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Moving horizon estimation and nonlinear model predictive control for autonomous agricultural vehicles

T. Kraus [a,*], H.J. Ferreau [b,c], E. Kayacan [a], H. Ramon [a], J. De Baerdemaeker [a], M. Diehl [b], W. Saeys [a]

[a] BIOSYST – MeBioS (Mechatronics, Biostatistics, Sensors), K.U. Leuven, Kasteelpark Arenberg 30, B-3001 Leuven, Belgium
[b] Electrical Engineering Department (ESAT-SCD), K.U. Leuven, Kasteelpark Arenberg 10, B-3001 Leuven, Belgium
[c] ABB Corporate Research, Segelhofstrasse 1K, CH-5405 Baden-Dättwil, Switzerland

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ABSTRACT

Controllers working in uncertain environments are often required to adapt themselves continuously to changing conditions to avoid steady-state errors, oscillations at the output or even instability of the closed loop system. The moving horizon estimation (MHE)–nonlinear model predictive control (NMPC) framework being proposed combines these two optimization-based methods to control field vehicles utilizing an adaptive nonlinear kinematic model. The full system state, including two unknown slip parameters and the unmeasurable vehicle orientation, is estimated by the MHE after each new measurement and fed afterwards to the NMPC routine which provides a wheel velocity and a steering rate to follow arbitrary time-based reference trajectories in difficult environmental conditions. This control problem occurs in modern agriculture e.g. in planting or mechanical weeding while slippery conditions make these operation difficult and off-track navigation results in plant damage. The experimental results show accurate reference tracking performance of the MHE–NMPC framework on a wet and bumpy grass field. The feedback times lie in the range of 0.6–1.6 ms when the `ACADO Code Generation` tool is used, which is part of the open-source software toolkit `ACADO`.

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1. Introduction

A large number of recent innovations in agricultural machinery design is focused on automation, i.e. taking tasks away from the operator and leaving it to intelligent control mechanisms. Such controllers have the purpose to operate whole machines or at least subprocesses autonomously and – desirably – in an optimal way. In agriculture, the preference for autonomous machines is not only related to the high expenses for manpower. Especially, in temperate climate zones farmers often have narrow time windows for planting and harvesting only. During these periods certain tasks regularly have to be carried out under an immense time pressure to remain profitable. Typically, this results in workloads that are difficult to handle with the available manpower which makes automation so necessary.

One important automation problem that many agricultural applications have in common – and this study deals with – is the challenge of autonomous navigation. The development of satellite-based positioning devices puts today's farmers in a comfortable position to choose among many accurate commercial positioning devices, with real-time kinematic variants of global positioning systems (RTK-GPS) yielding centimeter precision. Comparisons of commercial solutions for autonomous navigation in terms of precision are difficult since there is a lack of uniformity in the provided information of the manufacturers in terms of scale of the controlled vehicle, reference trajectories and velocities.

While the first studies covering the topic of autonomous navigation date back to the 1920s (Willrodt, 1924), prototype guidance systems based on machine vision techniques have been actively developed since the 1980s, whereas the first GPS-based systems saw the light in the 1990s. An overview of these developments can be found in Wilson (2000) or in Li et al. (2009). Also, it is evident that the development of autonomous vehicles has not been restricted to the agricultural sector. In automotive research, advanced model-based control techniques relying on sophisticated models including tire dynamics have been investigated actively for the last decade, see e.g. Gerdts (2005) or Borrelli et al. (2005). Recent publications also address the problem of varying tire–soil interactions by applying model-based control techniques for field vehicles (Lenain et al., 2006). Snider (2009) compared the most common approaches to tackle the path-tracking problem: geometric approaches, control laws derived directly from kinematic models and model-based approaches being "optimal control-like" such as the linear quadratic regulator. According to Snider (2009)

* Corresponding author.
E-mail addresses: tom.kraus@biw.kuleuven.be (T. Kraus), joachim.ferreau@ch.abb.com (H.J. Ferreau), erdal.kayacan@biw.kuleuven.be (E. Kayacan), herman.ramon@biw.kuleuven.be (H. Ramon), josse.debaerdemaeker@biw.kuleuven.be (J. De Baerdemaeker), moritz.diehl@esat.kuleuven.be (M. Diehl), wouter.saeys@biw.kuleuven.be (W. Saeys).

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"following a straight path is something all of the methods are pretty good at", and no method has been found in that study that always outperforms all the others on curved paths. The nonlinear model predictive control (NMPC) approach was labeled as the next logical step to take, even though the author remained skeptical about limitations of computational resources. A promising approach in applying NMPC to the autonomous navigation problem practically has been proposed in Backman et al. (2012) for a tractor with a steerable implement. Here, the tractor-implement system has been equipped with a high-precision sensing system including a GPS yielding an accuracy of 1 cm for position measurements. The measured yaw angle or heading is filtered with an extended Kalman Filter (EKF) while other measurements are fed directly to the NMPC routine. The article, however, did not provide information on the computation times, the actual shape of the trajectory or possible causes for the remaining tracking errors. A survey on further publications on linear model predictive control for agricultural vehicles can also be found in Backman et al. (2012).

The contribution of this study is the real-time implementation of NMPC for the trajectory tracking problem on an agricultural tractor utilizing an adaptive nonlinear kinematic model and similar sensors as the ones used in commercial systems for autonomous tractor guidance. An accurate online estimation of slip parameters is crucial in this context as the tire–soil interactions are expected to change when operating in difficult, slippery conditions. Instead of employing the popular EKF as done Lenain et al. (2006) or Backman et al. (2012), a nonlinear moving horizon estimation (MHE) is used as the online state and parameter estimation method in this study. The MHE method can handle state and parameter constraints which could be imposed for physical reasons in contrast to the EKF, which has been shown to be beneficial for the quality of estimates in literature (see e.g. Haseltine and Rawlings (2005) or Rao et al. (2003)). Furthermore, the MHE typically provides more accurate model parameter estimates due to a clear distinction in the internal treatment of model parameters and time-variant states within the estimation horizon in the nonlinear case (see e.g. Kühl et al. (2011)). Due to the incorporation of a nonlinear measurement function and two unknown model parameters which have to be identified online in this study, the nonlinearity of the process model is increased when compared to similar works (Backman et al., 2012) where also the accurate measurement of the yaw angle is a costly requirement. In this study the measurement of the yaw angle is not required since it can be treated as an unknown state also being estimated accurately with the MHE method. For the solution of the MHE and the NMPC problems, the code generation tool of the ACADO toolkit for (A)utomatic (C)ontrol (A)nd (D)ynamic (O)ptimization has been used which allows to export highly efficient C-code to solve the optimization problems resulting from the user-specified MHE and NMPC formulations (Houska et al., 2011; Ferreau et al., 2012). It allows for feedback times in the millisecond range for the tractor navigation problem as shown in simulation for a simplified process model in Ferreau et al. (2012). The study at hand represents the real-time implementation of the fully optimization-based solution approach on the actual vehicle and shows that real-time feasibility is clearly maintained.

