Optimal Control of Towing Kites

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***Abstract*— In this paper we present a challenging application of periodic optimal control. A kite that is towing a ship into a given target direction should fly optimal loops. We show how to find the maximum average tractive force by controlling the roll angle of the towing kite taking into account that the wind is increasing with the altitude over the sea. The optimal control problem for this highly nonlinear and unstable system has periodicity constraints, free initial values, and a free cycle duration. For its solution, we use MUSCOD-II, an optimal control package based on the direct multiple shooting method. Finally, we discuss the influence of an important design parameter, the effective glide ratio of the kite.**

Key words: **Periodic Optimal Control, Kite Modeling, Ship Propulsion.**

I. Introduction

Towing kites, patented by the SkySails company [7, 14], use wind energy to pull a ship into a given target direction. An important advantage in comparison to usual sails is that kites can use the powerful wind at high altitudes. In addition, they can fly periodic loops in crosswind direction to considerably increase the tractive power, as first discovered by Loyd [11]. The SkySails concept is already close to industrial use.

In this paper, we are interested in the optimization of these loops to achieve a maximum average tractive force in the ship's target direction. For this aim, we use the robust optimal control package MUSCOD-II, that is based on the direct multiple shooting method and uses a full space SQP-algorithm [8, 10].

To formulate and implement our optimal control problem, we improve a kite model, that was originally proposed in [3], [4] and [5]. To include the dependence of the wind on the altitude over the sea, we use a wind shear model [12]. Furthermore, we take into account that not only the aerodynamic drag of the kite but also the friction of the cable will slow the system down. The objective function is the average tractive force, while the state constraints are periodic boundary conditions to achieve a periodic loop simulation.

The selection of the parameters for our towing kite model allows us to assess the possibility of a powerful high-speed kite including a critical discussion of the technical feasibility. The result is considered in dependence on the angle between the wind and the ship's target direction. Motivated by the fundamental estimations in [11] we also determine the influence of the effective glide ratio on the performance of a towing kite.

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We first show, in Section II, how the differential equations for a towing kite can be found by collecting the forces on the kite. In Section III the optimal control problem is formulated. The results of our numerical studies are presented in Section IV, where we also discuss the influence of the effective glide ratio. Finally, we conclude the paper in section V.

II. The towing kite model

We consider a ship sailing with velocity ve_x, where e_x is a unit vector x-direction. Let e_z be an unit vector pointing to the sky and $e_y := e_z \times e_x$, such that $\{e_x, e_y, e_z\}$ is an orthonormal right-handed basis of the 3-dimensional Euclidean space. Furthermore, a kite is connected to the ship by a tight cable with length r, as illustrated in Fig. 1.

To derive the kite's equation of motion, we assume the ship to be an inertial system. Additionally, we assume that the wind has a constant horizontal direction forming an angle δ to the ship's target direction. Using the wind shear model, as it is proposed in [12], the wind velocity w is depending on the altitude h over the sea:

$$w(h) = \frac{\ln\left(\frac{h}{h_r}\right)}{\ln\left(\frac{h_0}{h_r}\right)} v_0 \,, \tag{1}$$

where v_0 is the wind velocity at a suitable altitude h_0 and h_r the roughness length. Now, the effective wind vector w_e^s in the ship's system is given by

$$w_e^s = (w\cos(\delta) - v)\, e_x + w\sin(\delta) e_y \,. \tag{2}$$

For our purposes it is useful to introduce spherical coordinates r, ϕ and θ to describe the position and the orientation of the kite. We define the local orthonormal basis $\{e_r, e_\phi, e_\theta\}$ by

$$\begin{aligned} e_r &:= \sin(\theta)\cos(\phi) e_x + \sin(\theta)\sin(\phi) e_y + \cos(\theta) e_z \,, \\ e_\phi &:= -\sin(\phi) e_x + \cos(\phi) e_y \,, \\ e_\theta &:= -\cos(\theta)\cos(\phi) e_x - \cos(\theta)\sin(\phi) e_y + \sin(\theta) e_z \,. \end{aligned}$$

Henceforth, we will write all vectors with respect to this local basis - including the acceleration of the kite given by Newton's laws of motion:

$$\frac{F - e_r \|F_c\|}{m} = \begin{pmatrix} -r\dot\theta^2 - r\sin^2(\theta)\dot\phi^2 \\ r\sin(\theta)\ddot\phi + 2r\cos(\theta)\dot\phi\dot\theta \\ -r\ddot\theta + r\sin(\theta)\cos(\theta)\dot\phi^2 \end{pmatrix} \tag{3}$$

