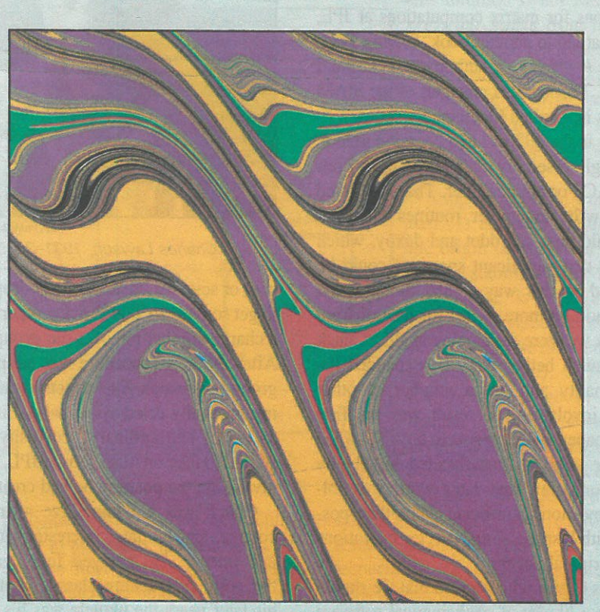
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In the article on page 8, Evelyn Sander and James A. Yorke tackle the topic of chaos and its various manifestations and mathematical definitions.

Figure 2 (above) from the article shows eight basins of attraction for the forced-damped pendulum. The geometic complexity of the basins results from the chaos on their boundaries.

Tensor Decompositions in Smart Patient Monitoring

By Sabine Van Huffel

Substantial progress in both data recording technologies and information processing in recent years has enabled the acquisition and analysis of large amounts of biomedical data. Extraction of the underlying information patterns relevant to patient health, called "sources," is a core problem in biomedical information processing. Blind source separation (BSS) is the task of finding such source signals and the mixing mechanism, given only the raw signals, hence "blind." Rapid advances in healthcare diagnostics and medical technologies are opening new challenges for information processing.

Biomedical signal processing has moved away from vector processing (classical single-channel time- and frequency-domain analysis, despite the continued use in medical practice) to matrix processing. At that level, the BSS problem is solved by decomposition of the data matrix into a sum of interpretable rank-1 terms. This problem is underdetermined, however, unless strong assumptions on the sources are imposed, such as statistical independence (with use of independent component analysis (ICA) methods) or nonnegative components (with use of nonnegative matrix factorization (NMF) methods). Matrix-based BSS methods have become increasingly popular for artifact removal [4] and even for separating stimulus-related activity in multimodal data, such as electro-encephalography (EEG) and functional magnetic resonance imaging (fMRI) [9]. These methods are too restrictive, however. It may be possible to develop more powerful information systems by facing the following challenges:

■ **From matrix to tensor.** For maintaining structural information, higher-order tensors are very attractive. They generalize vectors and matrices to multiway tables of numbers [1]. The BSS problem is solved via tensor decomposition. Most well known is the canonical polyadic decomposition (CPD; also known as canonical decomposition or parallel factor decomposition) [6], which decomposes a tensor in rank-1 terms (Figure 1 on page 5, top). CPD is unique under mild conditions, which makes it a broadly applicable key tool for BSS. The assumptions needed for source extraction are very natural.

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Control, Intuition, Existence, and Regularity

By Francis Clarke

Two concentric rings are dipped in a soapy solution so as to form a surface of revolution (see Figure 1). What is the profile x(·) of the resulting soap film? This famous *minimal surface* problem amounts to identifying the function $x(t)$ that minimizes the integral functional

$$x(\cdot) \mapsto \int_a^b x(t)\sqrt{1+x'(t)^2}\,dt$$

under the constraint that x have prescribed values at a and b. Leonhard Euler solved this instance of the basic problem in the *calculus of variations* in 1744, finding that the (physically observed) curve is a *catenary*. This is a smooth function, and the soap film visibly exists; thus, no issues of regularity or existence would seem to arise at all.

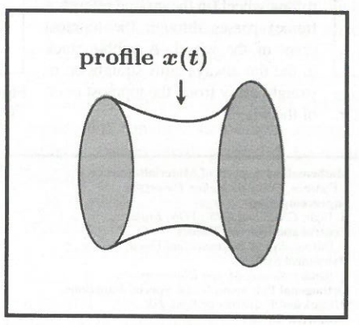


Figure 1. *A soap film spanned by two concentric rings.*

But what happens if we gradually increase the distance between the two rings? At some point, our physical intuition tells us, the bubble pops. Why so? Has the solution to the problem simply ceased to exist?

