Learning-Based Risk-Averse Model Predictive Control for Adaptive Cruise Control with Stochastic Driver Models

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1 Background and motivation

In recent decades, the usage of adaptive cruise control (ACC) systems has become widespread in the automotive research and industry, as they have demonstrated numerous benefits in terms of safety, fuel efficiency, passenger comfort, etc. The term ACC generally refers to longitudinal control systems that are aimed at maintaining a user-specified velocity, while avoiding collisions with preceding vehicles (see Figure 1).

Due to the inherent uncertainty about the future behavior of the preceding vehicle, stochastic model predictive control (MPC) has been a particularly popular strategy towards this application [1, 3]. Here, the lead vehicle behavior is commonly modelled by a combination of continuous physics-based dynamics with a discrete, stochastic decision model for the driver (e.g., [2, 1]). We follow this line of reasoning and model the preceding vehicle using double integrator dynamics, where the driver's inputs are generated by a Markov chain. The vehicle pair is thus described by a discrete-time Markov jump linear system with dynamics of the form

$$x_{t+1} = A(w_{t+1})x_t + B(w_{t+1})u_t + p(w_{t+1}),$$

where $x_t \in \mathbb{R}^{n_x}$ is the state vector $u_t \in \mathbb{R}^{n_u}$ is the input and $(w_t)_{t\in\mathbb{N}}$ is a Markov chain with transition matrix $P \in \mathbb{R}^{M\times M}$, with elements $P_{i,j} = \mathbb{P}[w_t = j \mid w_{t-1} = i]$.

A major shortcoming of stochastic MPC approaches, however, is their reliance on accurate knowledge of P, as in practice, only a data-driven estimate $\hat{P}$ is available. This results in uncertainty on the probability distributions governing the stochastic process, often referred to as *ambiguity*. Due to this ambiguity, stochastic controllers may perform unreliably with respect to the true distributions.

2 Risk-averse model predictive control formulation

We generalize the stochastic MPC methodology for ACC systems by adopting a distributionally robust approach which accounts for data-driven ambiguity. We define an *ambiguity set* $\mathscr{A}_i$ for each row $\hat{P}_i \in \mathbb{R}^M$ of the estimated transition matrix $\hat{P}$ as

$$\mathscr{A}_i := \left\{ \mu \in \mathbb{R}^M \;\middle|\; \begin{array}{c} \|\mu - \hat{P}_i\|_1 \leq r_i, \\ \sum_{j=1}^M \mu_j = 1, \mu \geq 0 \end{array} \right\}, \qquad (1)$$

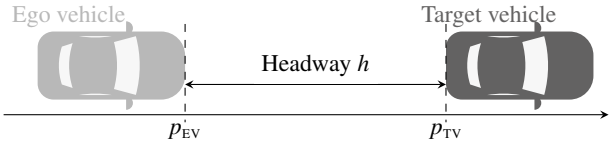


Figure 1: Illustration of the ACC problem.

where r_i is computed such that $\mathbb{P}[P_i \in \mathscr{A}_i] \geq 1-\beta$, with $\beta \in (0,1)$ an arbitrary confidence level [4].

By minimizing the expected cost subject to chance constraints with respect to the worst-case distributions in these ambiguity sets, we obtain a so-called multi-stage risk-averse risk-constrained optimal control problem (OCP). By virtue of the polytopic structure of (1), this can be cast to a convex conic optimization problem [5], which can be efficiently solved. When $\mathscr{A}_i = \{P_i\}, \forall i \in \mathbb{N}_{[1,M]}$, the problem reduces to the original stochastic OCP.

The closed-loop MPC controller is obtained by resolving the risk-averse OCP in every time step, where for all visited modes i, the radii r_i of the ambiguity sets $\mathscr{A}_i$ are decreased over time as more data is gathered. As a result, the obtained controller gradually becomes less conservative during closed-loop operation. Finally, we derive an inner approximation of the maximal robust control invariant set, which we use as a terminal constraint set for all realizations of the stochastic system. This allows us to establish recursive feasibility of the MPC scheme, and thus guarantee safe operation while using observed data to improve performance.

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References

[1] M. Bichi, G. Ripaccioli, S. D. Cairano, D. Bernardini, A. Bemporad, and I. V. Kolmanovsky. Stochastic model predictive control with driver behavior learning for improved powertrain control. In *49th IEEE Conference on Decision and Control (CDC)*, pages 6077–6082, Dec. 2010.

[2] U. Kiencke, R. Majjad, and S. Kramer. Modeling and performance analysis of a hybrid driver model. *Control Engineering Practice*, 7(8):985–991, 1999.

[3] D. Moser, R. Schmied, H. Waschl, and L. d. Re. Flexible Spacing Adaptive Cruise Control Using Stochastic Model Predictive Control. *IEEE Transactions on Control Systems Technology*, 26(1):114–127, Jan. 2018.

[4] M. Schuurmans, P. Sopasakis, and P. Patrinos. Safe learning-based control of stochastic jump linear systems: a distributionally robust approach. *arXiv preprint arXiv:1903.10040*, 2019.

[5] P. Sopasakis, M. Schuurmans, and P. Patrinos. Risk-averse risk-constrained optimal control. In *2019 18th European Control Conference (ECC)*, pages 375–380, June 2019.