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A Proposed Scheme for Adapting PWM Frequency and PID controller to Temperature Changing (Internal Report)

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The experiment results have shown that when temperature goes to below $0°C$, the second circuit (maybe also the primary circuit) in the valve body may not work. The reason for this is simple: both oil viscosity and density in low temperature is very high so that the output of PWM with frequency 60 Hz is very small. Consequently, the time constant of the whole system is very big. To solve this problem, both PWM frequency and PID control parameters should be adapted to different oil viscosity and density which are functions of temperature.

1 Analysis of the hydraulic system

It is difficult to analyze quantitatively the exact behavior of the second circuit of valve body when temperature changes because of the complex of the mathematical model. However it is possible to analyze qualitatively this behavior by taking two assumptions: the mechanical dynamics in valve body is negligible and the system is linear. Then by using the measured data from the tests, it is possible to find the way to adapt PWM frequency and PID controller to different temperature

By the first assumption, the second circuit in valve body can be presented by two parts: the pure hydraulic part and the pure PWM part which is just a two state valve, see Fig. 1. For the pure hydraulic part, the transfer function may be written as:

$$H_{hydro}(s) = K(\nu,\rho)\frac{\Pi(\frac{s}{z_i(\nu,\rho)}-1)}{\Pi(\frac{s}{p_j(\nu,\rho)}-1)} \tag{1}$$

where ν and ρ are oil viscosity and density, $z_i(\nu,\rho)$ and $p_j(\nu,\rho)$ are zeros and poles, $K(\nu,\rho)$ is a proportional factor. These zeros, poles and proportional factor are the functions of the oil viscosity and density. Now we can analyze how these zeros and the poles are composed.

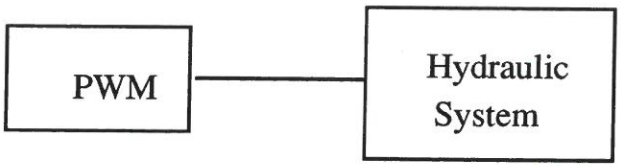

Figure 1: the secondary circuit representation

The Laplace variable, s, is with the unit of $1/sec.$. So zeros $z_i(\nu,\rho)$ and pole $p_i(\nu,\rho)$ are also with the unit of $1/sec.$. In this pure hydraulic system, there are only two kinds of elements which determine the zeros and poles: resistors with the unit $kgf_sec./m^5$ and capacitors with the unit of m^5/kgf. Thus, the obvious result is that each zero or pole has following form:

$$R_{eq}C_{eq} \tag{2}$$

where $R_{eq}(\nu,\rho)$ is equivalent resistance with the unit $kgf_sec./m^5$ and $C_{eq}(\nu,\rho)$ is equivalent capacitance with the unit m^5/kgf. Both R_{eq} and C_{eq} are rational combinations of the resistors and capacitors in the hydraulic part. We take assumption that the resistance for each hydraulic resistor is:

$$r = \frac{K_r}{\nu\rho} \tag{3}$$

where K_r is constant. This assumption actually is to make linearization of the resistance function with respect to ν and ρ. It is known that the capacitance for each of capacitors is constant when ν and ρ are changing. Then finally we may derive the expression for each zero or pole as a function of viscosity and density:

$$\frac{K_{req}C_{eq}}{\nu\rho} \tag{4}$$

where K_{req} and C_{eq} are constant. Then, Eq. (1) may be rewritten as:

$$H_{hydro}(s) = K(\nu\rho)\frac{\Pi(\frac{s\nu\rho}{K_{reqi}C_{eqi}} - 1)}{\Pi(\frac{s\nu\rho}{K_{reqj}C_{eqj}} - 1)} \tag{5}$$

Let $H^*_{hydro}(s)$ denotes the transfer function $H_{hydro}(s)$ when $\nu = \nu^*$ and $\rho = \rho^*$. Then, from Eq. (5), we can find

$$H_{hydro}(s) = \frac{K(\nu\rho)}{K(\nu^*\rho^*)}H^*_{hydro}(\frac{\nu\rho}{\nu^*\rho^*}s) \tag{6}$$

This is the equation we will use for deriving the laws of adaptation of PWM frequency and PID controller parameters. The physical meaning of the Eq. (6) may be explained as following. Term $K(\nu\rho)/K(\nu^*\rho^*)$ is the ratio of the static gains when the viscosity and density change from ν and ρ to ν^* and ρ^* respectively. From the test results, see Fig. 3