The model that is used in this study is presented in Section 2. The moving horizon state estimation and the model predictive control problems are described in Sections 3 and 4, respectively. The experimental setup is presented in Section 5 with the experimental results and their discussion following in Sections 6 and 7. The conclusions from this study are summarized in Section 8.

2. Vehicle model

The kinematic ordinary differential equation (ODE) model is a slight adaptation of the well-known bicycle model. Although basically representing a model for 2-wheeled vehicles it is known to approximate also 4-wheeled vehicle behavior accurately and yields a good representation when working at low speeds (Cameron and Probert, 1994; LaValle, 2006; Coen et al., 2008; Snider, 2009). The modification of the model equations done in this study consists in the incorporation of two additional factors: one for longitudinal slip and one for side-slipping. It is assumed that side-slipping affects only the steering wheels in the front. The second model modification has also been done in Backman et al. (2012).

The equations of motion are as follows:

$$\begin{bmatrix} \dot{x}_{\text{pos}} \\ \dot{y}_{\text{pos}} \\ \dot{\beta} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \kappa u_1 \cos\beta \\ \kappa u_1 \sin\beta \\ \kappa u_1 \frac{\tan(\mu\cdot\delta)}{l} \\ u_2 \end{bmatrix} \tag{1}$$

Here, x_{pos} [m] and y_{pos} [m] are differential states representing the position of the rear wheel in global cartesian coordinates, in case of a 2-wheeled vehicle as shown in Fig. 1. In case of a 4-wheeled vehicle this location corresponds to the center of the rear axle. Further differential states are representing the yaw angle or heading β [rad] and the realized steering angle δ [rad]. The control variables are the wheel speed u_1 [m/s] and the steering rate u_2 [m/s].

Furthermore, two model parameters typically being difficult to measure have been included in the dynamic system equations:

- the longitudinal slip factor κ to relate wheel speed to ground speed (see Fig. 2),
- the side-slipping factor μ being caused by inertia and approximately relative to the steering angle (Backman et al., 2012).

It is important to note here that both κ and μ are slip *parameters* which vary between zero and one. No forward or side slip occurs if $\kappa = \mu = 1$. Otherwise, the percentaged longitudinal slip can be given as $1 - \kappa$ and the percentaged side slip as $1 - \mu$. It also needs to be stressed that the side slip here must be understood as a "steering slip" affecting the front wheels. Respectively, it is assumed that only a fraction of the realized steering angle δ translates into actual vehicle motion. This fraction is given by μ and determines the *effective* steering angle $\mu \cdot \delta$. To avoid bias, these parameters should be estimated along with the full system state in each iteration based on a number of past measurements. The parameter $l = 1.4$ m represents the wheel base and is purely geometry-dependent.

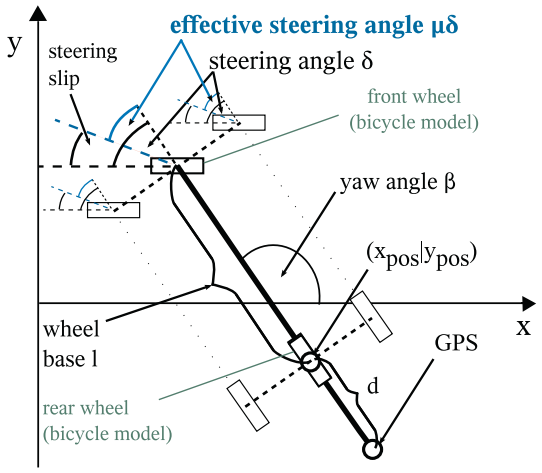

Fig. 1. Schematic illustration of adapted bicycle model with side slip affecting the front wheel.

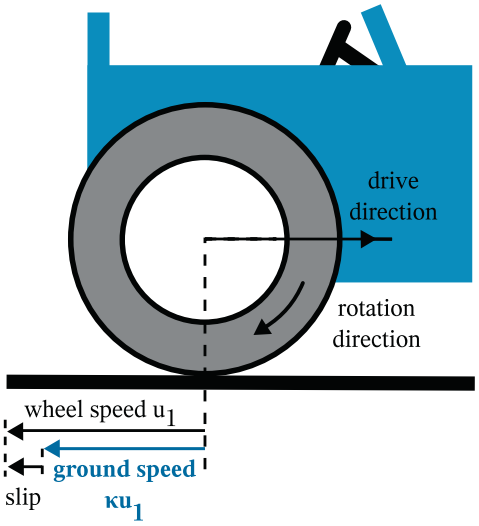

Fig. 2. Illustration of the longitudinal slip.

3. Moving horizon estimation

In practical applications, the number of available measurements is typically smaller than the number of states within the models to describe the dynamics of the respective system. Therefore, it is often necessary to estimate states or unknown model parameters online when working with advanced control methods such as NMPC. Usually, an Extended Kalman Filter (EKF) is used for the state and parameter estimation. This is justified for linear (or mildly nonlinear systems) when disturbances can be assumed Gaussian and constraints do not play a role. In this case, the linear Kalman Filter provides bias-free estimates of minimum variance. In the more general nonlinear case this is no longer true. Furthermore, constraints on states or parameters as they are often imposed for physical reasons cannot be incorporated in any Kalman Filtering framework, e.g. that the slip factors cannot be bigger than 1. Other approaches like particle filtering methods become practically unsolvable for a large number of states. A comprehensive overview on nonlinear state estimation methods is given in Daum (2005). The optimization-based moving horizon estimation (MHE) represents a powerful estimation technique with which many of the problems encountered with other estimators can be circumvented. Initially, the concept of unconstrained MHE was proposed by Thomas (1975) and Kwon et al. (1983) – although Jang et al. (1986) were the first to propose it as an online optimization strategy. In the following years, many publications extended this work. MHE treats the state and the parameter estimation within one problem and also constraints can be incorporated. For an introduction to constrained optimization-based estimation see e.g. Allgöwer et al. (1999). For a more detailed discussion on constrained MHE the reader is referred to Robertson and Lee (1995), Rao (2000), Rao et al. (2003) or Kühl et al. (2011). The choice of the employed numerical optimization schemes is crucial for its performance, especially for nonlinear systems.