Here, we took into account that r is constant, thus the first equation in (3) is redundant for the dynamics of the kite, but

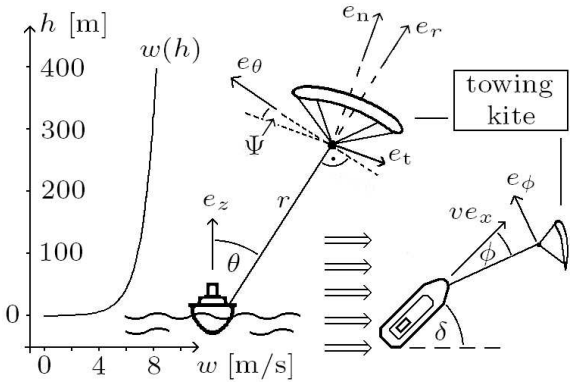

Fig. 1. The towing kite in back and top view

useful to calculate the radial constraint force $F_c = \|F_c\| e_r$ on the cable. For a detailed derivation of (3) we refer to [3].

The total force F in the kite's equation of motion is the sum of all forces but F_c acting on the effective inertial mass m of the kite. We will consider these forces in the next subsection.

A. Forces on the kite

The kite with position $p = re_r$ has in the ship's system the velocity $\dot{p} = r\sin(\theta)\dot{\phi}e_\phi - r\dot{\theta}e_\theta$. Thus, the effective wind vector w_e in the kite's system is given by

$$w_e = w_e^s - \dot{p} = w_e^s - r\sin(\theta)\dot{\phi}e_\phi + r\dot{\theta}e_\theta \quad . \tag{4}$$

We assume that the transversal axis of the kite is always perpendicular to this effective wind vector w_e by suitable yaw control. Furthermore, we introduce a controllable roll angle Ψ defined by

$$\sin(\Psi) := e_t \cdot e_r \, , \tag{5}$$

where e_t is a unit vector pointing from the left to the right wing tip (Fig. 1.). Using the projection w_e^p of the effective wind vector w_e onto the tangent plane spanned by e_θ and e_ϕ,

$$w_e^p := e_\theta(e_\theta \cdot w_e) + e_\phi(e_\phi \cdot w_e) = w_e - e_r(e_r \cdot w_e),$$

we can define the orthogonal unit vectors

$$e_w := \frac{w_e^p}{\|w_e^p\|} \quad \text{and} \quad e_o := e_r \times e_w,$$

so that (e_w, e_o, e_r) form an orthogonal right-handed coordinate basis. Note that in this basis the effective wind w_e has no component in e_o direction, as

$$w_e = \|w_e^p\| e_w + (w_e \cdot e_r) e_r.$$

In [3] is shown how to calculate the unit vector e_t under these conditions. The vector

$$e_t = e_w(-\cos(\psi)\sin(\eta)) + e_o(\cos(\psi)\cos(\eta)) + e_r\sin(\psi)$$

with

$$\eta := \arcsin\left(\frac{w_e \cdot e_r}{\|w_e^p\|}\tan(\psi)\right)$$

satisfies the requirements (5). Note that the considerations would break down if the effective wind w_e would be equal to zero, or if

$$\left|\frac{w_e \cdot e_r}{w_e \cdot e_w}\tan(\psi)\right| > 1.$$

Using this result, we can find the direction e_n of the aerodynamic lift:

$$e_n = \frac{w_e}{\|w_e\|} \times e_t \, . \tag{6}$$

Now, we can collect the forces on the kite: The sum of the gravitational force and the lifting force is

$$F_g := (V\rho - \overline{m})g \begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} , \tag{7}$$

where V is the volume, $\overline{m}$ the effective gravitational mass of the kite, g the gravitational constant and ρ the air density. Note, that the effective gravitational mass $\overline{m}$ in equation (7) and the effective inertial mass m in equation (3) are not necessary equal. Example: If we have a kite with mass m_k and a homogeneous cable with mass m_c, we find $m = m_k + \frac{m_c}{3}$ but $\overline{m} = m_k + \frac{m_c}{2}$.