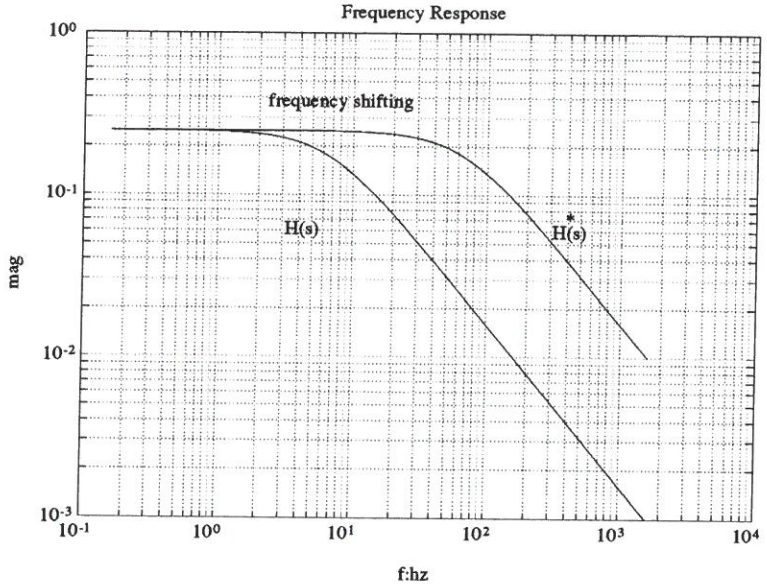

Figure 2: the frequency response shifting when ν and ρ changes

to 6, this static gains, corresponding the average values, are almost unchanged, in other word, the value of $K(\nu\rho)/K(\nu^*\rho^*)$ is 1. Then Eq. (6) becomes

$$H_{hydro}(s) = H^*_{hydro}(\frac{\nu\rho}{\nu^*\rho^*}s) \tag{7}$$

Thus, in frequency domain, the frequency response $H_{hydro}(j\omega)$ can be obtained by shifting the frequency response $H^*_{hydro}(s)$ by $-20log(\nu\rho/\nu^*\rho^*)$ db. This can be depicted in Fig. 2.

2 PWM frequency adaptation

Now we are at the point to derive PWM frequency adapting law. Take the inverse Laplace transfer to Eq. (7), we have the time impulse responses:

$$h_{hydro}(t) = \frac{\nu^*\rho^*}{\nu\rho}h^*_{hydro}(\frac{\nu^*\rho^*}{\nu\rho}t) \tag{8}$$

where $h_{hydro}(t) = L^{-1}[H_{hydro}(s)]$ and $h^*_{hydro}(t) = L^{-1}[(H^*_{hydro}(s)]$. Let the input be $u^*(t)$ when the viscosity and density are ν^* and ρ^*, then the output, denoted by $y^*(t)$, is the convolution:

$$y^*(t) = \int_0^t u^*(\tau)h^*_{hydro}(t-\tau)d\tau \tag{9}$$

For arbitrary viscosity ν and density ρ, an input $u(t)$ will cause the output $y(t)$ as:

$$y(t) = \int_0^t u(\tau) h_{hydro}(t-\tau) d\tau = \int_0^t u(\tau) \frac{\nu^*\rho^*}{\nu\rho} h^*_{hydro}[\frac{\nu^*\rho^*}{\nu\rho}(t-\tau)] d\tau \tag{10}$$

Let

$$u(t) = u^*(\frac{\nu^*\rho^*}{\nu\rho}t) \tag{11}$$

then, $y(t)$ becomes:

$$y(t) = \int_0^{\frac{\nu^*\rho^*}{\nu\rho}t} u(\sigma) h^*_{hydro}(\frac{\nu^*\rho^*}{\nu\rho}t - \sigma) d\sigma = y^*(\frac{\nu^*\rho^*}{\nu\rho}t) \tag{12}$$

This equation shows that with different viscosity and density, the outputs have exact dynamics except that the time scaling is changed by $\frac{\nu^*\rho^*}{\nu\rho}$. If this is what we want, then Eq. (11) actually gives the adapting law for PWM frequency:

If for certain viscosity ν^* and density ρ^*, we use the PWM frequency f^* to get the response $y^*(t)$ and if we want to get same response but only with a different time scaling, or more precisely, $y^*(\frac{\nu^*\rho^*}{\nu\rho}t)$ for arbitrary viscosity ν and density ρ, then the PWM frequency should be

$$f = \frac{\nu\rho}{\nu^*\rho^*} f^* \tag{13}$$

For example, let ν^{50} and ρ^{50} correspond to the viscosity and density at the temperature of $50°C$ and the PWM frequency be 60 hz. Then at the temperature of $0°C$ if we want the output has the same dynamics as at $50°C$ with the time scaling $\frac{\nu^*\rho^*}{\nu\rho}t$, the PWM frequency should be

$$f = \frac{\nu^0\rho^0}{\nu^{50}\rho^{50}} 60$$

If we adapt PWM frequency f by Eq. (13), then we may combine pure PWM part and the pure hydraulic part in Fig. 1 to form the whole second circuit. The transfer function has following character:

$$H(s) = H^*(\frac{\nu\rho}{\nu^*\rho^*}s) \tag{14}$$

where $H^*(s)$ is the transfer function when the viscosity and density are ν^* and ρ^*.