3.1. MHE optimization problem

The MHE problem is formulated as follows: At current time t_k there shall be M measurements $y_{k-M+1}, \dots, y_k \in \mathbb{R}^{n_y}$ available, associated to the time instants $t_{k-M+1} < \cdots < t_k$ in the past. If it is assumed that the earliest available measurement corresponds to the time instant t_0, this requires that $k \geqslant M-1$ with M bounded. $T_E = t_k - t_{k-M+1}$ shall be the length of the estimation horizon. Hence, the constrained state and parameter estimation problem to be solved at time t_k – given a dynamic model f and a measurement model h as well as the measurement data y_j for $j = k-M+1$, $k-M+2, \dots, K$ and regularization data $(\bar{x}, \bar{p}, \overline{Q})$ – looks as follows:

$$\min_{x(\cdot),p,u(\cdot)} \left\{ \left\| \begin{matrix} x(t_{k-M+1}) - \bar{x} \\ p - \bar{p} \end{matrix} \right\|^2_{\overline{Q}} + \sum_{j=k-M+1}^{k} \|y_j - h(x(t_j), u(t_j), p)\|^2_{R_j} \right\} \quad (2)$$

$$\textbf{subject to} \quad \left. \begin{aligned} \dot{x}(t) &= f(x(t), u(t), p) \\ x_{\min} &\leqslant x(t) \leqslant x_{\max}, \\ p_{\min} &\leqslant p \leqslant p_{\max}, \end{aligned} \right\} \; t \in [t_j, t_{j+1}],$$

(for the sake of a compact notation the norm $\|a\|^2_A := a^T A a$ is defined)

Here, $t \in \mathcal{I} := [t_{k-M+1}, t_k] \in \mathbb{R}$ represents time and $x : \mathcal{I} \to \mathbb{R}^{n_x}$ an unknown function to describe the differential state of the system which is of dimension n_x. The right-hand side of the ODE system is denoted by $f : \mathbb{R}^{n_x+n_u+n_p} \to \mathbb{R}^{n_x}$ and represents the dynamic model of the process. It describes the propagation of the system state on a continuous time scale under the influence of the system state x itself, time–variant control variables $u : \mathcal{I} \to \mathbb{R}^{n_u}$ and model parameters p. Here, n_u denotes the number of control variables and n_p the number of model parameters. As the differential system state has dimension n_x the ODE system comprises n_x equations. For the practical experiments with the tractor, the model Eq. (1) described in the previous section will be inserted here (with $n_x = 4, n_u = 2$ and $n_p = 2$).

The (possibly nonlinear) measurement function to predict measurements is denoted by $h(x(t), u(t), p)$ with $h : \mathbb{R}^{n_x+n_u+n_p} \to \mathbb{R}^{n_y}$ where n_y denotes the number of measurements per sample. The measurement function is normally assumed not to be involved in the process dynamics and should only be determined by the properties of the actual measurement process. Since in practice usually discrete-time measurements are encountered a formulation has been chosen in the objective function where the measurement function is also evaluated at the respective measurement instants only. It should be stressed here that h can also depend on control variables $u(t)$. Practically, $u(t)$ represents a parameterized process input which is typically assumed piecewise constant within each subinterval. In the case when a controller provides the input $u(t)$ there is knowledge about this (past) process actuation available which is crucial to reconstruct the evolution of the dynamic states within the estimation horizon. Therefore, it must either be assumed to be a perfectly-known input or assumed to be given as a measurement. In the first case h does not depend on $u(t)$. In the latter case, it can be incorporated in the measurement function h to filter out measurement noise or to account for actuation errors. In the practical experiments, the process input given as the wheel speed and the steering rate will be assumed to be measured. It is assumed here that components of h including control variables do not depend on state or parameter quantities at the same time. The number of components in h including control variables shall be labeled by n_{yu} for later use. Additionally, upper and lower bounds for states and parameters can be imposed to define the feasible domain where the minimizing arguments must reside.

In practice, measurements and the outputs of the measurement function will never be in accordance for a large value for M. Roughly speaking, it is the task of the estimation to find state and parameter estimates for x and p such that the measurement function h reflects the considered measurements best. After the minimization of (2) the resulting estimate of $(x(t_k), p)$ represents the sought-after system state estimate at time t_k. In this study it is defined already by the estimate $(x(t_{k-M+1}), p)$ if the process input $u(t)$ is known since process noise is not explicitly included here in contrast to standard MHE formulations. However, it could always be introduced through the incorporation of additional control variables.

Although MHE is a deterministic method at the heart due to the underlying least-squares approach, it can also be interpreted statistically. If it is assumed that the measurement noise is white and normally distributed (with zero mean and known covariance matrices $V_j \in \mathbb{R}^{n_y \times n_y}$ a minimization of (2) under the neglection of constraints with $R_j = V_j^{-1}$ can be interpreted as a maximum likelihood estimation (Cox, 1964). Whether this property is preserved when constraints are incorporated depends on the modeling (see Robertson and Lee (1996)).

Although the amount of acquired measurement data increases with time, the size of the estimation problem (2) is bounded. At each measurement instant, the number of measurements explicitly incorporated in the estimation is kept constant, since for each new measurement vector entering the set of measurements $y_{k-M+1}, \dots, y_k$ a previous measurement y_{k-M+1} becomes y_{k-M} and drops out which explains the concept of a *moving horizon*.