The sum of the aerodynamic lift and drag is

$$F_{aer} := \frac{1}{2}\rho c_L A \|w_e\|^2 e_n + \frac{1}{2}\rho c_D A \|w_e\| w_e \, , \tag{8}$$

where c_L and c_D are the lift and drag coefficient respectively and A the area of the kite. Additionally, the air friction of the cable slows the kite down. Unfortunately, the air friction of the cable slows the kite down. We estimate the angular momentum of this friction by:

$$\begin{aligned} M_f &:= \int_0^r (se_r) \times \frac{\rho c_{D,C} d_c}{2}\left(\frac{s\|w_e\|}{r}\right)^2 \frac{w_e}{\|w_e\|} ds \\ &= (re_r) \times \frac{c_{D,C}\rho A_c}{8}\|w_e\| w_e \, , \end{aligned} \tag{9}$$

where d_c is the diameter, A_c the cross wind area and $c_{D,C}$ the drag coefficient of the cable. The associated friction force is:

$$F_f := \frac{c_{D,C}\rho A_c}{8}\|w_e\| w_e \, . \tag{10}$$

Finally, the total force F in equation (3) is given by

$$F = F_g + F_{aer} + F_f \quad . \tag{11}$$

III. The formulation of the optimal control problem

We are interested in the tractive force F_t in the ship's target direction. The first equation in (3) yields the force on the cable[1]

$$F_c = \left[F \cdot e_r + mr\left(\dot{\theta}^2 + \sin^2(\theta)\dot{\phi}^2\right)\right] e_r \, . \tag{12}$$

Now, F_t is simply the projection of F_c onto e_x :

$$\|F_t\| = \|F_c\| \sin(\theta)\cos(\phi) \, . \tag{13}$$

[1] Alternatively, Equation (12) can be found more directly: The force F_c is the sum of the projection of the total force F in e_r-direction and the centrifugal force $F_{centrifugal} = \frac{m\|\dot{p}\|^2}{r} e_r = mr\left(\dot{\theta}^2 + \sin^2(\theta)\dot{\phi}^2\right) e_r$.

The average of this tractive force over one cycle is our objective function, that we would like to maximize:

$$\overline{F_t} := \frac{1}{T}\int_0^T ||F_t||dt\,. \tag{14}$$

Here, T is the time the kite needs for one loop. Let us define the differential state vector x and the control u by

$$\begin{aligned} x &:= \left(\phi\,,\theta\,,\dot{\phi}\,,\dot{\theta}\,,\Psi\,,I\right)^T \\ \text{and } u &:= \dot{\Psi}\,. \end{aligned}$$

Here we control the derivative $u := \dot{\Psi}$ of the roll angle with respect to the time while Ψ is a differential state. This is a way to keep Ψ continuous and we can simply require additional constraints of the form $|u| \le u_{\max}$ to avoid too fast changes of Ψ, that might be hard to realize.

Furthermore, the easiest way to express the objective function (14) is as a Mayer term, since T is unknown. Thus, we use an additional state I given by $\dot{I} := \|F_t\|$ to evaluate the integral in equation (14).

Now, the differential equation for the state vector x can be rewritten as $\dot{x} = f(x,u)$ by using equation (3), where all parameters are listed in Table I:

$$f(x,u) := \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \frac{F\cdot e_\phi}{mr\sin\theta} - 2\cot(\theta)\dot{\phi}\dot{\theta} \\ -\frac{F\cdot e_\theta}{mr} + \sin(\theta)\cos(\theta)\dot{\phi}^2 \\ u \\ \|F_t\| \end{pmatrix}. \tag{15}$$

To achieve that the kite flies closed loops, we have to require periodic boundary conditions for all differential states but I :

$$r(x(0)\,,x(T)\,) := \begin{pmatrix} \phi(T)-\phi(0) \\ \theta(T)-\theta(0) \\ \dot{\phi}(T)-\dot{\phi}(0) \\ \dot{\theta}(T)-\dot{\theta}(0) \\ \Psi(T)-\Psi(0) \\ \dot{\phi}(0) \\ I(0) \end{pmatrix} = 0\,. \tag{16}$$

The additional condition $\dot{\phi}(0) = 0$ is introduced in order to remove the indefiniteness due to the symmetry of periodic orbits with regard to phase shifts. It is necessary to keep the problem well defined. And $I(0) = 0$ is a necessary initial condition such that $I(T) = \int_0^T ||F_t||dt$ holds. Now, we define $\overline{F_t}(x(t),u(t),T) := \frac{I(T)}{T}$ and summarize our optimal control problem in the following common form:

$$\begin{aligned} &\underset{x(\cdot),u(\cdot),T}{\text{maximize}} && \overline{F_t}(x(t),u(t),T) \\ &\text{subject to:} && \\ &\forall t\in[0,T]: && \dot{x}(t) = f(x(t)\,,u(t)\,) \\ &\forall t\in[0,T]: && |u(t)| \le u_{\max} \\ & && 0 = r(x(0)\,,x(T)\,) \end{aligned} \tag{17}$$

This optimal control problem must be solved numerically. We will treat this point in the next section.