3 PID parameters adaptation

Let $C(s)$ denotes the PID controller transfer function, then the close loop transfer function is

$$H_{close}(s) = \frac{C(s)H(s)}{1 + C(s)H(s)} \tag{15}$$

Let $C^*(s)$ and $H^*_{close}(s)$ denote the controller transfer function and corresponding close loop transfer function for viscosity ν^* and density ρ^*. If we want the close loop dynamics with any viscosity ν and density ρ is same as that with viscosity ν^* and density ρ^* except the time scaling, or more precisely:

$$H_{close}(s) = H^*_{close}(\frac{\nu\rho}{\nu^*\rho^*}s) \tag{16}$$

then, from Eq. (14) and (15), we have the control adaptive law:

$$C(s) = C^*(\frac{\nu\rho}{\nu^*\rho^*}s) \tag{17}$$

For PID controller,

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{18}$$

Let K^*_p, K^*_i and K^*_d denote the PID controller values, then, from Eq.(17) and (18) we have

$$C(s) = K^*_p + \frac{\nu^*\rho^*}{\nu\rho}\frac{K^*_i}{s} + \frac{\nu\rho}{\nu^*\rho^*}K^*_d s \tag{19}$$

The corresponding parameter adaptive laws are

$$\begin{aligned} K_p &= K^*_p \\ K_i &= \frac{\nu^*\rho^*}{\nu\rho}K^*_i \\ K_d &= \frac{\nu\rho}{\nu^*\rho^*}K^*_d \end{aligned} \tag{20}$$

For a digital PID controller, there is a very interesting result. The digital form of the PID controller given by Eq.(18 is

$$C(z^{-1}) = K_p(1 - z^{-1}) + hK_i + \frac{K_d}{h}(1 - 2z^{-1} + z^{-2}) \tag{21}$$

where h is the sampling period. If we define the digital PID controller parameters same as that in [1]:

$$\begin{aligned} p_{gain} &= K_p \\ i_{gain} &= hK_i \\ d_{gain} &= \frac{K_d}{h} \end{aligned} \tag{22}$$

then the adaptive laws for these parameters are:

$$\begin{aligned} p_{gain} &= K_p = K_p^* \\ i_{gain} &= hK_i = h\frac{\nu^*\rho^*}{\nu\rho}K_i^* \\ d_{gain} &= \frac{K_d}{h} = \frac{1}{h}\frac{\nu\rho}{\nu^*\rho^*}K_d^* \end{aligned} \tag{23}$$

Now we also adapt the sampling period h to any viscosity ν and density ρ such that:

$$h = \frac{\nu\rho}{\nu^*\rho^*}h^* \tag{24}$$

where h^* is the sampling period at viscosity ν^* and density ρ^*. Let p_{gain}^*, i_{gain}^* and d_{gain}^* denote the digital PID controller parameters with sampling period h^* at the viscosity ν^* and density ρ^*, then, from Eq. (23) and (24),

$$\begin{aligned} p_{gain} &= p_{gain}^* \\ i_{gain} &= i_{gain}^* \\ d_{gain} &= d_{gain}^* \end{aligned} \tag{25}$$

This means that if we use Eq. (24) to adapt the sampling period h then the digital PID controller parameters will be constant for any viscosity and density.

4 Practical Consideration

From previous discussion, we have known that the close loop of the second circuit and the PID controller has same dynamics but only with different time scaling for different oil viscosity and density, by adapting PWM frequency via Eq. (13) and sampling period via Eq. (24). What we need for these adaptive procedures is the value:

$$\frac{\nu\rho}{\nu^*\rho^*}$$

Theoretically, this value is a function of the temperature because both oil viscosity and density are functions of the temperature, but it seems difficult to calculate it practically. This value, however, may be obtained from some tests on the hydraulic part indirectly. The test is done in following way: Let duty cycle be 50%, then change the PWM frequency from 2 Hz to 100 Hz, repeat this for different temperature. Fig. 3 to 6 shows some of the test results. What we can get from these tests is frequency responses for different temperature. As we have already discussed earlier, the difference between two frequency responses at two different temperature points is frequency shifting by $-20log(\nu\rho/\nu^*\rho^*)$, see Eq. (7). To calculate the frequency shift, we need frequency response identification from the test

