{SB} \tag{22}$$

with $\Delta = \text{diag}(\mu_m)$ a diagonal matrix with subband dependent stepsizes. The subband errors are passed through the synthesis bank twice before they are combined with the far–end signal x.

[3]We assume that all filters are FIR.

In a standard subband adaptation scheme the subband errors are minimised, leading to

$$\min_{\mathbf{w}_k} \mathcal{E}\left\{||\mathbf{E}_k^{SB}||_2^2\right\}. \tag{23}$$

The gradient with respect to $\mathbf{w}_k$ is

$$\nabla\mathbf{w}_k = 2\mathbf{X}_k^H(\mathbf{X}_k\mathbf{w}_k - \mathbf{D}_k). \tag{24}$$

Therefore, the optimal $\mathbf{w}$ can be found in an iterative way by optimising the subband errors :

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 2\Delta\mathbf{X}_k^H\mathbf{E}_k^{SB}. \tag{25}$$

Equation 25 corresponds to a standard subband weights updating algorithm using a block–LMS algorithm in each band. The compensation matrix $\mathbf{S}^*\mathbf{S}^T$ is absent here. It is hoped that employing the fullband error adaptation scheme i.o. subband error adaptation will lead to improved performance.

4.2 PBFDAF weight updating

The alternative adaptation scheme studied in paragraph 4.1, leading to a weight updating equation based on fullband errors (Eq. 22) is now applied to the PBFDAF.
For DFT modulated filter banks the synthesis bank polyphase matrix $\mathbf{G}(z)$ can be written as

$$\mathbf{F}^{-T}\mathbf{C}^T(z)J. \tag{26}$$

For the PBFDAF it can be proven [2] that when P is a multiple of L

$$\mathbf{C}(z) = \underbrace{\begin{bmatrix} \mathbf{I}_L & \mathbf{0}_L & \dots & \mathbf{0}_L & \mathbf{0} \end{bmatrix}}_{M} = \mathbf{C} \tag{27}$$

is independent of z and hence also $\mathbf{G}(z)$ is independent of z. This means the synthesis polyphase filters have length $L_f = 1$. $\mathbf{S}$ as defined in Eq. 14 then equals $\mathbf{G}(z) = \mathbf{S} = \mathbf{F}^{-T}\mathbf{C}^TJ$. For the PBFDAF the update equation 22 then becomes

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 2\frac{\Delta}{M}\mathbf{X}_k^H\mathbf{F}\mathbf{C}^T\mathbf{C}\mathbf{F}^{-1}(\mathbf{D}_k - \mathbf{X}_k\mathbf{w}_k) \tag{28}$$

$$= \mathbf{w}_k + 2\frac{\Delta}{M}\mathbf{X}_k^H\mathbf{F}\begin{bmatrix} \mathbf{I}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{M-L} \end{bmatrix}\mathbf{F}^{-1}\mathbf{E}_k^{SB} \tag{29}$$

$\mathbf{E}_k^{SB}$ are the subband errors, $\mathbf{F}\mathbf{C}^T\mathbf{C}\mathbf{F}^{-1}$ does the error correction. It can be proven (see [4]) that the weight update equation of a so–called unconstrained PBFDAF for which P is a multiple of L (Eq. 10) is closely related to equation 29, derived here based on fullband error feedback.
One can observe that the subband filters are adapted

based on corrected subband errors. The correction implies passing the subband errors through the synthesis filter bank in order to obtain a fullband, i.e. an aliasing–free, error signal. This in fact refers to the overlap–save or overlap–add procedure upon which frequency–domain techniques are based. It is believed that a more general class of subband adaptive systems which are based on a filter bank set fulfilling 3 realisation conditions of frequency selectivity, perfect reconstruction and time–invariance modelling [2], and provided with the adaptation mechanism presented in paragraph 4.1, could approximate the performance of the frequency–domain approach.

