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A RELATION BETWEEN SUBBAND AND FREQUENCY-DOMAIN ADAPTIVE FILTERING

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Abstract- Modelling acoustic impulse responses with lengths up to 250 ms is required for high quality echo cancellation in strongly reverberating environments, leading to FIR adaptive filters with several thousands of taps. Classical LMS-based solutions then clearly fail as they exceed the computational capabilities of present-day DSPs. Cheaper alternative solutions have been proposed and are mainly based on either subband or frequency-domain techniques. Subband filter implementations have many interesting properties but their inherent delay and residual errors have made them unattractive for real-time applications up till now. On the other hand, frequency-domain adaptive filters do not suffer from these problems despite being (nearly) equivalent to subband adaptive filters. In this paper we specify 3 realisation conditions for DFT modulated subband schemes. Standard subband adaptive filters cannot fulfil all conditions. We explain the operation of the frequency-domain techniques in the 'subband jargon' and show that the realisation conditions can be fulfilled in this case.

1. INTRODUCTION

For high quality echo cancellation long acoustic echoes need to be suppressed. Acoustic echo paths are characterised by FIR filters with lengths up to 250 ms. Filters clocked at a rate of, say, 10 kHz then require several thousands of filters taps to be identified. Classical LMS-based echo cancellers are unattractive for real-time processing as their computational requirements clearly exceed the capabilities of present-day DSPs. Moreover, speech signals have a coloured spectrum and it is well known that the performance of the LMS algorithm is suboptimal in that case, especially when extremely long FIR filters are being adapted. Therefore alternative solutions have been proposed and they are mainly based on either subband or frequency-domain techniques. Such multirate adaptive echo cancellation schemes have been a topic of interest for many years now. Subband adaptive filters and frequency-domain based techniques are mostly considered as being different approaches. In this paper we will consider the frequency-domain approach as a special case of subband adaptive filtering having some desired properties and point out why frequency-domain techniques are better from certain perspectives, or -at least- are able to compete with subband schemes. It is hoped that this study will eventually lead to improved subband techniques, which is a topic of current further research.

2. SUBBAND ADAPTIVE FILTERING

The general setup for a subband acoustic echo canceller is shown in figure 1. The loudspeaker and microphone signal are fed into identical M-band analysis filter banks. After subsampling with a factor L, (mostly LMS-based) adaptive filtering is done in each subband and finally the outputs of the subband adaptive filters are recombined in the synthesis filter bank. Due to aliasing effects, this setup will only work for $M \geq L$. The ideal frequency amplitude characteristics of the analysis bank filters H_i and synthesis bank filters G_i are shown (ideal bandpass filters).

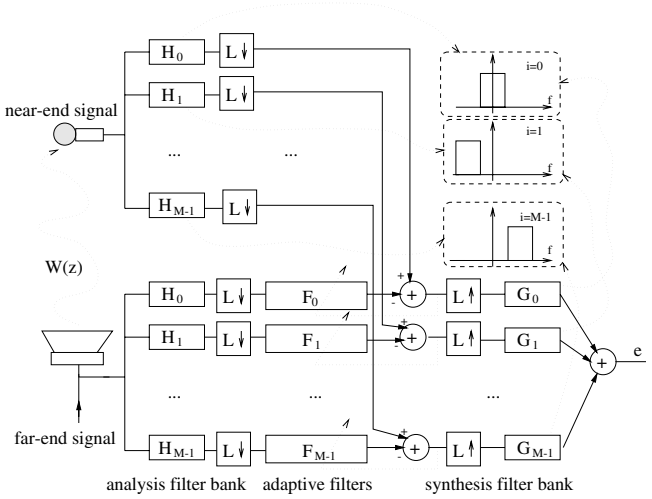


Fig. 1. subband adaptive echo canceller

If L is chosen equal to M a critically downsampled subband adaptive filter is being implemented. Such subband systems are attractive because optimal computational savings can be obtained when L is as high as possible. In [1] it is shown that critically downsampled subband systems lead to a residual modelling error which is considerable unless cross filters are included between neighbouring subbands. Cross filters again increase the complexity, which is unwanted. Furthermore, cross filters fail to converge quickly to the optimum solution. This suggests the use of oversampled subband schemes where $M > L$.