The first summand of the objective function (initial weight term) in the MHE problem (2) is supposed to summarize information prior to t_{k-M+1} representing the start of the estimation horizon. The incorporation of the cost for the whole measurement past would theoretically be ideal to summarize this "arrival cost" of the problem:

$$\min_{x(\cdot),p,u(\cdot)} \left\{ \sum_{j=0}^{k-M} \|y_k - h(x(t_j), u(t_j), p)\|^2_{R_j} \right\}, \tag{3}$$

Note that if the first term of the objective function of (2) is replaced with (3), a least-squares problem which incorporates all measurements from t_0 would be the result. It is obvious, that this would represent an MHE with the horizon length k. In the limiting case when k grows to infinity, this MHE with infinite horizon length is impossible to solve in the general nonlinear case. A suitable way to avoid this is to approximate the arrival costs given in (3) and, accordingly, keep the MHE horizon length fixed. However, these arrival costs can only be calculated exactly in the unconstrained case and when the models are linear. A suitable approach in the general, nonlinear case is to approximate the arrival costs recursively with a *smoothed* EKF update. This represents an EKF – running at time t_{k-M+1} and utilizing only measurements y_{k-M+1} – with a beneficial choice of the linearization point to update Kalman state estimates $\bar{x}$, $\bar{p}$ and the inverted Kalman covariance $\bar{Q} \in \mathbb{R}^{n_x+n_p \times n_x+n_p}$ in advance of each MHE optimization (the reader is referred to (Robertson and Lee, 1996) for a detailed description of the update formulas in standard Kalman Filter notation). As can be verified in the given reference, the inputs for this smoothed EKF update are the same as the ones which have to be provided for a conventional EKF. Thus, only for the first MHE iteration an initial guess for the system state $\left(\bar{x}, \bar{p}\right)$ needs to be available along with a measure of confidence that we have in this guess in form of a matrix $P_0 \in \mathbb{R}^{n_x+n_p \times n_x+n_p}$. Obviously, P_0 is also a guess. It is typically chosen as a diagonal matrix with high values to reflect the general case that the initial guess for the system state is of poor quality. Accordingly, we set $\bar{Q} = P_0^{-1}$ and run the first MHE iteration with the provided guesses by solving problem (2). Together with the solution for the MHE problem also the respective Jacobian or sensitivity matrices, which are generally necessary to calculate the time update of any EKF variant, can be received if a Newton-type optimization method is used. Additionally, the covariance matrix $V_j^{\text{update}} \in \mathbb{R}^{n_y - n_{yu} \times n_y - n_{yu}}$ describing the error distributions for measurement noise (related to state- and parameter-dependent components) as well as a nominal covariance matrix $W^{\text{update}} \in \mathbb{R}^{n_x+n_p \times n_x+n_p}$ describing process noise distributions must be available. With these inputs it is possible to calculate the regularization data $(\bar{x}, \bar{p}, \bar{Q})$ for the subsequent MHE problem by setting $\bar{x}$ and $\bar{p}$ to the received EKF estimates for states and parameters and using the inverse of the received Kalman covariance P as the matrix $\bar{Q} = P^{-1}$. This routine is repeated until the estimation is stopped by the user. Within this study a numerically more stable square-root variant of the EKF update formulas has been employed and the update is performed directly on the inverted Kalman covariance $\bar{Q}$, see Kühl et al. (2011).

When the process model is linear, the Kalman Filter update gives a stable estimator – no matter if constraints are present, see Rao et al. (2001). For general nonlinear models stability of the MHE is given for a sufficiently "good" approximation of the arrival costs. As long as certain technical conditions are satisfied, nondivergence or stability are guaranteed, see Rao et al. (2003). From a practical point of view, it is necessary to ensure that the arrival costs are bounded. since if past data are weighted too heavily, the arrival cost might grow to infinity. In the chosen arrival cost approximation this is accounted for since the linearized contributions of past measurements to the matrix $\bar{Q}$ are downweighted by a "state noise covariance matrix" $W \in \mathbb{R}^{n_x \times n_x}$ and a "parameter noise covariance matrix" $W^p \in \mathbb{R}^{n_p \times n_p}$ representing the block diagonal entries of the process noise covariance matrix W^{update} required for any Kalman Filter formulation in order to account for linearization errors and possible disturbances in the model equations:

$$W^{\text{update}} = \begin{pmatrix} W & 0 \\ 0 & W^p \end{pmatrix} \tag{4}$$

It is noted here that while the explicit modeling of disturbances on states and parameters within the MHE formulation is omitted (as W^{update} is also not an input for the MHE), these process noise covariances must always be provided for any EKF formulation. It is possible to show that the inverse Kalman covariance matrix $\bar{Q}$ computed by this approach is upper bounded by $(W^{\text{update}})^{-1}$ in the sense that $(W^{\text{update}})^{-1} - \bar{Q}$ is positive semidefinite.

The choice of the horizon length M has to be provided from extern and is problem-specific. There is no global strategy available how to choose this quantity. Typically, it is a trade-off between computational complexity, which we want to keep low but grows with M, and the desired quality of the estimation which typically increases with M to a certain extent. However, also the process model might make the choice of a too high M undesirable, since plant-model mismatches deteriorate the significance of model predictions and, therefore, might have an adverse effect on the estimation quality again. For example, if there are unknown model parameters to be estimated, which are assumed constant within the estimation horizon, but are actually slowly-drifting, the plant-model mismatch increases with the horizon length M. This makes the choice of M a problem-specific choice depending on the dimension and the structure of the specific process model and the desired estimation accuracy.

4. Nonlinear model predictive control

A brief introduction to model predictive control for tracking problems is given for consistency reasons. A much more profound description can be found e.g. in Magni et al. (2009) and Qin and Badgwell (2003) as well as in the references therein.

In comparison to single-input single-output (SISO) control schemes, that are usually designed to fit single state values separately to given requirements by manipulating single allocated control variables, the model predictive control approach being a multiple-input multiple-output (MIMO) control scheme usually considers process-wide models. Such process models often are nonlinear, but not seldomly give a more suitable description of process dependencies than their linearized counterparts. However, when dealing with nonlinear process models, certain formulations

for model predictive control – e.g. using transfer functions which apply only to linear models – are not suited anymore. Therefore, more general formulations capable of treating such nonlinear models are explicitly referred to as *nonlinear* model predictive control (NMPC) formulations, although effectively the case of general, i.e. linear *and* nonlinear process models is treated. In this article, the chosen formulation is supposed to cover the general case.

In NMPC one is seldomly concerned to predict the resulting costs of applying a certain control signal up to an *infinite* end time because it is computationally too demanding and usually even impossible. Also, such an (open-loop) control is only applicable to a system under the assumption of having no disturbances and no plant-model mismatch. However, in practice, significant disturbances and usually also plant-model mismatches must be expected. Thus, calculated control trajectories for a predicted system state far in the future are usually no longer suitable since real states usually differ significantly from their predictions in this case. So, what is usually done is to consider a fixed size time window $[t_k, t_k + T_c] = [t_k, t_{k+N}]$ for which an optimal control trajectory is calculated by the minimization of a control cost function. Here, T_c denotes the length of the control horizon which is determined by the number of incorporated prediction intervals N whose length typically coincide with the sampling time. For the practical choice of N similar arguments hold as for the horizon length of the MHE. There is no general strategy available how to choose this quantity. Typically, a suited problem-specific choice is a trade-off between the desired quality of the control performance, which typically improves with N, and the computational demand, which we want to keep low but increases with N.

