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MULTI-MODE SYSTEM IDENTIFICATION

Erik I. Verriest*, Bart De Moor**

* *School of ECE*
Georgia Institute of Technology
Atlanta, GA 30332-0250
Fax : +1 404 894-2997 and e-mail : erik.verriest@ee.gatech.edu

** *Department of Electrical Engineering - ESAT/SISTA*
Katholieke Universiteit Leuven
B-3001 Leuven, Belgium

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Abstract

The subspace identification method for time-invariant linear systems is extended to the class of multi-mode or hybrid systems. Under an excitation criterion, the parameters in all modes are simultaneously estimated, and combined with a detection scheme for the mode switching. In turn, parameters of the mode switching mechanism, whether deterministic or markovian, can be estimated as well. The method is readily applicable to fault detection and identification.

1 Introduction

The problem of detecting changes in the parameters of a dynamic system has been studied via different methods during the past decades. There are too many references to cite them all. The main ideas have been gathered in the books [6, 3], and the surveys [8, 4, 5]. As discussed in recent papers [2, 9], many problems can be categorized under the detection framework. Fault detection and model validation are closely related. In fault detection one checks whether all parameters (of sensors, actuators and process dynamics) coincide with their nominal values, whereas in model validation, one wants to determine if a preconceived model is consistent with the observed input output data. It leaves blank the modeling of the nominal behavior itself. The present paper combines the identification step with the change point detection. In fact it goes further by allowing the identification of the system parameters once the fault occurred. Generalizing the problem even more, it will be assumed that the nominal system itself can operate in several (but finitely many) modes. This brings the problem to the realm of hybrid or multi-mode systems. Particularly important subclasses are the periodic systems (mode switching in a deterministic periodic pattern), hybrid systems (there is no unique definition, we shall here consider a system that shows a Markovian mode switching behavior as hybrid) and simply multi-mode systems, if the mode switching is deterministic (albeit unknown to the observer) as e.g., controlled by an independent actor. Preliminary results were reported in [11].

In this paper it is assumed that a system may operate in several possible modes. The objective is the identification of the system parameters in each mode. As usual, the final objective may be the ability to predict future behavior of the system or to control its behavior.

It is thus desired to combine parameter estimation (identification) with detection of mode transitions. A simple approach would be to use any of the classical on-line identification methods perhaps followed by validation and change detection. With or without change detection, the identification algorithm will keep tracking the parameters, but quite some time may lapse before the new values are identified. This inertia is essentially due to the large amount of memory that is associated with the identification method. Consequently good tracking performance can only be obtained if the system parameters are slowly varying. In hybrid systems, the change is assumed to be very abrupt, and in the case of fast mode switching it may be unreasonable to expect a classical identification algorithm to work. What is proposed in this paper is a method which enables identification of all modes *in parallel* so to speak. Of course this can only be feasible if all modes have already occurred. So one should expect some type of *persistency of excitation* condition.

At once the change point detection is performed at the same time, basically using the same operations that are necessary for the mode identification. The identification and detection are not inseparable: one helps the other.

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In section 2 the basic principle in the identification of deterministic ARMA models is explained. In order to set the ideas we focused on a two-mode system. Extensions to a larger number of modes are straightforward in principle, but increase the computational complexity. State space models with discretely varying parameters are discussed in the section 3. Some examples are presented in Section 4, where also some general comments on accuracy will be given.

2 Multi Mode ARMA Identification

For simplicity we consider a single input single output system operating in two possible modes. The observed data consists of a sequence of pairs $h_k = [u_k, y_k]'$. Once the method is explained, it will be obvious how to extend the results for an M-mode system. Consider a representation of the system dynamics in ARMA form,

$$y_k = -a_1^{(m)} y_{k-1} - a_2^{(m)} y_{k-2} - \cdots - a_n^{(m)} y_{k-n} + \\ + b_1^{(m)} u_{k-1} + b_2^{(m)} u_{k-2} + \cdots + b_n^{(m)} u_{k-n} \quad (1)$$