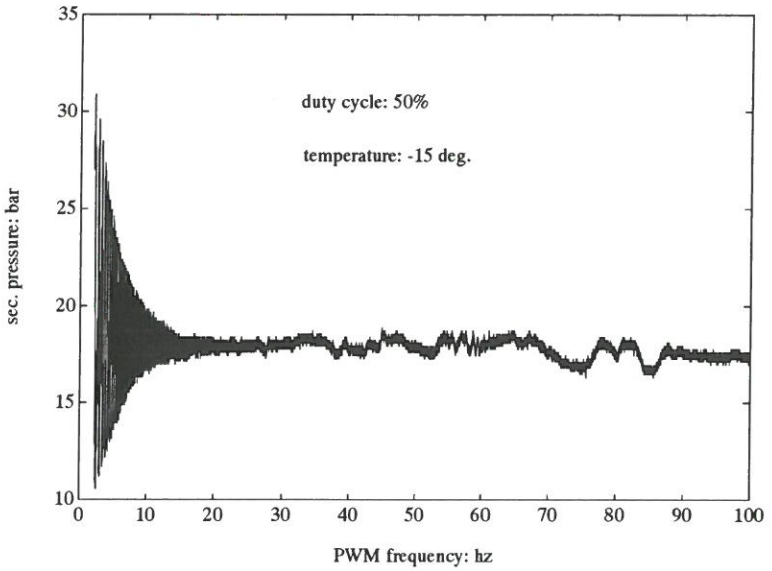

Figure 3: second pressure for duty cycle 50%, temperature $-15°C$

data. A more practical way which may directly give the PWM frequency is proposed by VCST based on following observation. When duty cycle is 50%, the ripple of 0.5 bar in the second pressure is accepted. Then the PWM frequency adaptive law is: for an certain temperature, the PWM frequency is the frequency at which the ripple in second pressure is 0.5 bar when the duty cycle is 50%. From the test data, the PWM frequency with respect to temperature values is shown in Fig. 7. For the PID controller, we may always choose the sampling frequency same as the PWM frequency and remain the PID parameters constant.

5 Conclusion

In this report, we derived the PWM frequency and sampling period adaptive law such that, without changing PID parameters, the close loop response has same dynamics, but with different time scaling, for different temperature. It should be emphasized here is that whether several assumptions in the derivation, for instance, linearity and neglecting of the mechanical dynamics which may not change with the temperature, are valid practically or not remains unknown. More tests are necessary.

References

[1] Yi Cheng, "Notes on VOLVO PID Control Algorithm", Internal Report, ESAT, K.U. Leuven, Jan. 14, 1992