4.1. NMPC optimization problem

The NMPC approach at time t_k consists in the effort of finding a function $u(t)$ that solves the following optimal control problem:

$$\min_{u_j, x(\cdot)} \left\{ \sum_{j=k+1}^{k+N-1} \|s_j - m(x(t_j), u(t_j))\|_D^2 + \|s_{k+N} - m(x(t_{k+N}), u(t_{k+N}))\|_E^2 \right\} \quad (5)$$

$$\textbf{subject to} \quad \left.\begin{array}{l} \dot{x} = f(x(t), u(t), p) \\ x_{\min} \leqslant x(t) \leqslant x_{\max} \\ u_{\min} \leqslant u(t) \leqslant u_{\max}, \end{array}\right\} \quad t \in [t_j, t_{j+1}],$$

where D and E are weighting matrices being symmetric and (semi) positive definite ensuring that all appearing terms are nonnegative. The first summand sums up the costs for the deviations of the model response $m(x(t), u(t))$ to discrete-time reference trajectories s_j where $j = k + 1, \ldots, k + N - 1$ in the time interval $t_k \leqslant t \leqslant t_{k+N-1}$. Naturally, these time-based references s_j must be provided from extern. In the tractor case these will incorporate sets of reference points in global cartesian coordinates which are assigned to the corresponding time instants. The last summand evaluates the final costs raised by the controlled variables at the given end time t_{k+N}. This term is usually referred to as the *terminal penalty term* and is often stated for stability reasons. For a detailed discussion of stability issues in NMPC the reader is referred to Mayne et al. (2000) and Diehl (2001).

The control function minimizing the NMPC problem (5) is denoted as the optimal control $u^*(t)$. Because of the additional dependence of the objective function on the state x_k and model parameters p the optimal control is also dependent on the initial system state which is denoted by the notation $u^*(t, x_k, p)$. Therefore, the full initial state (x_k, p) must be known. This information either must be measured directly or determined by a state estimator as described in Section 3.1. To solve the NMPC problem (5) practically, it is necessary to parameterize the control function $u(t)$ on the considered intervals. The most common parameterization, which we also will use in the practical experiments, is to assume the control output $u(t_j)$ as piecewise constant on each interval $[t_j, t_{j+1})$.

The fixed size time window is usually referred to as the *control horizon*. Once the optimal control problem at t_k has been solved the calculated values for the control variables are applied to the real system for a short time period ξ only. The period ξ typically coincides with the sampling time. After that, a new optimal control problem is solved for a fixed size window $[t_k + \xi, t_k + T + \xi] = [t_{k+1}, t_{k+N+1}]$, i.e. the time window is moved forward. If applicable, a new full state measurement or estimate is used as the initial value. Thus, a sequence of optimization problems is formulated and solved in real–time providing the possibility of reacting to disturbances.

4.2. Solution methods

The similarity between the NMPC problem (5) and the MHE problem (2) described in Section 3 is apparent. The summed differences between reference and model response resemble the summed differences between the measurements and the measurement function. The arrival cost approximation of the MHE has its counterpart in the terminal penalty term in NMPC that can also be seen as an approximation of the infinite horizon costs. It is usually referred to as the *cost to go* in this context. However, structurally the cost to go as well as the arrival cost just represent an additional least squares term in the respective optimization problems. Therefore, it is not surprising that the same solution methods can be applied for both the presented quadratic MHE and NMPC problem at each measurement instant. It is known that optimization problems with such least-squares objectives and subject to nonlinear dynamic systems and constraints can be solved in different ways. The three most popular solution approaches are simultaneous collocation, single-shooting and multiple-shooting. The structural differences between collocation and shooting methods result in a different distribution of computational workloads. While in collocation approaches the complexity is oriented towards the factorization of the Karush–Kuhn–Tucker (KKT) matrix, in single and multiple shooting it is shifted towards evaluating derivative information. In the given implementation the multiple shooting method is incorporated with a generalized Gauss–Newton method. This method is a special variant of the classical Newton method being tailored for least-squares problems with the advantage that second derivatives which are typically difficult to compute are not needed. However, as it is an iterative method, in general the time – i.e. the amount of iterations – necessary to reach a desired accuracy cannot be determined beforehand. This is problematic in the real–time context. To tackle this problem the real-time iteration scheme (Diehl et al., 2002) was proposed. The main idea behind it is to constrain the amount of Gauss–Newton iterations to 1 to have quick process feedback while each new optimization problem is initialized intelligently with the outputs of the previous one. This improves the convergence properties of the method and typically delivers very similar results compared to the classical method if given some (typically very short) time when started far off the solution – but with the advantage of minimum feedback delay. While originally developed for NMPC problems the method has also been adapted for MHE (Kühl et al., 2011). The reader is referred to Diehl (2001) for a detailed discussion of solution methods, not only for least squares problems but also for general nonlinear objective functions exploiting structures and sparsity.

The core of the software used to solve the MHE and NMPC optimization problems in this study is an extension of the `ACADO Code Generation` tool that allows the user to export customized real-time iteration MHE and NMPC algorithms (Houska et al., 2011; Ferreau et al., 2012). The tool automatically generates highly efficient, self-contained C-code based on a user-specified symbolic MHE or NMPC problem formulation. It is capable of running on

embedded hardware. The exported code contains only the essential algorithmic components, exploits problem-specific structures as dimensions, sparsity patterns etc. and avoids unnecessary computations. The `ACADO Code Generation` tool is released as open-source software under the LGPL license and can be freely downloaded at http://www.acadotoolkit.org.

5. Experimental setup

The objective in the real-time experiments is to follow a user-defined time-based trajectory with a small-scale agricultural tractor (TZ25DA, New Holland) while facing difficult and varying soil conditions which are represented by a bumpy and wet grass field in this study. This control problem occurs e.g. in planting or mechanical weeding where the loose soil introduces wheel slip while a high navigation accuracy is desired to obtain a constant inter-row distance to avoid crop damage.