4.3 Computational complexity

To update the weights with the fullband error adaptation scheme, the subband errors are passed through the synthesis bank twice (Eq. 22). The corresponding cost is therefore rather high. As $\mathbf{S}$ is an $ML_f \times L$ matrix, the extra cost compared to a subband–error based adaptation will be

LML_f cplx MACs	$\mathbf{S}^T$∗vector
LML_f cplx MACs	$\mathbf{S}^*$∗vector

cplx MAC stands for a complex multiply–accumulate operation and comes down to a complex multiplication and a complex addition, which is equivalent to 4 real multiplications and 4 real additions.
The cost reduces however when the filter banks are DFT modulated and e is a real signal. Only half of the subbands need to be computed as they are pairwisely complex conjugated. Furthermore, $\mathbf{S}^*\mathbf{S}^T$ can be simplified. For DFT modulated filter banks the transpose of the synthesis bank polyphase matrix can be written as

$$\mathbf{G}^T(z) = J\mathbf{C}(z)\mathbf{F}^{-1}. \tag{30}$$

$\mathbf{S}^T$ as defined in Eq. 14 and 17 is closely related to $\mathbf{G}^T(z)$ and can be expressed as

$$\mathbf{S}^T = J\mathbf{Q}\hat{\mathbf{F}}^{-1}. \tag{31}$$

$\hat{\mathbf{F}}$ is an extended DFT–matrix and is defined as[4]

$$\hat{\mathbf{F}} = \mathbf{F} \otimes I_{L_f}. \tag{32}$$

$\mathbf{Q}$ is a block $L \times M$ matrix and is constructed from $\mathbf{C}(z)$ in the same way as $\mathbf{S}^T$ was derived from $\mathbf{G}^T(z)$. Hence, each block element of $\mathbf{Q}$ is a real–valued $1 \times L_f$ vector such that $\mathbf{Q}$ becomes

$$\mathbf{Q} = \begin{bmatrix} C_{0,0} & \dots & C_{0,M-1} \\ \vdots & \ddots & \vdots \\ C_{L-1,0} & \dots & C_{L-1,M-1} \end{bmatrix} \tag{33}$$

[4] $\otimes$ is the Kronecker product. I_{L_f} is the identity matrix of order L_f.

with

$$C_{n,m} = \begin{bmatrix} c_{n,m}[L_f-1] & \dots & c_{n,m}[0] \end{bmatrix}. \tag{34}$$

$c_{n,m}$ refers to element (n,m) of $\mathbf{C}(z)$, which is equal to $c_{n,m}[0]+z^{-1}c_{n,m}[1]+\ldots+z^{-L_f+1}c_{n,m}[L_f-1]$. Equation 22 now becomes

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{2}{M}\Delta\mathbf{X}_k^H\hat{\mathbf{F}}\mathbf{Q}^T\mathbf{Q}\hat{\mathbf{F}}^{-1}\mathbf{E}_k^{SB}. \tag{35}$$

The matrix–vector product $\hat{\mathbf{F}}^{-1}\mathbf{E}_k^{SB}$ comes down to a set of IFFT–operations : for each of the L_f time–lags in $\mathbf{E}_k^{SB}$ an M–point IFFT needs to be computed. The algorithm can be further simplified as L_f-1 IFFTs can be taken from a previous block. Although this simplification is only approximate, the performance stays more or less the same.
For DFT modulated filter banks $\mathbf{C}(z)$ typically has the same structure as $\mathbf{B}(z)$ and hence it is a sparse matrix [1][2]. As a result only about LL_f of the LML_f elements of $\mathbf{Q}$ are non–zero. The cost associated with $\frac{1}{M}\hat{\mathbf{F}}\mathbf{Q}^T\mathbf{Q}\hat{\mathbf{F}}^{-1}$ therefore is

$2M\log_2 M$ ops	1 M–point real–output IFFT
LL_f real MACs	$\mathbf{Q}$∗vector
LL_f real MACs	$\mathbf{Q}^T$∗vector
$L_f(2M\log_2 M - 4M + 6)$ ops	L_f M–point real–input FFTs