2.1. DFT modulated Subband Schemes

Oversampled subband acoustic echo cancellers are mainly based on DFT modulated filter banks. M

subband filters are derived by frequency shifting a well-designed prototype lowpass filter $h_0(k)$. DFT modulated filter banks lead to efficient implementation by using polyphase decomposition and fast signal transforms. In [2] a general framework for oversampled DFT modulated subband systems was proposed. A DFT modulated filter bank with L-fold downsampling can be implemented as a tapped delay line of size L followed by a structured $M \times L$-matrix $\mathbf{B}(z)$, containing polyphase components of the prototype h_0, and an $M \times M$-DFT matrix $\mathbf{F}$. In case of DFT modulated filter banks, figure 1 can be redrawn resulting in figure 2. It can be shown that element (i,j) of $\mathbf{B}(z)$ is given by :

$$B_{ij}(z) = z^{-l} E_{(j+lL):K}(z^J) \tag{1}$$

$$i,j \geq 0,\ (j+lL) \bmod M = i,\ (i-j) \bmod g = 0,$$
$$J = \frac{M}{g},\ g = \gcd(M,L),\ K = \frac{ML}{g}$$

$E_{k:K}(z)$ is the k-th K-th order polyphase component of the prototype filter h_0. The synthesis bank is constructed in a similar fashion with matrix $\mathbf{C}(z)$.
A DFT modulated analysis/synthesis filter bank set is (preferably) designed such that the following 3 realisation conditions are met :

- the analysis filters are frequency selective. This prevents inserting too much inter-subband aliasing components by downsampling as it has an inhibitive effect on the convergence of the adaptive filters.
- the analysis/synthesis filter bank set is (nearly) perfect reconstructing i.e. a near-end source signal is not distorted by the analysis/synthesis system. A condition ensuring perfect reconstruction is

$$\mathbf{C}(z)\mathbf{B}(z) = I \tag{2}$$

- the acoustic path can be modelled by finite-length adaptive subband filters. In general, the lower branch in fig. 2 models a time-varying periodic system (see [3]). Only when

$$\mathbf{C}(z)\mathbf{F}^{-1}\mathrm{diag}\{F_i(z)\}\mathbf{F}\mathbf{B}(z) =$$

$$\underbrace{\begin{bmatrix} W_0 & W_1 & \dots & W_{L-1} \\ z^{-1}W_{L-1} & W_0 & \dots & W_{L-2} \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}}_{=\text{pseudo-circulant}} \tag{3}$$

the following time-invariant path is being modelled:

$$W(z) = \sum_{l=0}^{L-1} z^{-l} W_l(z^L) \tag{4}$$

The above conditions are necessary conditions to ensure complete modelling with a set of finite-length adaptive filters.

2.2. Subband Echo Cancellation : Performance

Splitting signals into subbands seems very promising since for coloured input spectra, fullband convergence is slow due to ill-conditioned covariance matrices. In the subband case, each subband signal will have a flatter spectrum after appropriate subsampling, leading to improved convergence. Instead of a single fullband N-taps FIR filter, M subband filters of, say, $\frac{N}{L}$ taps are used to model the acoustic path. As the adaptive computations as well as the filter bank convolutions can be done at a reduced sampling rate, this subband approach is supposed to give a better performance at a lower cost.