In the above equation, the superscript, m refers to the particular mode the system is operating in. It is assumed that the order n is the same for all modes. In order to fix the ideas, suppose further that mode switching occurred at time $k = 0$. Thus for $k \leq 0$, the data vectors satisfy

$$d_k\theta_1 = [y_k, y_{k-1}, \cdots, y_{k-n}; u_{k-1}, u_{k-2}, \cdots, u_{k-n}]\theta_1 = 0, \quad (2)$$

with

$$\theta_1 = [1, -a_1^{(1)}, \cdots, -a_n^{(1)}; b_1^{(1)}, \cdots, b_n^{(1)}]'. \quad (3)$$

For $k \geq 1$, the data vectors satisfy

$$d_k\theta_2 = [y_k, y_{k-1}, \cdots, y_{k-n}; u_{k-1}, u_{k-2}, \cdots, u_{k-n}]\theta_2 = 0 \quad (4)$$

with

$$\theta_2 = [1, -a_1^{(2)}, \cdots, -a_n^{(2)}; b_1^{(2)}, \cdots, b_n^{(2)}]'. \quad (5)$$

Stacking T consecutive data vectors $d_k, d_{k+1}, \ldots, d_{k+T-1}$ in a data matrix

$$\begin{bmatrix} d_k \\ \vdots \\ d_{k+T-1} \end{bmatrix} = D_{k+T-1}^k, \quad (6)$$

we find thus

$$\begin{aligned} D_0^{-N}\theta_1 &= 0 \\ D_T^1\theta_2 &= 0 \end{aligned} \quad (7)$$

However the combination

$$\begin{bmatrix} D^- \\ D_+ \end{bmatrix} \theta = 0 \quad (8)$$

will not be satisfied for either $\theta = \theta_1$ nor $\theta = \theta_2$. As the classical identification algorithms [10] are based on the (approximate) solution of $D\theta = 0$, a relatively long time will be required for data crossover, i.e., before one settles into $\theta \approx \theta_2$.

Consider now again mode switching at arbitrary times. Let N consecutive data vectors, $d_k; k = 1, \ldots, N$, be collected. The main novel idea which makes fast identification of both models possible is the observation of the logic

$$\{ d_k'\theta_1 = 0 \} \vee \{ d_k'\theta_2 = 0 \} - \{ d_k'\theta_1 \cdot d_k'\theta_2 = 0 \} \quad (9)$$

The right hand side can be reorganized as

$$d'\theta_1 \cdot d'\theta_2 = \mathrm{Tr}\, dd'\theta_1\theta_2' = \mathrm{vec}\,(dd')' \cdot \mathrm{vec}\,(\theta_1\theta_2') = d^{(2)}\theta^{(2)}. \quad (10)$$

The datavectors $d^{(2)} = \mathrm{vec}\,(dd')$ are quadratic in the original data. Stacking these consecutive data vectors $d^{(2)}$ again in a datamatrix

$$D^{(2)} \stackrel{\mathrm{def}}{=} \begin{bmatrix} d_1^{(2)} \\ \vdots \\ d_N^{(2)} \end{bmatrix} = \begin{bmatrix} \mathrm{vec}\,(d_1 d_1')' \\ \vdots \\ \mathrm{vec}\,(d_N d_N')' \end{bmatrix} \quad (11)$$

gives the system of equations

$$D^{(2)}\theta^{(2)} = 0. \quad (12)$$

to be solved for the parameters $\theta^{(2)}$. Hence the problem is reduced to a ***linear problem in quadratic data.*** The identification problem boils thus down to a 3 step procedure:

1) Preprocessing of data: this prepares the datamatrix $D^{(2)}$ from consecutive inputs and outputs.
2) Find the right singular vector(s) of $D^{(2)}$, call them $\theta^{(2)}$.
3) Solve for the mode parameters θ_1 and θ_2.

The identification can be combined with a detection step. Assume one obtained at time k the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$. Defining the residuals

$$\epsilon_i = y_k - d_k\hat{\theta}_i \qquad i = 1, 2 \quad (13)$$

the ***data association*** may, proceed as follows:
Decide that at time k mode 1 was on if $|\epsilon_1| \leq |\epsilon_2|$. For future reference, one introduces the extended datavector as the input and output datum, together with the estimated mode vector $\hat{\theta}$. The generalization to M modes should now be clear: The preprocessor constructs data matrices of order M in the original input output data. The components of the right singular vectors are combinations (symmetric functions) of the components of the M mode parameters. The mode parameters associated with each mode are determined (see below). Finally the detection associates the mode vector which minimimes the residual to the data at time k . Some fine tuning of the algorithm is required to deal with particular cases.