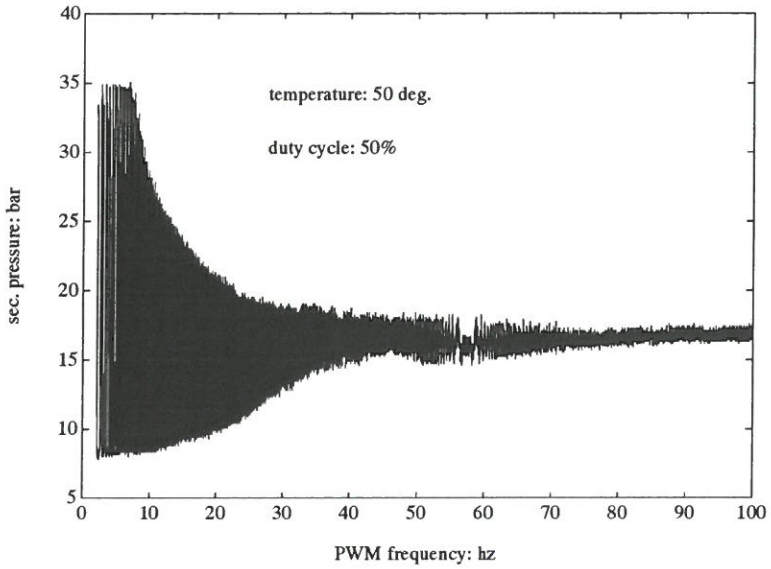

Figure 4: second pressure for duty cycle 50%, temperature $50°C$

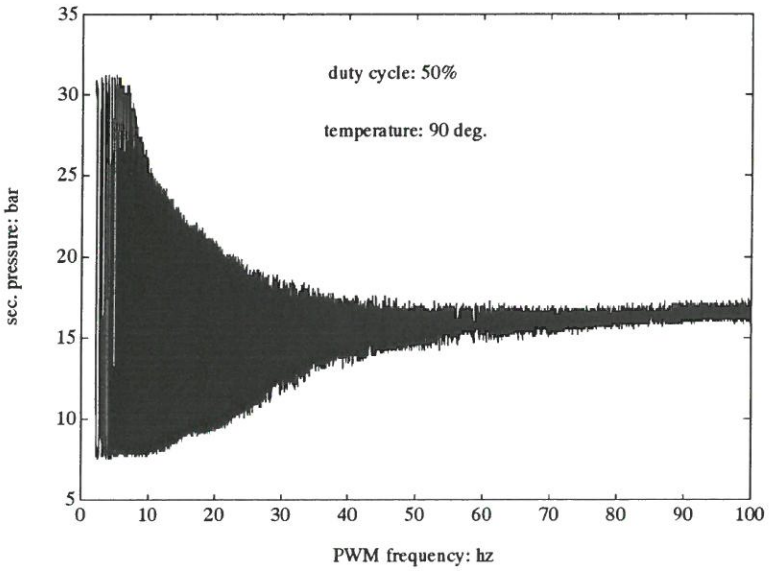

Figure 5: second pressure for duty cycle 50%, temperature $90°C$

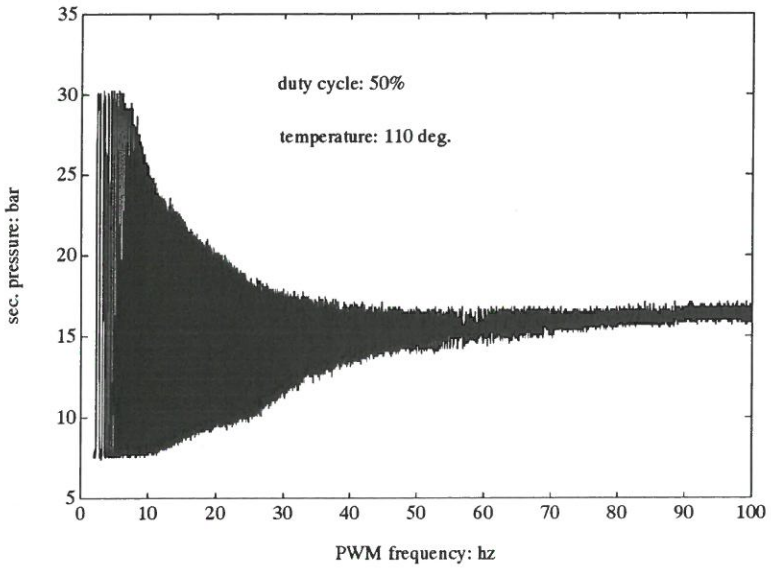

Figure 6: second pressure for duty cycle 50%, temperature $110°C$

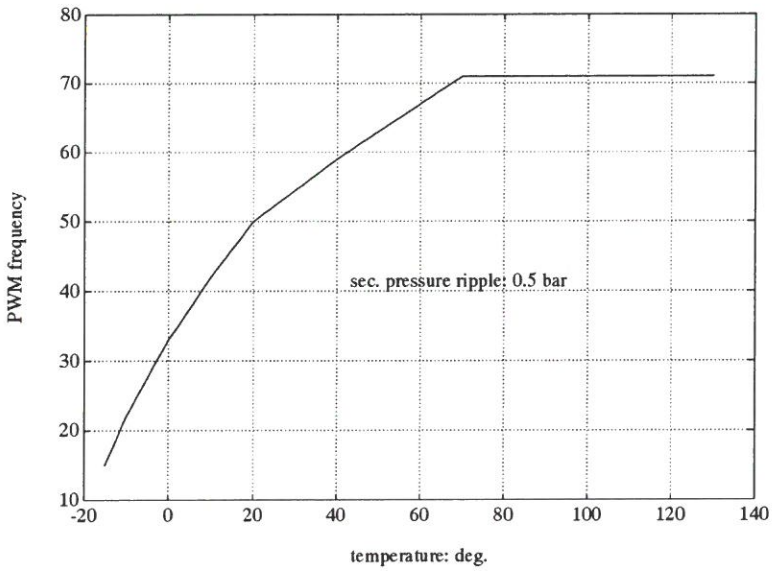

Figure 7: PWM frequency with ripple 0.5 bar for duty cycle 50%