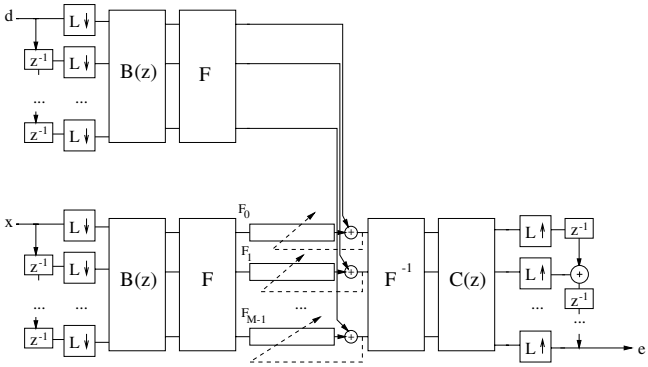


Fig. 2. DFT modulated subband adaptive echo canceller

It is clear that this picture is certainly too optimistic. The assumption of having M subband filters with reduced length $\frac{N}{L}$ seems to be quite wrong. It appears that in the case of M-band, L-fold downsampled ideally frequency selective filter banks the adaptive filters should converge to an L-fold downsampled convolution of the acoustic path and a double-sided sinc. In fact, this corresponds to an interpolation operation. The adaptive identification process therefore has to track more than $\frac{N}{L}$ samples and due to the spreading out in both directions of the time axis, an extra delay has to be inserted in the near-end signal path [4]. Neglecting the additional subband filter length due to these sinc-effects strongly limits the convergence of the adaptive filters and leads to a residual under-modelling error.
Furthermore, delay constraints also make subband schemes unattractive. Selective filter banks are needed to avoid aliasing distortion within subbands. They introduce a substantial processing delay and thus put a constraint on the downsampling factor L. However, the implementation cost is more or less inversely proportional to L.
Filter bank sets can be designed such that the first 2 realisation conditions are met. As indicated, condition 3 can not be met with finite-length filters in this standard subband approach.

3. FREQUENCY-DOMAIN ADAPTIVE FILTERS

As a cheaper alternative to LMS, the frequency-domain adaptive filter (FDAF) was introduced, which is a direct translation of Block LMS in frequency domain [5]. Correlation (weight updating) and convolution (filtering) operations are expensive but in the case of block processing, they may be implemented more efficiently in frequency domain. Instead of a linear convolution/correlation a circular operation is

performed. This requires some 'restore' operations which can be of the overlap-save or overlap-add type. If only the convolution operation is corrected a so-called unconstrained FDAF is obtained requiring 3 FFTs. Two more FFTs are needed for the gradient estimate correction resulting in a constrained FDAF. A major drawback concerning standard frequency-domain adaptive filters is the inherent delay.

3.1. Partitioned Block FDAF

By splitting the acoustic impulse response in equal parts, a kind of mixed time and frequency convolution canceller is obtained, called Partitioned Block Frequency-Domain Adaptive Filter (PBFDAF) [6]. Here block lengths can be adjusted, resulting in a cheap echo canceller with acceptable processing delay.

3.2. PBFDAF as a special case of Subband Adaptive Filtering

The PBFDAF scheme can be put into the oversampled subband framework proposed in [2]. Call $X(z)$ the far-end signal and $D(z)$ the near-end signal, so

$$D(z) = S(z) + W(z)X(z) \tag{5}$$

$S(z)$ is the contribution of a near-end source. The acoustic impulse response $W(z)$ can be split up in its P-th order polyphase components

$$D(z) = S(z) + \sum_{p=0}^{P-1} z^{-p} W_p(z^P) X(z) \tag{6}$$

This leads to equation 7 ($M = P + L - 1$), which is rewritten as

$$\mathbf{D}(z) = \mathbf{S}(z) + \mathbf{M}(z)\mathbf{X}(z) \tag{8}$$

Transfer matrix $\mathbf{M}(z)$ was made circulant so that it can be transformed into a diagonal matrix by means of DFT operations, i.e., $\mathbf{F}\mathbf{M}(z)\mathbf{F}^{-1} = \mathrm{diag}\{\hat{W}_i(z^P)\}$. $\hat{W}_i(z)$ are related to the DFT coefficients of the first column of $\mathbf{M}(z)$ and therefore they are of finite length. Instead of identifying a "half-full" matrix $\mathbf{M}(z)$, a diagonal matrix can be tracked in frequency domain. An adaptive identification process trying to match $W(z)$ in frequency domain based on the above formulas is depicted in figure 3.