One such particular case occurs if all observed data is generated from within the same mode. Also it needs to be resolved how the symmetric equations lead to a unique solution.

2.1 Determination of the mode parameters

Consider again the equivalence for all k in equation (9)

$$(d_k\theta_1)(d_k\theta_2) = \mathrm{Tr}\, d_k' d_k \theta_2\theta_1' = \mathrm{Tr}\, d_k' d_k \theta_1\theta_2' = 0. \quad (14)$$

Thus not only is $\mathrm{vec}\,(\theta_1\theta_2')$ in the nullspace of the second order datamatrix $D^{(2)}$, but also $\mathrm{vec}\,(\theta_2\theta_1')$. Assuming that θ_1 and θ_2 are truly the only modes of the system and that the system satisfies a persistency of excitation condition, the nullspace of $D^{(2)}$ will be two dimensional, of the form

$$\mathcal{N}(D^{(2)}) = \{\alpha\,\mathrm{vec}\,(\theta_1\theta_2') + \beta\,\mathrm{vec}\,(\theta_2\theta_1') \mid \alpha,\beta \in \mathbb{R}\}. \quad (15)$$

Thus let u_1 and u_2 be two mutually orthogonal (normalized) right singular vectors of $D^{(2)}$. Then

$$u_i = \alpha_i\,\mathrm{vec}\,(\theta_1\theta_2') + \beta_i\,\mathrm{vec}\,(\theta_2\theta_1') \quad ; \quad i = 1,2. \quad (16)$$

Given u_1, u_2, how can we reconstruct θ_1 and θ_2? Noting that the components of $\mathrm{vec}\,(\theta_2\theta_1')$ are a specific permutation $\mathcal{P}$ of the components of $\mathrm{vec}\,(\theta_1\theta_2')$, the problem is restated as

Problem 1: Given orthonormal vectors, x and y, determine a vector z and scalars $\alpha_i, \beta_i; i = 1,2$, such that

$$\begin{aligned} x &= (\alpha_1 I + \beta_1\mathcal{P})z \\ y &= (\alpha_2 I + \beta_2\mathcal{P})z \end{aligned} \quad (17)$$

Equivalently, we have

Problem 1': Let $\mathcal{N}(D^{(2)})$ be the (two dimensional) nullspace of $D^{(2)}$. Given orthonormal vectors, u_1 and u_2, such that $\mathrm{Span}\,(u_1,u_2) = \mathcal{N}(D^{(2)})$, determine θ such that

$$\mathrm{Span}\,(u_1,u_2) = \mathrm{Span}\,(\theta, \mathcal{P}\theta) \quad (18)$$

In order to solve this problem, let us first investigate the structure of the nullspace $\mathcal{N}(D^{(2)})$. Introduce "Mat" as the inverse of the vector stacking operator, i.e., for any square matrix X, there holds $\mathrm{Mat}\,\mathrm{vec}\,(X) = X$. By (15), $p = \alpha\,\mathrm{vec}\,(\theta_1\theta_2') + \beta\,\mathrm{vec}\,(\theta_2\theta_1') \in \mathcal{N}(D^{(2)})$ implies that also $q = \alpha\,\mathrm{vec}\,(\theta_2\theta_1') + \beta\,\mathrm{vec}\,(\theta_1\theta_2') \in \mathcal{N}(D^{(2)})$. Note now that $\mathrm{Mat}\,q = (\mathrm{Mat}\,p)'$. Consequently,

$$\mathrm{Mat}\,p + \mathrm{Mat}\,q = (\alpha+\beta)(\theta_1\theta_2' + \theta_2\theta_1'). \quad (19)$$

The matrix on the right of (19) has interesting structure.