The reality is more complex. Almost a century later, Goldschmidt explained (in 1831) that, in fact, the minimal surface has folded *onto* the rings and has become the union of two disks—that is, the surface of revolution generated by the "broken curve" shown in Figure 2, one that has two "corners" (points of nondifferentiability). To our knowledge, this heralds the first appearance in analysis of a nonsmooth function (and irreversible dynamics).

There is no element of "control" in the problem above; that is, there is no structure whereby the state $x(\cdot)$ corresponds to the choice of a control function $u(\cdot)$ via certain dynamics, such as that of a *standard control system*

$$x'(t) = f\big(x(t),u(t)\big) \text{ a.e.},$$
$$u(t) \in U \text{ a.e.} \qquad (1)$$

(For the shape of soap bubbles, nature does the controlling, so to speak.) But the insight of Goldschmidt prefigures two of the central topics in control theory, a descendant of the calculus of variations: existence and regularity.

Let's examine these issues for an optimal control problem that arises in modeling renewable resources. There are two state variables: x (which we may think of as a measure of a fish population) and y (available boats, which are subject to depreciation). We choose directly two controls: $u(t)$ (boats sent out at time t to catch fish, a value between 0 and $y(t)$), and $I(t)$ (investment at time t in new boats). It is assumed that investment can have immediate effect; thus I may be an impulse control. The dynamics linking the choice of the control functions (u, I) to the resulting states (x, y) (that is, the function f in (1)), as well as the (net infinite-horizon discounted) return that one seeks to maximize, will not be made explicit here; see [1,3]. Instead, let's take a look at what turns out to be the answer. It has the form of a *feedback synthesis*, in which the choice of control values depends on the current state.

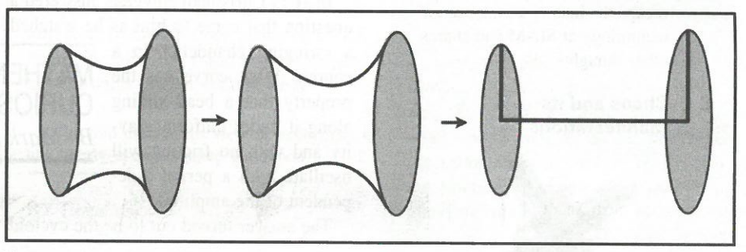


Figure 2. *Does the soap film disappear?*

The optimal synthesis is indicated in Figure 3 (see page 3), for the case in which the initial values of x (fish) and y (boats) lie at the point designated by the letter **a** (many fish, few boats). It makes sense to invest in boats (at a certain cost, of course) from such a point: We move to the point **b** via an impulse purchase. (The dotted curve, whose provenance is explained later, determines **b**.) Subsequently, between **b** and **c**, we use all the boats we have ($u = y$); the number of boats is decreasing through wear and tear (depreciation). Once **c** is reached, there is a change of tactic: We cease to use all available boats ($u < y$) in order to maintain the fish population at a certain level x_S.

As experience shows, some economists will grumble at this stage that it must have been wasteful to buy so many boats initially, since some are not being used now. However, the level x_S, it turns out, corresponds to a *short-term* equilibrium that is a known (and economically accepted) feature of the solution when investment and depreciation are absent; so there is an economic argument for it. At the point **d**, however, where the boat level y_S is attained, economic intuition is even more seriously challenged: We change our minds about

See **Control** *on page 3*

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Tensor Decompositions

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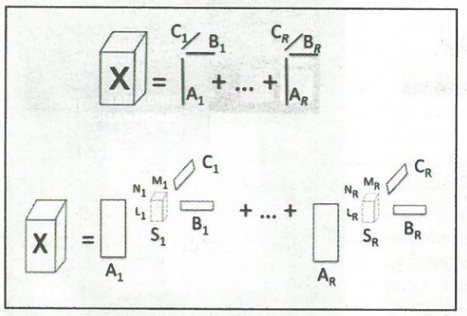


Figure 1. *Decomposition of a third-order tensor. Top, CPD, atomic terms; bottom, BTD, molecular terms. Adapted from [3].*

■ **From rank-1 to low-rank.** It is important to observe that a rank-1 structure is, in fact, very restrictive. Except for strength, no other source variations can be modeled. The decomposition may still be unique when it involves terms that are more general and/or realistic than rank-1 terms. We denote such generalized decompositions as block term decompositions (BTD) [3]. (See Figure 1, bottom.) BTDs make it possible to model more variation (shape, delay, . . .) and provide more detail. In addition, BTDs allow broad source modelling [2].