Looking closer, fig. 3 can be cast in the oversampled subband framework of fig. 2, i.e. with size L (instead of size M) tapped delay lines. The $\mathbf{B}(z)$-matrix for an M-band L-fold oversampled DFT modulated analysis filter bank is a structured matrix satisfying Eq. 1. The filter bank used here is a simple DFT filter bank for which only $E_{k:K}(z) = 1$, $k < M$, are non-zero i.e.

$$\underbrace{\mathbf{B}(z)}_{M\times L} = \begin{bmatrix} \mathbf{I}_L \\ z^{-1}\mathbf{I}_L \\ \vdots \\ z^{-(\lfloor \frac{M-1}{L} \rfloor - 1)}\mathbf{I}_L \\ \mathbf{R}(z) \end{bmatrix} \tag{9}$$

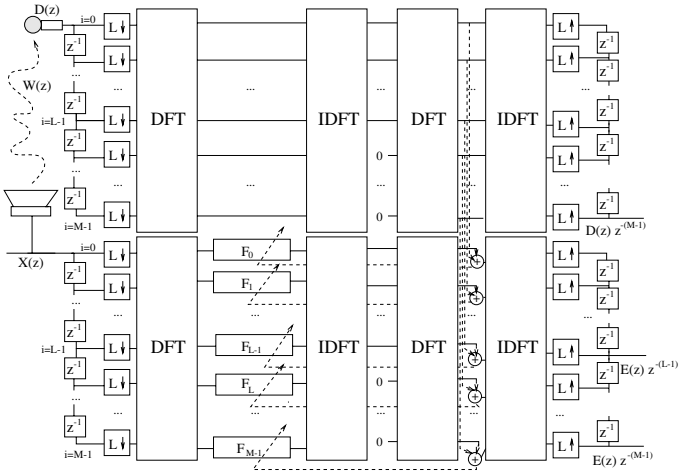


Fig. 3. Partitioned block frequency-domain adaptive filtering

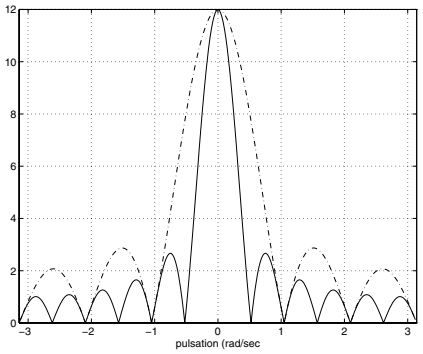


Fig. 4. analysis and synthesis bank prototype filters

and $\mathbf{R}(z)$ is an $(M - L\lfloor \frac{M-1}{L} \rfloor) \times L$-matrix :

$$\mathbf{R}(z) = \begin{bmatrix} z^{-\lfloor \frac{M-1}{L} \rfloor} & \dots & 0 \\ \vdots & \ddots & \vdots \\ \dots & z^{-\lfloor \frac{M-1}{L} \rfloor} & \dots \end{bmatrix} \tag{10}$$

The prototype frequency response has a sinc-like shape with a low frequency selectivity. The analysis prototype frequency amplitude response in shown in figure 4 in full line for M=12 and L=6.

Also the synthesis part can be fit into the subband filter approach. The synthesis bank $\mathbf{C}(z)$-matrix is given by :

$$\underbrace{\mathbf{C}(z)}_{L\times M} = \begin{bmatrix} \mathbf{I}_L & \mathbf{0}_L & \dots & \mathbf{0}_L & \mathbf{0} \end{bmatrix} \tag{11}$$

The synthesis filters are time-reversed and complex conjugated versions of a DFT modulated filter bank with prototype polyphase components $E_{k:K}(z) = 1$, $k < L$. The other polyphase components are zero. The synthesis prototype frequency response for M=12 and L=6 is shown in fig. 4 in dashed dotted line. Its frequency response is twice as wide as the analysis equivalent.