A picture of the tractor is given in Fig. 3 while a diagram in Fig. 4 shows the respective inputs and outputs of the MHE–NMPC framework. Changes in soil–tire interaction have to be detected through online parameter identification along with the full system state at a sampling time of 200 ms using the MHE algorithm. The measurements being available at this sampling rate are the global cartesian GPS coordinates, the steering angle δ, the wheel velocity u_1 and the steering rate u_2. The sampling time was selected to match typical CAN-based data acquisition frameworks, as they are used e.g. for combine harvesters. New actuation values, however, can be fed to the tractor at a rate of 5 ms. Since the RTK-GPS (AsteRx2eH, Septentrio, Leuven, Belgium) is not mounted at the center of the rear axle but on the roll bar, positioned $d = 0.4$ m towards the back along the longitudinal vehicle axis, the measurement function h includes two terms depending nonlinearly on the unmeasured yaw angle β. A GPRS device (Digi Connect WAN 3G Dual Antenna, Navigation Solutions, Culemborg, Netherlands) employing the 3G technology is used to send uncorrected and receive corrected GPS data (Flepos, host: ntrip.flepos.be) in order to minimize the GPS error. The generally non-Gaussian measurement errors of the GPS are given by the manufacturer as $\tilde{\sigma}_x = \tilde{\sigma}_y = 0.03$ m. The steering angle δ is a state being measured directly by a potentiometer (533-540-J00A3X0-0, Mobil Elektronik, Langenbeutingen, Germany). The measurements were



Fig. 3. Picture of the New Holland tractor TZ25DA.

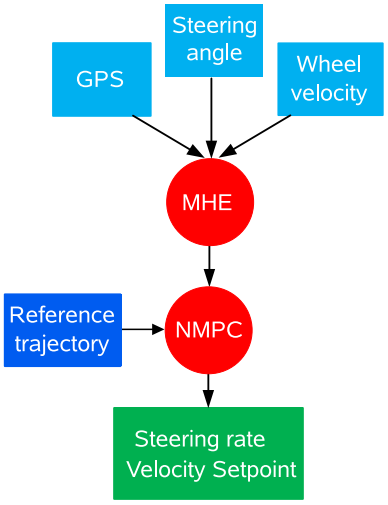

Fig. 4. Diagram showing the practical MHE–NMPC framework.

found to be perturbed by Gaussian noise with standard deviations of $\sigma_\delta = 0.01745$ rad according to analysis of the steering angle measurement. The control variables are the wheel speed and the steering angle rate. The low-level wheel speed control of the tractor is done through a PID type controller by adjusting a linear pedal position actuator (LA12, Linak, Nordborg, Denmark). The steering actuator (OSPC50-LS/EH-20, Danfoss, Nordborg, Denmark) allows for a relatively accurate manipulation of the steering rate. However, as it is unrealistic to assume that the applied controls coincide with the real actuation it must also be measured. Analysis of the actuation measurements yielded a standard deviation of $\sigma_{u,1} = 0.1$ m/s and $\sigma_{u,2} = 0.1$ rad/s for the measurement errors. Therefore, the measurement function looks as follows

$$h(x,u) = (x_{\text{pos}} - d \cdot \cos\beta, y_{\text{pos}} - d \cdot \sin\beta, \delta, u_1, u_2)^T \tag{6}$$

Since information about the accuracy for all measurements is available it can be used to set up the estimator accordingly: the diagonal matrix with the reciprocal values of the squared GPS accuracy and the measurement variances as entries is the natural choice for the MHE weighting matrix $R_j = \text{diag}\left(\tilde{\sigma}_{x,j}^2, \tilde{\sigma}_{y,j}^2, \sigma_\delta^2, \sigma_{u,1}^2, \sigma_{u,2}^2\right)^{-1} \in \mathbb{R}^{5\times5}$ at measurement instance t_j. In the case when the output of the GPS is not valid or timed-out, $\tilde{\sigma}_{x,j}^2$ and $\tilde{\sigma}_{y,j}^2$ are chosen differently from their nominal value and set to infinity for the respective measurement at time t_j resulting in a zero weight for the corresponding component in R_j. The EKF weighting matrices $V_j^{\text{update}} \in \mathbb{R}^{3\times3}$ and $W^{\text{update}} \in \mathbb{R}^{6\times6}$ being necessary for the smoothed EKF-update are chosen as $V_j^{\text{update}} = \text{diag}\left(\tilde{\sigma}_{x,j}^2, \tilde{\sigma}_{y,j}^2, \sigma_\delta^2\right)$ and $W^{\text{update}} = \text{diag}(10.0, 10.0, 0.1, 0.1745, 0.001, 0.25)$. The horizon lengths M and N were both set to 15 for both the MHE and the NMPC algorithm, which means that the respective horizons stretch across 3 s of past measurements (for MHE) and across 3 s of future prediction (for NMPC). As explained in the Sections 3 and 4, there is no globally valid strategy for choosing M and N available and, therefore, these values were chosen rather heuristically providing decent sizes for the estimation and control horizons while keeping the computational complexity low.

The model response $m(x,u) = (x_{\text{pos}}, y_{\text{pos}}, u_1, u_2)^T$ employed by the NMPC routine incorporates the states describing the x–y position as well as the wheel velocity u_1 and the steering rate u_2 in this study. The utilized reference vectors $s_j = \left(x_{\text{ref}}^{(j)}, y_{\text{ref}}^{(j)}, \hat{u}_{1,k}, 0\right)^T$ combine the desired time-varying position references $x_{\text{ref}}^{(j)}$ and $y_{\text{ref}}^{(j)}$

in global Cartesian coordinates for the future time instance t_j, the most recent MHE estimate for wheel velocity $\hat{u}_{1,k}$ and the time-constant reference of 0 rad/s for the steering rate into vectors s_j with $j = k+1, \ldots, k+N$. Here, the usage of the recent wheel velocity estimate $\hat{u}_{1,k}$ in the objective function provided a simple possibility to penalize variations of the wheel velocity at the control output from timestep to timestep. The same argument holds also for the incorporation of the zero reference for the steering rate. However, it needs to be stressed that both controls are not penalized heavily and that, therefore, the NMPC controller keeps sufficient freedom to vary both for an accurate tracking of the GPS position references.

Upper and lower bounds have been imposed on both the steering rate u_2 (±35°/s) and the steering angle δ (±35°). The respective (constant) weighting matrix is $D = \mathrm{diag}(1.0, 1.0, 5.0, 5.0)$. The weighting matrix at the end node of the control horizon is set to $E = 10 \cdot D$. This means that a deviation of the predicted position at the end of the horizon from its reference is penalized ten times more within the NMPC cost function than for previous points. As the control variables u_1 and u_2 are assumed piecewise constant within subintervals and defined by the value at the start of the interval, the penalty end term effectively penalizes the dynamic states in m only.