We state first a lemma about the eigen problem of a symmetrized rank one matrix of form (19):

Lemma: Let x and y be linearly independent vectors in $\mathbb{R}^n$ and $M_2 = xy' + yx'$, then the nonzero eigenvalues of M_2 are

$$\lambda_\pm = x'y \pm \|x\|\|y\|. \quad (20)$$

A set of associated eigenvectors is

$$z_\pm = \|y\|x \pm \|x\|y \quad (21)$$

Proof: From $(xy'+yx')z = \lambda z$, we get $(y'z)x+(x'z)y = \lambda z$, so that z is a linear combination of the vectors x and y. Thus let

$$z = \alpha x + \beta y.$$

Substituting in the eigen equation, one deduces the system

$$\begin{bmatrix} y'x & y'y \\ x'x & x'y \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \lambda \begin{bmatrix} \alpha \\ \beta \end{bmatrix}.$$

The characteristic equation is

$$\|x\|^2\|y\|^2 - (x'y - \lambda)^2 = 0$$

from which the eigenvalues follow. With $\lambda_+ = x'y + \|x\|\|y\|$ corresponds

$$[x'x\ ,\ -\|x\|\|y\|]\begin{bmatrix} \alpha_+ \\ \beta_+ \end{bmatrix} = 0,$$

or

$$[\|x\|\ ,\ -\|y\|]\begin{bmatrix} \alpha_+ \\ \beta_+ \end{bmatrix} = 0.$$

With λ_+ corresponds then

$$\begin{bmatrix} \alpha_+ \\ \beta_+ \end{bmatrix} = \begin{bmatrix} \|y\| \\ \|x\| \end{bmatrix},$$

determining the corresponding eigenspace.
Likewise, with $\lambda_- = x'y - \|x\|\|y\|$ correspond the scalars

$$\begin{bmatrix} \alpha_- \\ \beta_- \end{bmatrix} = \begin{bmatrix} \|y\| \\ -\|x\| \end{bmatrix},$$

thus proving the lemma. □

Combining this lemma with the observation preceeding it, any singular vector $u \in \mathcal{N}(D^{(2)})$, has the property that $\mathrm{Mat}\,(u) + (\mathrm{Mat}\,(u))'$ is of the form in the lemma. This will be the key in the proof of the following theorem

Theorem: The modal parameters of the multi-mode systemar obtained by the following procedure:

1. Generate the quadratic data matrix $D^{(2)}$ from the input and output measurements.

2. The SVD of $D^{(2)}$ yields the nullspace $\mathcal{N}(D^{(2)})$

$$D^{(2)} = USV'$$

where S has corank (nullity) 2, and contains the singular values on its diagonal, and $V = [V_1, V_2]$, with $\dim \mathrm{Span}\,(V_2) = 2$.

3. Let $V_2 = [v_1, v_2]$. Generate the matrix $T = \mathrm{Mat}\,(v_1) + (\mathrm{Mat}\,(v_1)')$.

4. By the lemma, the eigenvectors corresponding to the nonzero eigenvalues of T are

$$z_1 = \|y\|x - \|x\|y \quad \text{and} \quad z_2 = \|y\|x + \|x\|y.$$

5. The vectors θ_1 and θ_2 are obtained from

$$\theta_1 = \frac{z_1 + z_2}{(z_1 + z_2)_1} \quad \text{and} \quad \theta_2 = \frac{z_2 - z_1}{(z_2 - z_1)_1}.$$

This scaling is done to ensure that the first component of the θ vector i.e., the multiplier of y_k in the ARMA model, is one.

It is possible to put the above algorithm in a more explicit form:

Problem 1": Given u_1 and u_2 orthonormal in $\mathbb{R}^n$. Let

$$u_i = (\alpha_i I + \beta_i \mathcal{P})\mathrm{vec}\,(\theta_1\theta_2'), \tag{22}$$

Determine θ_i, $i = 1, 2$.

Solution: Isolate in equation (1), the y_k terms from the rest. i.e.,

$$(y_k - \delta_k\theta_1)(y_k - \delta_k\theta_2) = 0 \tag{23}$$

with some abuse of notation, we used the same symbol θ to denote the $2n$-dimensional parameter vector $(\theta_1, \cdots \theta_{2n})'$ and denote $\delta_k = [y_{k-1}, \cdots, y_{k-n}; u_{k-1}, \cdots, u_{k-n}]'$. Thus

$$y_k^2 - (\delta_k'\theta_1 + \delta_k'\theta_2)y_k + (\delta_k'\theta_1)(\delta_k'\theta_2) = 0. \tag{24}$$

from which

$$y_k^2 - [\delta_k'(\theta_1 + \theta_2)]y_k + \mathrm{Tr}\,(\delta_k\delta_k')(\theta_2\theta_1') = 0. \tag{25}$$