TABLE I

THE KITE'S AND OTHER PARAMETERS

Name	Symbol	Value
inertial mass	m	900 kg
gravitational mass	$\overline{m}$	925 kg
area	A	500 m^2
volume	V	720 m^3
lift coefficient	c_L	0.96
drag coefficient	c_D	0.08
gravitational const.	g	9.81 $\frac{m}{s^2}$
air density	ρ	1.23 $\frac{kg}{m^3}$
cable length	r	1000 m
cable friction	$c_{D,C}$	0.4
cable diameter	d_c	0.05 m
reference wind	w_0	6 $\frac{m}{s}$
reference altitude	h_0	40 m
roughness length	h_r	0.1 m
velocity of the ship	v	2 $\frac{m}{s}$
control bound	$u_{\max}$	0.025 $\frac{rad}{s}$

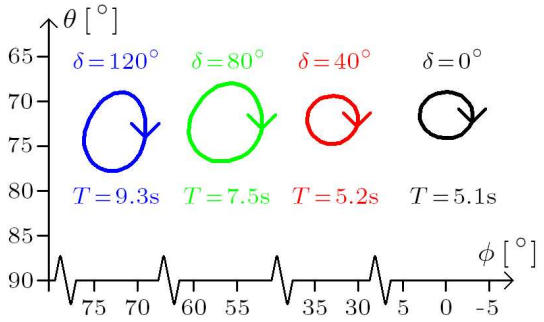

Fig. 2. Optimal loops in ϕ- and θ-coordinates from the ship's point of view (clockwise course) for some angle δ.

IV. NUMERICAL OPTIMIZATION STUDY

To study the numerical solution, we use the optimal control package MUSCOD-II [9] that is based on the direct multiple shooting method [2]. It allows us to find local solutions of the above optimal control problem (17). For more information about this package we refer to [9] and [10].

A. The tractive force in dependence on the wind direction

Let us consider the solution of the optimal control problem (17) in dependence on several values of the angle δ. Fig. 2 shows some optimal loops.

Since we have symmetry in δ, it is enough to analyze the result for positive δ. We discuss some possible trajectories reminding that MUSCOD-II can only find local but in general not the global solution of an optimal control problem. We tested "simple loops" and "skewed eights" each in clock- and counterclockwise direction. Fig. 3 shows the comparison between the results for the optimal "eight" and the optimal "simple loop" for $\delta = 0°$. We found that "simple loops" in clockwise direction are the best choice, since they provide an average tractive force that is 0.9% to 1.3% higher than those for "eights" and, depending on δ, 0.0% to 1.4% higher than the average tractive force for "simple loops" in counterclockwise direction. The only disadvantage of

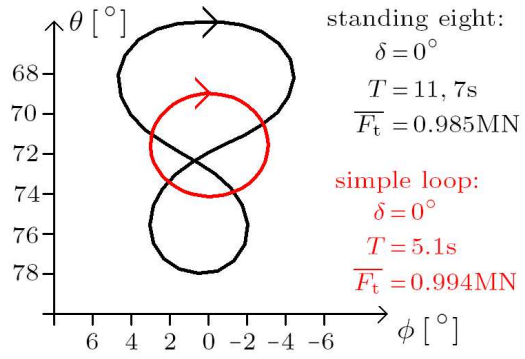

Fig. 3. Comparison between the optimal "simple loop" and the optimal "eight" for $\delta = 0°$.

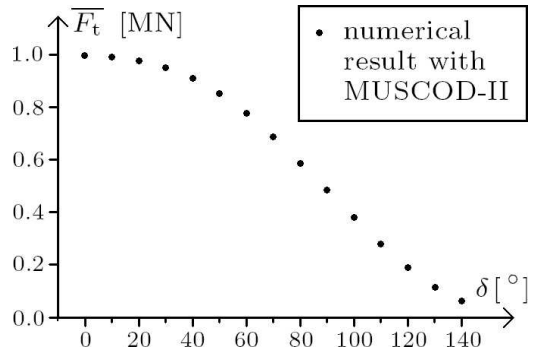

Fig. 4. The maximum average tractive force in dependence on δ

"simple loops" in comparison with "eights" seems to be that the cable is coiled up if we don't use a good bearing.