6. Experimental results

After being started off-track, the tractor finds its (constantly shifting) reference trajectory quickly and is capable to stay on-track afterwards (see Fig. 5). The NMPC output, the real actuation in case of the wheel speed and the respective steering angle are shown in Fig. 6. It can be observed that the calculated speeds and steering rates show relatively small changes from timestep to timestep. This is a result of the provided weighting. It is to be noted that the steering angle δ itself being a state of the model is only subject to minor corrections on slightly curved strips and subject to maximum steering action in the so-called headland turns, as also can be verified in Fig. 6.

The MHE state and parameter estimation is visualized in Fig. 7. The actual state estimation performance becomes visible on the basis of the yaw angle β. It is an example for a state typically being difficult to measure but playing an important role for the accuracy

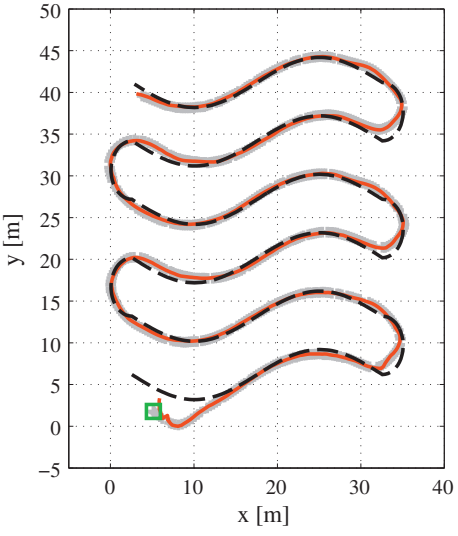

Fig. 5. The reference and the actual trajectory of the autonomous tractor; black dashed line: reference, red line: estimated center of rear axle, gray markers: GPS measurements, green square: starting point. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

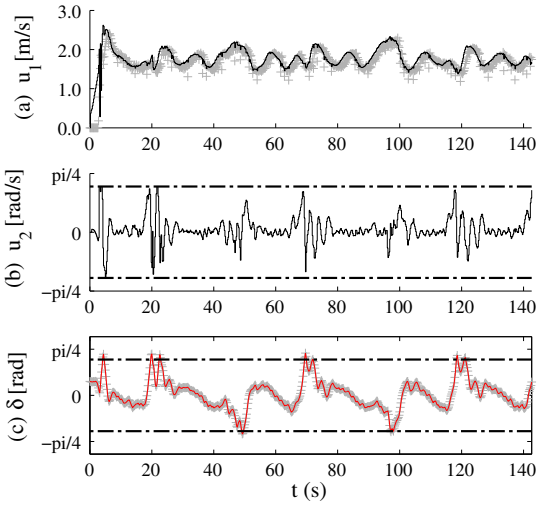

Fig. 6. NMPC output and steering angle; (a) black line: wheel speed, gray markers: measurements, (b) black line: steering rate, dashed line: imposed bounds ±35°/s, (c) red line: steering angle estimate, gray markers: measurements, dashed line: imposed bounds ±35°. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

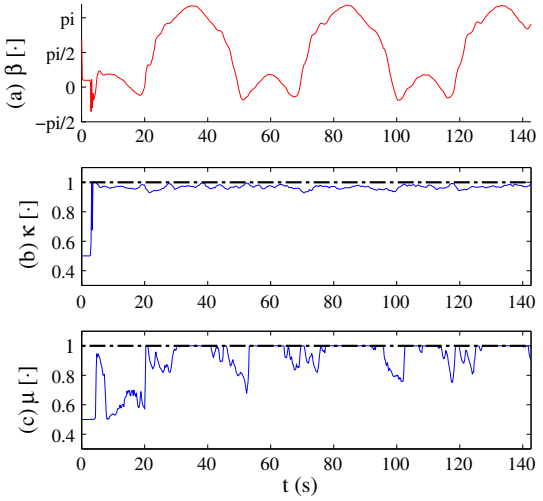

Fig. 7. MHE output; (a) red line: yaw angle estimate, (b) blue line: forward slip parameter estimate κ, dashed line: parameter bound of 1, (c) blue line: side slip parameter estimate μ, dashed line: parameter bound of 1.

of the process model. Not estimating such a state could easily deteriorate the model and render it useless for control purposes. The estimation of the traction parameter κ and the side-slipping parameter μ is also shown in Fig. 7. Naturally, the "true" values for both model parameters are difficult to determine. Although initialized with very bad initial guesses, after only a few iterations both model parameter estimates stabilize at certain values which secure a stable path-tracking.

In case of the longitudinal slip it is clear that there exists a dependency of the tire pressure on the wheel radius and, therefore, also on the wheel speed measurement and the longitudinal slip which makes an estimation of parameter κ necessary. This holds even when the parameter value is estimated to be close to 1, as observed here, which generally indicates a very low influence of slip within the tested field. In case of lateral slip it can be verified that μ only deviates from the value 1 in the headland turns. Furthermore, it has to be noted here that this parameter must be labeled as unobservable when the steering angle is zero or close-to-zero. In

the given application, the regularization for this parameter has been increased heuristically in these cases by multiplying the respective component of $\bar{Q}$ by a factor 10 when not at least two of the steering angle measurements were greater than 10°. This has the effect that the MHE parameter estimate of μ stays closer to the estimate provided by the smoothed EKF update on the slightly-curved parts of the trajectory, but without violating constraints. The bounds of 1 for both the maximum longitudinal and lateral slip parameters represent upper limits given for geometrical reasons.

The tracking error compared to the time-based reference is visualized in Fig. 8. It stabilizes around an average error of 0.39 m relatively fast when the parameters are estimated sufficiently well.

In the presented experiments one real-time iteration is performed for the MHE as well as for the NMPC step, which both consist of a typically much longer preparation and a shorter estimation/feedback phase. For the total feedback time of the MHE–NMPC framework only the estimation phase of the MHE and the feedback phase of the NMPC need to be summed up. This holds as both preparation phases do not require the most recent measurement and can, therefore, be done in advance of each MHE–NMPC iteration (Diehl, 2001; Kühl et al., 2011). Table 1 summarizes the computation times obtained with a Real-Time Controller supplied with a 2.26 GHz Intel Core 2 Quad Q9100 quad-core processor (NI PXI-8110, National Instruments, Austin, TX, USA). The average and the maximum runtimes of both the preparation as well as the estimation/feedback phase are given. The MHE and the NMPC routine had been assigned to one processor on which both run in serial. It is noted that employing these combined MHE–NMPC real-time algorithms always allowed to give optimized feedback to the tractor within 2 ms after the measurements had been obtained while the average feedback time is less than 1 ms.