This is using Kronecker product notation

$$y_k^2 - (y_k \otimes \delta_k)'(\theta_1 + \theta_2) + (\delta_k \otimes \delta_k)'(\theta_1 \otimes \theta_2) = 0. \tag{26}$$

Let a right singular vector of the data matrix be partitioned as $[1, -S, \Pi]$. Thus

$$[y_k^2, y_k\delta_k', \mathrm{vec}\,(\delta_k\delta_k')'] \begin{bmatrix} 1 \\ -S \\ \Pi \end{bmatrix} = 0. \tag{27}$$

or again in Kronecker product form

$$y_k^2 - (y_k \otimes \delta_k)'S + (\delta_k \otimes \delta_k)'P = 0 \tag{28}$$

where $P = \mathrm{Mat}\,\Pi$. Clearly

$$S_i = (\theta_1)_i + (\theta_2)_i \tag{29}$$
$$P_{ii} = (\theta_1)_i(\theta_2)_i. \tag{30}$$

leading to decoupled quadratic equations. The solution set for the i-th pair is

$$\left\{\frac{1}{2}\left[S_i + \sqrt{S_i^2 - 4P_{ii}}\right], \frac{1}{2}\left[S_i - \sqrt{S_i^2 - 4P_{ii}}\right]\right\} \tag{31}$$

but it still needs to be determined which of the two solutions in (31) is associated with mode 1. At this point observe that not all of the available information has yet been used. The ij-th component of P yields

$$(\theta_1)_i(\theta_2)_j + (\theta_2)_i(\theta_1)_j = 2P_{ij} \tag{32}$$

Introducing a signature vector σ, and assigning

$$(\theta_1)_i = \frac{1}{2}\left[S_i + \sigma_i\sqrt{S_i^2 - 4P_{ii}}\right]$$
$$(\theta_2)_i = \frac{1}{2}\left[S_i - \sigma_i\sqrt{S_i^2 - 4P_{ii}}\right] \tag{33}$$

then (33) in (32) gives

$$\sigma_i\sigma_j = \frac{S_iS_j - 4P_{ij}}{\sqrt{S_i^2 - 4P_{ii}}\sqrt{S_j^2 - 4P_{jj}}} \tag{34}$$

This matrix determines (up to a global sign) the signature vector σ and hence the association of the mode parameters. The global sign change simply permutes the two modes.

By computing for all k the residuals one estimates which mode was active at step k. This yields a *derived* (binary) sequence, from which either the deterministic (e.g., period and dutycycle) or stochastic mode switching parameters (e.g., transition matrix) can be estimated. Preliminary ideas were presented in [12].

3 Multi Mode State Space Identification

The main idea is to first reduce the problem to the one that was solved before. One obtains a multi-mode ARMA model, and based on the identified ARMA parameters, one determines with a realization algorithm the corresponding state space models.

To fix the ideas, let the state space relation in mode i be

$$x_{k+1} = A^{(i)}x_k + B^{(i)}u_k$$
$$y_k = c^{(i)}x_k. \tag{35}$$

Now it is assumed that for $k = 0$ the model changes from mode $i = 1$ to mode $i = 2$. If one uses the ARMA

method, a degradation is to be expeceted since strictly speaking, the bimodal state space realization does not lead to a bimodal ARMA relation in general. The I/O model (ARMA) exhibits a 'transition period', necessary to flush out the state. To illustrate what is meant, consider the following example.

Example: Let

$$A^{(1)} = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix} \qquad A^{(2)} = \begin{bmatrix} -1 & \\ & 1 \end{bmatrix},$$

with $b^{(1)} = b^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $c^{(1)} = c^{(2)} = [1 \quad , \quad 1]$. If the initial state at step $k = -3$ is $x_0 = [\xi, \eta]'$, we get the consecutive I/O relations:

$$\begin{array}{rcll} y_{-3} & = & \xi + \eta & \text{Initialization} \\ y_{-2} & = & \xi - \eta & \text{"} \\ y_{-1} & = & y_{-3} + 2u_{-3} & \text{Mode} - 1 \\ y_0 & = & y_{-2} + 2u_{-2} & \text{"} \\ & -- & & \text{MODESWITCHING} \\ y_1 & = & -y_{-3} - 2u_0 & \textit{Transition} \\ y_2 & = & y_0 - 2u_0 & \text{Mode} - 2 \\ y_3 & = & y_1 - 2y_1 & \text{"} \\ & \vdots & & \end{array}$$