4.4 Simulation example

A random FIR system of order 15 was generated and fed with white noise. A 6–band, 4–fold downsampled analysis/synthesis filter bank set was designed following the method described in [3] such that the perfect reconstruction property

$$\mathbf{C}(z)\mathbf{B}(z) = z^{-\delta}\mathbf{I}_L. \tag{36}$$

is fulfilled. Both the analysis and the synthesis filters are FIR, DFT modulated and are of order 27. The frequency characteristics of the analysis prototype filter are shown in figure 5. Based on this filter bank set, different subband adaptive algorithms are compared. They all have subband filters of length $\frac{L_{FB}}{L} = 4$. Figure 6 shows the time evolution of the error for the following subband adaptive filters : a standard subband adaptive system (cf. figure 1) with LMS–adapted subband filters (upper curve), a subband adaptive filter using block–LMS weight updating according to Eq. 25 (middle curve) and a subband implementation using the fullband error adaptation scheme presented in Eq. 22 (bottom curve). The fullband error scheme clearly outperforms the standard subband adaptive filter.

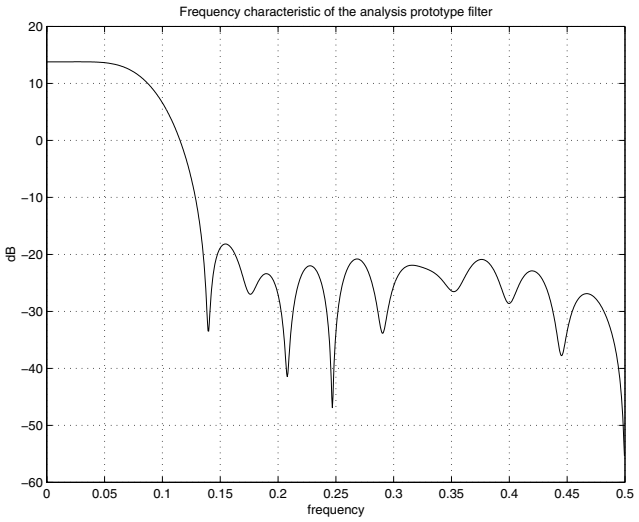


Figure 5: Frequency characteristics of the analysis prototype filter.

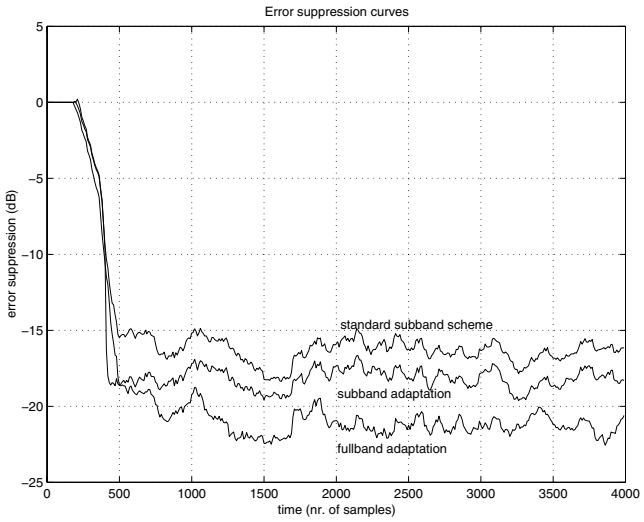


Figure 6: Convergence curves of 3 different subband adaptive systems. The FIR system to be identified is of order 15.

5 Conclusions

For the identification of high–order FIR systems efficient adaptive algorithms, such as subband and frequency–domain adaptive filters, are required. Frequency–domain adaptive algorithms turn out to have many desirable properties. We presented an adaptation scheme that feeds back the fullband error, which can be considered as a generalisation and extension of frequency–domain error correction techniques to subband adaptive systems.

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- Research Project F.W.O. nr. G.0295.97 ('Design and implementation of adaptive digital signal processing algorithms for broadband applications')

The scientific responsibility is assumed by its authors.

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