Energy-Efficient Double Relay Communication Protocol in Cellular Networks

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***Abstract*—Network densification is a promising solution to increase the spectral efficiency of cellular networks that comes at the cost of a larger energy expenditure. Relays have been proposed as an energy-efficient alternative to network densification. However, deploying relays requires additional planning and infrastructure, which can be costly or even prohibitive for very dense networks. The Double Relay Communication Protocol (DRCP) [1] increases the spectral efficiency by combining spatial diversity and network coding principles. In contrast to other schemes based on relays, DRCP does not require deploying additional relays since it uses base stations as relays. However, its potential for providing an energy-efficient solution has not been explored yet. Therefore, in this paper we propose a power control mechanism that increases the energy efficiency of DRCP, while still achieving a larger spectral efficiency compared to other state-of-the-art approaches.**

***Index Terms*—Energy efficiency, relaying, power control**

I. Introduction

Network densification through the deployment of additional base stations has been identified as a promising solution to increase the spectral efficiency of cellular networks [2]. However, this comes at the cost of a larger energy expenditure. Finding ways to reduce the energy consumption while maintaining a high spectral efficiency is hence one of the main challenges in cellular networks [3].

Relays have been proposed as an energy-efficient alternative to network densification [4]. They work as low-power nodes that receive signals from base stations and forward them to users in different time-slots, hence increasing the spectral efficiency at the cost of increasing the transmission time.

Network coding can be used to further improve the energy efficiency of relay schemes by reducing the transmission time [5], [6]. In its most basic form, network coding uses relays that receive signals from different users and forward a combined signal, hence reducing the number of time-slots required for transmission. However, the additional planning and infrastructure that comes with the relay deployment can be costly or even prohibitive for very dense networks.

In contrast to other schemes based on relays [7], [8], [9], the Double Relay Communication Protocol (DRCP) [1] can increase the spectral efficiency of cellular networks by efficiently combining signals from different users, without requiring additional infrastructure as it uses base stations as relays. It consists of a three-time-slot transmission strategy and a minimum mean square error (MMSE) reception strategy. It exploits spatial diversity and network coding principles by transmitting the original signals in the first and the second time-slots and then forwarding the relayed signals in the third time-slot.

Although the benefits of DRCP in terms of spectral efficiency have been demonstrated, its potential for providing an energy-efficient solution has not been explored yet. Therefore, in this paper we propose a power control mechanism that improves the energy efficiency of DRCP, while achieving a larger spectral efficiency compared to other state-of-the-art approaches.

The rest of the paper is organized as follows. Section II describes the system model. Section III presents the baseline approaches. Section IV presents the energy efficiency optimization of DRCP. Section V shows the performance evaluation. Finally, Section VI draws some conclusions.

II. System Model

Consider a system composed by 2 base stations, each of which has data to be delivered to a specific user. We denote as s_l the symbol from the l-th base station (BSl) to be delivered to the l-th user (Ul). Symbols for different users are assumed to be uncorrelated. It is also assumed that all the base stations and users are half-duplex (i.e. they cannot transmit and receive simultaneously) and that no backhaul link exists between the base stations. This last condition excludes the use of transmission schemes that require coordinated base stations

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or antennas, e.g. space-time block codes such as Alamouti codes. We also assume that each base station can overhear the transmission of the other base station in a reliable way, neglecting possible decoding errors[1].

We assume a standard base station power model consisting of the transmit power and the circuit power [10]. While the transmit power is radiated by the base station for data transmission, the circuit power represents the average energy consumption of electronic components.

Let us define $P_l^{(t)}$ as the transmit power of BSl in time-slot t and P_l^c as the circuit power of BSl. We also define $\gamma_{lm}^{(t)} = \frac{\sigma_{\mathrm{s}}^2}{\sigma_{\mathrm{n}}^2} P_l^{(t)} |h_{lm}|^2$, where h_{lm} is the channel gain of Um from BSl, $\sigma_{\mathrm{s}}^2 = \mathbb{E}\{|s_1|^2\} = \mathbb{E}\{|s_2|^2\}$, and σ_{n}^2 is the received noise power, assumed to be equal for both users. Without loss of generality, we assume $\sigma_{\mathrm{s}}^2 = \sigma_{\mathrm{n}}^2 = 1$.

The energy efficiency of the system is defined as the total amount of data sent by each user with a given amount of energy [10]. The energy efficiency ϵ can be expressed as a function of the spectral efficiency and the power used over T time-slots as [10], [11]

$$\epsilon = \frac{S_1}{T_1 P_1^c + \sum_{t=1}^{T_1} P_1^{(t)}} + \frac{S_2}{T_2 P_2^c + \sum_{t=1}^{T_2} P_2^{(t)}}, \tag{1}$$

where S_l is the spectral efficiency of Ul over T_l time-slots. By considering both the spectral efficiency and the power used over T time-slots, we can establish a fair comparison among schemes that use a different number of time-slots per transmission round.