Note that here in the one transition step from mode 1 to mode 2, a fixed input-output pattern occurs. This happens for y_1 above. In principle this could be detected as 'Mode 3', Likewise a 4-th mode for the reverse transition may be identified. A transition between two modes for an n-th order system will have a transition period between ARMA modes of $n-1$ steps. With the reverse transition inducing another fixed $n-1$ ARMA relations, a bimodal n-th order state space system would require $2n$ ARMA modes to be identified. This makes the method much more complex. If the average time between mode switches is larger than the system order, we expect that a simple bimodal ARMA identification will still work, albeit somewhat in a degraded way, because the quadratic model will not be satisfied at each step.

Another feature of this state space form is that not every change in the state space parameters leads to a change in the ARMA model. If the second mode $(A^{(2)}, b^{(2)}, c^{(2)})$ is related to the first by *similarity*, the same input-output model results. Detection would still be possible by the mismatch during the transition times, when the state is flushed out as explained above.

4 Examples

Some simple tests were run illustrating the feasibility of the proposed method. First for deterministic periodic modeswitching, input-output data was generated by a second order system switching mode every tenth step (thus the period of the modal behavior is 20). As input signal, white noise was chosen of unit variance. Then both input and output signal were corrupted by additive gaussian noise with RMS σ_u and σ_y. These corrupted signals were fed into the identification algorithm. As performance measure of the algorithm, we choose $J_{100} = \|\theta - \hat{\theta}_{100}\|/\|\theta\|$, where $\theta = [a_1^{(1)}, a_2^{(1)}, b_1^{(1)}, b_2^{(2)}]$ is the vector of ARMA parameters. The estimated parameter vector is the one obtained after the first 100 samples. Two different bi-modal systems were considered:

Example 1: Both modes stable, with transfer function respectively $H_1(z) = \frac{z-0.5}{z^2-z+0.5}$ and $H_2(z) = \frac{z-0.5}{z^2-z+0.75}$. The first mode has complex poles at $1/2 \pm 1/2j$, the second has real poles at $1/4$ and $3/4$. Good parameter tracking behavior was found despite the fact that the parameter vectors of both modes are relatively close. The dependence on input and output noise variance is

σ_u	σ_y	J
0	0	0.020
0.05	0	0.026
0	0.05	0.093
0.05	0.05	0.109
0.1	0.1	0.158

Example 2: Both modes stable, with transfer function respectively $H_1(z) = \frac{z}{z^2-z+0.5}$ and $H_2(z) = \frac{0.5z-0.5}{z^2-0.75z+0.75}$. The first mode has complex poles at $1/2 \pm 1/2j$ and zero at $z = 0$, the second has complex poles at $0.375 \pm 0.78j$. The parameter vectors for the two modes are more separated. This gave better parameter tracking behavior than in the first example. The dependence on input and output noise variance was found to bse

σ_u	σ_y	J
0	0	0.0004
0.1	0	0.0171
0	0.1	0.0816
0.1	0.1	0.1039

With purely random switching, faster convergence for the two modes was observed.

5 Conclusions

The benefits of a more refined knowledge of the structure of a system in estimation, prediction and control are well known. The book [1] gives a nice overview of the problems associated with jump linear systems. What we discussed in this paper is that such a hybrid structure can be identified from input and output data of the system by extending the known principles. The guiding principle is to form an M-fold product of input output data, essentially one (linear) factor per mode.

Alternatively, multi-mode systems may be identified, locally in time, using the above described two-mode identification method in case it is known that each mode turns on for some minimal time. Of course, if more that two modes are thoroughly mixed this will not work. Hence the two-mode identification may be a viable alternative for combined fault-detection / fault estimation problem. Finally, there are many ways in which the algorithm can be fine tuned and adapted to specific situations, following the richnesss of adaptives methods in these simple single mode case.

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