It is easily verified that the perfect reconstruction condition (Eq. 2) holds. The additional condition (Eq. 3) is also satisfied for $F_i(z) = \hat{W}_i(z^{\frac{P}{L}})$ if P is a multiple of L. It is known that in this case extra savings can be made since signal buffers can be recuperated from previous block cycles (see [6]).

$$\begin{bmatrix} D(z) \\ z^{-1}D(z) \\ \vdots \\ z^{-(L-1)}D(z) \\ * \\ \vdots \\ * \end{bmatrix} = \begin{bmatrix} S(z) \\ z^{-1}S(z) \\ \vdots \\ z^{-(L-1)}S(z) \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} W_0(z^P) & \dots & W_{P-1}(z^P) & 0 & \dots & 0 \\ 0 & W_0(z^P) & \dots & W_{P-1}(z^P) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & W_0(z^P) & \dots & W_{P-1}(z^P) \\ W_{P-1}(z^P) & \dots & 0 & 0 & W_0(z^P) & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ W_1(z^P) & \dots & 0 & 0 & \dots & W_0(z^P) \end{bmatrix} \begin{bmatrix} X(z) \\ z^{-1}X(z) \\ \vdots \\ z^{-(P-1)}X(z) \\ \vdots \\ z^{-(M-1)}X(z) \end{bmatrix} \tag{7}$$

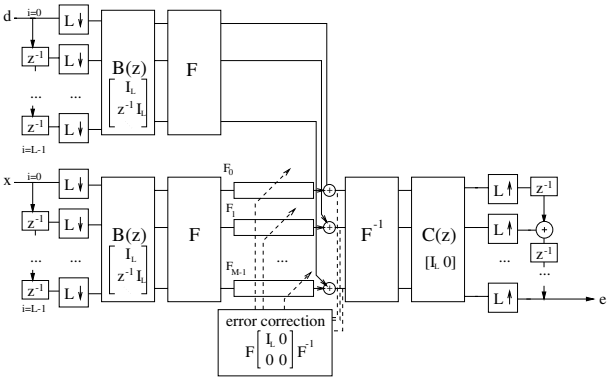


Fig. 5. General oversampled subband scheme

The implicit error 'restore' or projection operation in frequency-domain adaptive filtering consists of a transformation to time domain, zeroing of certain components and transformation back to frequency domain (see figure 3). A general scheme, shown in figure 5, now depicts the PBFDAF completely in the subband 'jargon'. An extra module called 'error correction' was included to do the circular-to-linear conversion as no projection operations are applied in the standard subband approach.

The PBFDAF turns out to be a special case of subband adaptive filtering. It implements a simple filter bank with low frequency selectivity. The PBFDAF satisfies all 3 realisation conditions, so there won't be any residual error as a complete and unique modelling can be done with finite-length filters. It is remarkable how an unselective filter bank can lead to satisfactory results.

In the previous, M was set equal to $P+L-1$. But of course, there is an extra degree of freedom of choosing $M > P+L-1$. This can have some advantages. If M is a power of 2 efficient FFT implementation is possible. Now, in this case an extra ambiguity occurs as the subband filters $F_i(z)$ are not uniquely defined anymore.

A random $\delta(z^P)$ can appear for instance as an extra $(P+1)$-th polyphase component provided it is compensated for at the first component (see Eq. 7):

$$W_0(z^P) \rightarrow W_0(z^P) - z^{-P}\delta(z^P) \tag{12}$$

Similar terms can be added to other polyphase components. The ambiguity can be removed by back-transforming $M-P+1$ filtered components instead of L. A more accurate gradient estimate is obtained as now $M-P+1$ past estimates are being averaged. Block length L has remained the same, so tracking performance has not gone down. Simply taking $M > P+L-1$ is thus inefficient in a sense as with a little amount of extra cost (some extra additions) performance can be approved. As $\delta(z^P)$ is random no control on its amplitude is possible, so there is a possibility of running into numerical trouble with unconstrained PBFDAF.

4. CONCLUSIONS

Cancelling long echo paths requires efficient adaptive algorithms. Both the subband and frequency-domain approach turn out to have their strong and weak points. We specified 3 conditions for appropriate subband modelling and showed that the PBFDAF fulfils all 3 conditions being a simple filter bank with low frequency selectivity.

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