De Lathauwer's team implemented most tensor decompositions, including CPD and BTD, into a powerful open-source, easy-to-use, optimization-based Matlab toolbox; see www.tensorlab.net. Moreover, factorizations can be coupled or fused with each other. Factors can be shared, and any structure can be imposed. Tensorlab is the core of our solution of tensor-based BSS problems. A few case studies illustrate the process.

Extracting the Epileptic Component from EEG

Because seizures exhibit oscillatory behavior that is almost stable in localization and frequency during an entire episode (2–10 sec), they fit a trilinear structure. In Figure 2, we construct a third-order tensor via wavelet transform of all EEG channels and show that CPD reliably identifies one epileptic source with $R = 2$ and no need for artifact removal [4].

When seizures are nonstationary (e.g., varying in space or frequency during the episode), CPD is too restrictive. In that case, use of BTD improves the extracted seizure component, as shown in [5]. We applied wavelet transform or Hankel expansion to organise the EEG data in a tensor. With the former approach, we were able to model nonstationary seizures evolving in either frequency or spatial distribution; the latter was useful for extracting the epileptic pattern obscured by severe artifacts. Nevertheless, successful use of this technique in practice depends on blind selection of appropriate model parameters.

Monitoring Neonatal Brain Recovery from EEG

We have worked to automate monitoring of the vulnerable brain in the Neonatal Intensive Care Unit following hypoxic insult or brain injury, after which seizures often emerge. We developed CPD-based algorithms similar to those described above and successfully extracted the onset of seizures [4]. In addition, we sought to monitor brain recovery through automated quantification of the abnormality in one-hour EEG segments (called "EEG grading"). Our proposed holistic approach is shown in Figure 3. When we simultaneously decomposed all such tensors in a training set of 33 neonates via higher-order discriminant analysis (HODA), more relevant features were selected, thereby improving EEG grading accuracy (89%) and increasing robustness to the presence of artifacts [7].

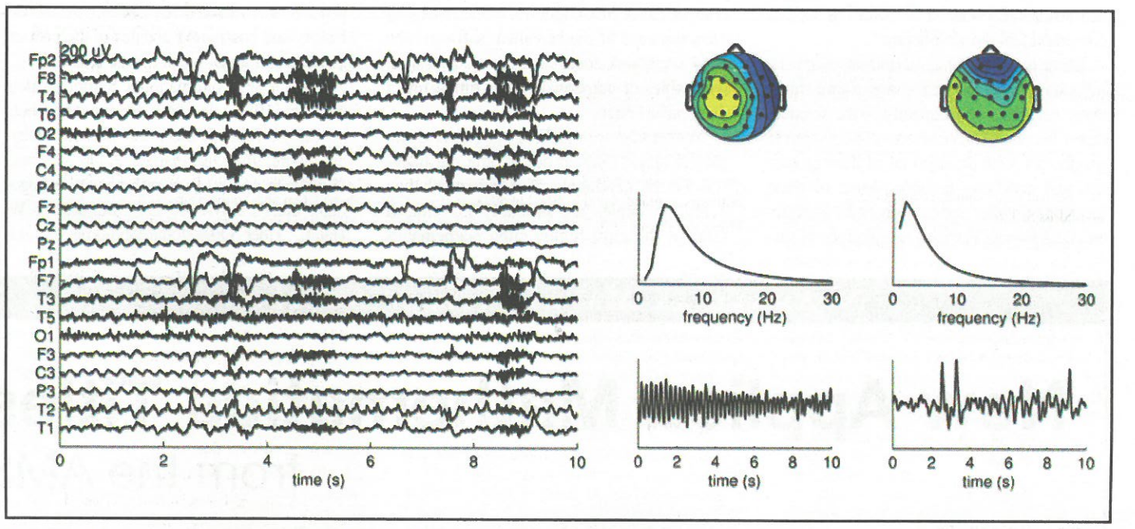


Figure 2. *At left, CPD of wavelet transformed EEG showing 10 sec onset of a right temporal lobe complex partial seizure. Muscle (at 5 and 9 sec) and eye blink artefacts (sharp wave) are visible. At right, the spatial (top), frequency (middle), and temporal modes (bottom) are shown. The epileptic atom is on the left, the eye blink artefact on the right. Adapted from [5].*

Combining EEG and fMRI to Study Cognitive Function

Symmetric, data-driven fusion of EEG and fMRI has the potential to reveal and characterize the consecutive steps of cognitive brain processing with high spatiotemporal resolution. The methodological challenge is to define meaningful yet efficient multiway representations, decompositions, and constraints for the coupling of the two modalities.