The loops in Fig. 2 have a diameter of 100 m to 180 m while the velocity of the kite is between $55\frac{\text{m}}{\text{s}}$ and $63\frac{\text{m}}{\text{s}}$. Noting that these loops are quite narrow in proportion to the dimension of our kite, we should think about a way to compensate yaw wise torques. Although a detailed consideration of such torques is beyond the scope of this paper, we like to mention that this compensation should be possible with effective rudders at the back of the kite.

In Fig. 4 the result for the average tractive force in dependence on the angle δ between the wind and the ship's target direction is presented. Although we could find convergent solutions for some values for δ greater than 140°, they are not interesting from a practical point of view, since the tractive force will be very low. If we are sailing downwind, the average tractive force attains almost 1 MN. Observing that the forces on the cable between the ship and the kite are quite large, we should vary the mounting point of the cable, such that the roll and pitch moments on the ship can be minimised [7, 14].

What are the parameters in Table I that mainly influence the performance of our kite? Of course, we can use bigger kites or hope for more wind. Rather, we like to discuss the dependence on the effective glide ratio of the kite, that will be introduced in the next subsection.

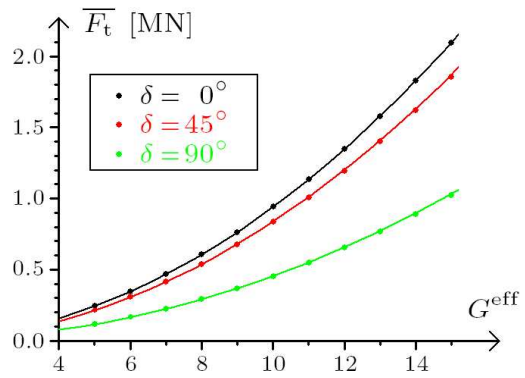

Fig. 5. The maximum average tractive force in dependence on the effective glide ratio: All investigated points are on parabolas

B. The tractive force in dependence on the effective glide ratio

Loyd [11] laid the foundation for the analysis of power-producing kites. Transferring his simplified considerations to our towing kite suggests that the tractive force is approximately proportional to the square of the kite's glide ratio c_L/c_D.

If we take into account that the friction of the cable is not negligible, it is useful to define the effective glide ratio by

$$G^\text{eff} := \frac{\|F_\text{L}\|}{\|F_\text{D} + F_\text{f}\|} = \frac{c_\text{L}}{c_\text{D} + \frac{1}{4}\frac{A_\text{c}}{A} c_\text{D,C}} \tag{18}$$

with F_D and F_f being the aerodynamic drag of the kite and the cable respectively and F_L being the aerodynamic lift (cf. equation (8) and (9)).

Fig. 5 shows the numerical evaluation of the influence of the effective glide ratio for some values for δ. To vary the effective glide ratio, we adjust the drag coefficient, while all the other parameters in Table I, including the lift coefficient, are fixed. In fact, we can confirm our hypothesis that the tractive force is approximately increasing with the square of the effective glide ratio. This can be seen in Fig. 5, where the numerical results are interpolated with parabolas.

Consequently, on the one hand, we have to improve the effective glide ratio as a design parameter of our kite that is mainly influencing the performance. On the other hand, we should try to realize a control that keeps even a high-speed kite close to the small optimal loops that we presented in the previous section IV-A. This might be done by Nonlinear Model Predictive Control (NMPC) [1], as it is investigated in [3, 5] for kite control.

V. Conclusions and Outlook

We have presented optimal periodic loops for a towing kite in dependence on the angle between the wind and the ship's target direction. Furthermore, we explained how to find the maximum average tractive force by controlling the time derivative of the kite's roll angle. We introduced a model and corresponding parameters for a powerful kite to study a realistic simulation. The optimal control problem was formulated with periodic boundary conditions, free initial

values and a free cycle duration. We found out that the best orbits are simple loops such that the kite flies in downward direction on the center side of the loop. Eight-shaped orbits are only slightly worse. Finally, we discussed the influence of the effective glide ratio as an important design parameter of the kite.

In the future, we like to improve our model considering not only the control of the roll angle but also of the yaw and pitch angle to guarantee always the maximum effective glide ratio. Also, the windage disturbance of the kite from one cycle to the next – though it is small for sufficiently large loops – should be modelled in order to evaluate the true performance limits of the system. Moreover, for a real world implementation of our loops, a feedback control must be designed in order to stabilize the system.

Finally, the use of wind energy with the means of kites is not limited to ship traction systems. We should also think of converting the large wind power in high altitudes into renewable electrical energy [6, 13]. We are currently performing optimization studies for power producing kites based on a similar set up as presented in this paper.

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