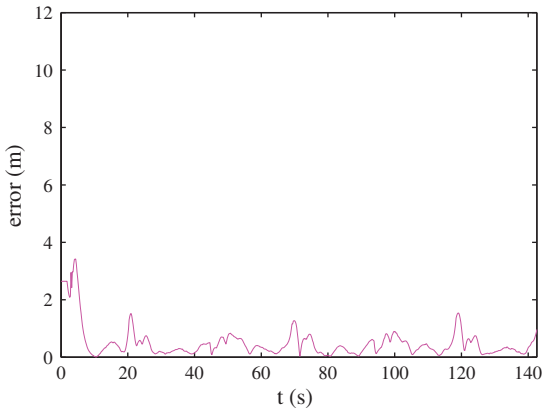

Fig. 8. Absolute tracking error (distance between estimate for the center of the rear axle and time-based reference).

Table 1
Execution times of the auto-generated code of the MHE and the NMPC on the real-time device.

	Average (ms)	Max. (ms)
MHE CPU time		
Preparation	0.83	0.87
Estimation	0.17	0.43
Overall	1.00	1.30
NMPC CPU time		
Preparation	1.47	2.03
Feedback	0.43	1.13
Overall	1.90	3.16

7. Discussion

Despite the difficult soil conditions the challenging reference trajectory is followed with reasonable accuracy by the MHE–NMPC scheme. It needs to be stressed that no inclinometer has been installed to correct for noise in the GPS measurement due to the bumpiness of the grass field. If the tracking errors shortly before and after the so-called headland turns and on the slightly curved trajectories in between are considered separately, an average deviation of 0.26 m and a maximum deviation of 0.68 m is found for the slightly curved strips while the average error lies at 0.60 m and the maximum error at 1.54 m for the headland turns. These numbers describe the deviation between the state estimate for the global position of the mid point of the rear axle and the corresponding time-based reference trajectory as provided to the NMPC controller. The tracking error with respect to the spatial reference are smaller. As mentioned the controller is showing relatively small variations at the output from timestep to timestep which can be seen in Fig. 5. If such a control behavior is not desired the "aggressiveness" of the controller could be increased in a straight-forward manner by increasing the entries corresponding to the x–y positions in the weighting matrix D which is likely to improve the tracking accuracy still.

For both the maximum longitudinal and lateral slip parameters there are natural bounds given. In the experiments, the online parameter estimates for κ and μ always stay very close to these bounds (compare Fig. 7) without violating them. It is most probable that a purely EKF-based estimation with the given data would yield parameter estimates exceeding these values frequently as constraints cannot be incorporated in Kalman Filter frameworks. Such behavior could also be observed in the smoothed EKF updates of $\bar{p}$, which represents an EKF with a better linearization than the original variant. Although these violations were seldomly higher than 5% they could be observed in multiple experiments. But since the parameter estimates play a crucial role for the NMPC performance, such physically wrong estimates can be expected to deteriorate the controller performance significantly. On the other hand, it is to be noted that determining these parameters for a certain tire–soil interaction and hard-coding them would always provide a more accurate path-tracking. However, this is only valid as long as such an offline parameter identification is possible and the corresponding environmental conditions are stable afterwards. For the considered setup where an adaptive control framework is desired the MHE performance enables a good path-tracking performance of the NMPC controller by providing accurate estimates of these model parameters.

Within the MHE, a higher regularization for the parameter μ was applied when only small steering angles were measured within the estimation horizon, as described in the previous section. This is due to a reduced observability of μ in this case. Increasing the trust in the EKF update in such a situation prevents the MHE from varying the parameter estimate excessively to correct for measurement noise. If this is omitted a too low estimate for μ is likely to result in a larger NMPC steering angle output than necessary. Although this self-imposed disturbance will allow for a better estimation of μ after the following measurement again and provide decent NMPC output as long as respective measurements remain in the estimation horizon, such disturbances can be avoided by the described regularization. Possible alternative and less heuristic counter-measures would be to use a Levenberg-Marquardt algorithm instead of the Gauss–Newton algorithm, the explicit incorporation of process noise in the MHE formulation and/or an elongation of the estimation horizon. Another approach is

described in Sui and Johansen (2011), where a more elaborate method to evaluate the observability of system components due to the given process excitation is proposed while the MHE weighting matrices are adapted in case of limited observability.

Given the computation times it would be an obvious step to reduce the sampling time to allow for a faster process feedback. It is clear that this would also require more intervals within the MHE and NMPC horizons if the same time windows should be covered. Practically, this would increase the computational complexity but it is also likely to improve the controller performance still despite the relatively slow process dynamics. For instance, the influence of actuation errors, as they are visible in Fig. 6(c), on the controller performance could be reduced within the current implementation. In the figure, it can be observed that the imposed steering angle constraint of ±35° gets violated occasionally due to actuation errors while the controller reacts with a immediate counter-steering action to get δ in the feasible domain again. In case of a smaller sampling time, the NMPC controller could react to such actuation errors faster and, therefore, reduce the violations. At the same time, when constraint violations occur the duration of such counter-steering is shortened due to the shorter sampling time also.

It is to be noted that the standard bicycle model has been used in previous studies for adaptive model-based control employing more traditional estimation methods like Kalman Filtering. However, in this study the nonlinearity of the system is increased. This is mainly due to the additional presence of a nonlinear measurement function and two unknown model parameters which have to be identified alongside. Also, the reduced amount of measurements makes the resulting navigation problem more interesting from a practical point of view and more challenging as an estimation and control problem despite its relatively small size.

8. Conclusions

An MHE–NMPC framework for autonomous tractor navigation based on an adaptive bicycle model with longitudinal and lateral slip factors has been elaborated in this study. The MHE allows to estimate unknown states and model parameters being unknown due to varying soil conditions online. The NMPC provides close-to-optimal control output based on the estimation results provided by the MHE. For both the estimation and the control the full nonlinear model as well as constraints have been incorporated. Thanks to the automatic code generation of the ACADO toolkit for both the MHE and the NMPC routine the customized real-time iteration code provided feedback times in a millisecond range. Apart from clearly maintaining real-time-feasibility the fully optimization-based solution approach has yielded reasonable tracking quality with average deviations from the time-based reference trajectory of 0.26 m on slightly curved strips and average deviations of 0.60 m in headland turns on a wet and bumpy grass field.

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