Using a well-known visual detection task, we successfully applied Joint-ICA: structuring of EEG and fMRI data together in one matrix, followed by joint decomposition with ICA [8]. This algorithm was extended to more complicated task paradigms (e.g., contour integration) and to a multichannel set-up. Extensions to a tensor-based framework are currently under investigation.

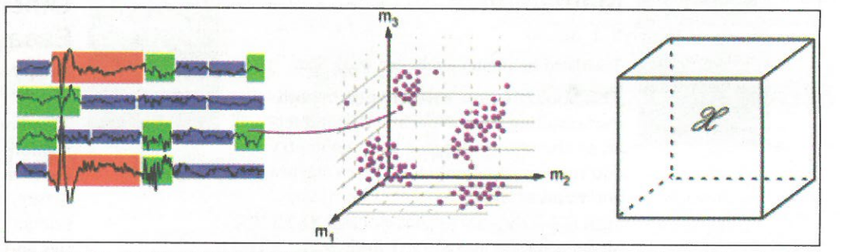


Figure 3. *The algorithm performs adaptive segmentation, according to the EEG amplitude (left: red, green, and blue indicate respectively high, moderate, and low amplitude EEG) and maps the segments' features (amplitude, duration, spatial distribution) into a multidimensional histogram (middle) which is stored in a tensor (right). Courtesy of Vladimir Matic.*

In summary, we have shown that tensor decompositions can be highly useful in biomedical data processing. CPD is now the main approach; the advantages of BTD are emerging. Tensors still have great unexplored potential in smart patient monitoring.

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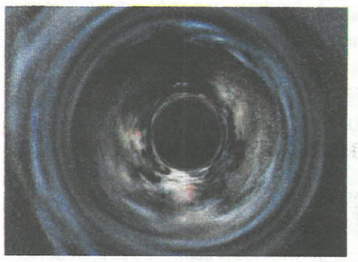
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Sabine Van Huffel is a professor in the Department of Electrical Engineering (ESAT), KU Leuven, and is principal investigator at iMinds Medical IT Department, Leuven, Belgium.

Interstellar

continued from page 4

Thorne describes some of the experimental evidence confirming this aspect of the general theory. When director Nolan asked Thorne if time on a certain planet could be slowed to the extent that a single hour on its surface could correspond to seven years on Earth, Thorne initially doubted that it could. However, after playing around with the field equations, Thorne discovered such slowing to be possible in the environs of a black hole spinning at 99.8% of the limiting rate of spin imposed by the fact that nothing can exceed the speed of light. In time, Thorne was able to develop a specialized set of field equations that enabled the project's technically sophisticated special effects team to produce a picture of a massive and rapidly spinning black hole silhouetted against a star-studded patch of night sky (see image at left).



The stars in Gargantua's galaxy as seen around the black hole's shadow, depicted in Figure 3.3 in the book. The black hole bends the light rays coming from each star, "gravitationally lensing" the galaxy.

This, according to Thorne, is new science! It shows with great precision how "gravity lensing" would distort any picture one might take of the firmament beyond such an object. Indeed, Thorne is currently writing a technical paper describing the new science discovered during the making of *Interstellar*.

Thorne cofounded the Laser Interferometer Gravitational Wave Observatory (LIGO) project in 1983 along with Rainer Weiss at MIT and Ronald Drever at Caltech. The project he envisioned and spent two decades bringing to fruition is now an international collaboration of 900 scientists in 17 nations. With gravitational wave detectors in Hanford, Washington, and Livingston, Louisiana, in the U.S., a third location planned in India, and similar detectors under construction in Italy and Japan, it will form a worldwide network to explore the universe using gravitational waves. In the book, Thorne explains in particular how the movie's NASA personnel might have used LIGO measurements to detect the presence of a wormhole near Saturn.

As interesting as these recent discoveries are, they fall well short of explaining how the hero of the story, a NASA-trained space pilot/engineer named Cooper—played onscreen by Matthew McConaughey—manages to survive his fall into the black hole Gargantua and return through the wormhole in a tesseract (a four-dimensional hypercube) provided by "them" in time to witness his daughter's death on a space station orbiting Saturn. It does even less to explain how Cooper manages to send data recorded inside the black hole backward through time to that very daughter (a celebrated physicist in adulthood) in time for her to figure out how one might exploit "gravitational anomalies" to propel county-sized space stations into solar orbit. Thorne supplies the missing explanations in the final chapters of this well-written book, a must read for all who refuse to abandon the dream of travel through interstellar space.

Jim Case writes from Baltimore, Maryland.