III. Baseline Approaches

This section presents the analysis of the baseline approaches, which will be used as a benchmark.

A. TDMA

In a basic time division multiple access (TDMA) approach, the communication is done in turns, i.e. first BS1 transmits s_1 to U1 while BS2 is inactive, and then BS2 transmits s_2 to U2 while BS1 is inactive, hence requiring 2 time-slots. We refer to this approach as TDMA.

The energy efficiency of TDMA can be directly computed as

$$\epsilon^{\mathrm{TDMA}} = \frac{S_1^{\mathrm{TDMA}}}{P_1^c + P_1^{(1)}} + \frac{S_2^{\mathrm{TDMA}}}{P_2^c + P_2^{(2)}}, \tag{2}$$

where

$$\begin{aligned} S_1^{\mathrm{TDMA}} &= \log_2\left(1 + \gamma_{11}^{(1)}\right) \\ S_2^{\mathrm{TDMA}} &= \log_2\left(1 + \gamma_{22}^{(2)}\right). \end{aligned} \tag{3}$$

In this paper we assume that the circuit power is much smaller than the transmit power for all of the approaches, i.e. $P_1^c \ll P_1^{(t)}$ and $P_2^c \ll P_2^{(t)}$ $\forall t$. This is typical for cellular networks, which need to compensate for the large path loss of communication over long distances.[2] Under this assumption, a quick inspection of the Hessians $\frac{\partial^2 \epsilon^{\mathrm{TDMA}}}{\partial \left(P_1^{(1)}\right)^2}$ and $\frac{\partial^2 \epsilon^{\mathrm{TDMA}}}{\partial \left(P_2^{(2)}\right)^2}$ shows that equation (2) is strictly decreasing with both $P_1^{(1)}$ and $P_2^{(2)}$. Therefore, the maximization of the energy efficiency corresponds to using the minimum transmit power[3] for both base stations, similar to [10].

B. Diversity

One way of increasing the spatial diversity is to exploit the overhearing capabilities of the system in order to share the transmitted symbols between the base stations. In this way, each symbol can reach its destination following more than one signal path. For instance, BS1 transmits s_1 to both U1 and BS2 in time-slot 1, then BS2 transmits s_2 to both U2 and BS1 in time-slot 2. Then, in time-slot 3 BS1 transmits s_2 and in time-slot 4 BS2 transmits s_1. We refer to this approach as *Diversity* (DIV).

The energy efficiency of DIV (using maximal ratio combining) can be directly computed as

$$\epsilon^{\mathrm{DIV}} = \frac{S_1^{\mathrm{DIV}}}{2P_1^c + P_1^{(1)} + P_1^{(3)}} + \frac{S_2^{\mathrm{DIV}}}{2P_2^c + P_2^{(2)} + P_2^{(4)}}, \tag{4}$$

where

$$\begin{aligned} S_1^{\mathrm{DIV}} &= \log_2\left(1 + \gamma_{11}^{(1)} + \gamma_{21}^{(4)}\right) \\ S_2^{\mathrm{DIV}} &= \log_2\left(1 + \gamma_{22}^{(2)} + \gamma_{12}^{(3)}\right). \end{aligned} \tag{5}$$

Assuming that $P_1^{(1)} = P_1^{(3)}$ and $P_2^{(2)} = P_2^{(4)}$, a quick inspection of the Hessians $\frac{\partial^2 \epsilon^{\mathrm{DIV}}}{\partial \left(P_1^{(1)}\right)^2}$ and $\frac{\partial^2 \epsilon^{\mathrm{DIV}}}{\partial \left(P_2^{(2)}\right)^2}$ shows that equation (4) is strictly decreasing with both $P_1^{(1)}$ and $P_2^{(2)}$. Therefore, assuming that $P_1^c \ll P_1^{(t)}$ for $t = 1, 3$ and $P_2^c \ll P_2^{(t)}$ for $t = 2, 4$, the maximization of the energy efficiency corresponds to using the minimum transmit power for both base stations.

C. Interference

This approach consists in both base stations transmitting simultaneously regardless of the interference that they cause to the other user. We refer to this approach as *Interference* (INTF).

The energy efficiency of this approach can be directly computed as

$$\epsilon^{\mathrm{INTF}} = \frac{S_1^{\mathrm{INTF}}}{P_1^c + P_1^{(1)}} + \frac{S_2^{\mathrm{INTF}}}{P_2^c + P_2^{(1)}}, \tag{6}$$

[1]These errors are less relevant than the errors that occur at the users' side since base stations are usually equipped with more powerful receivers (i.e. with greater sensitivity and smaller noise figure) and they often count with line-of-sight (LOS) between them.

[2]When the circuit power dominates the transmit power, maximizing the energy efficiency of TDMA is equivalent to maximizing the sum spectral efficiency, which corresponds to a binary power control where both users transmit with full power or only one with full power and the other with minimum power [12].

[3]Minimum transmit power refers in this paper to the transmit power, larger than the circuit power, sufficient to transmit the minimum amount of data.

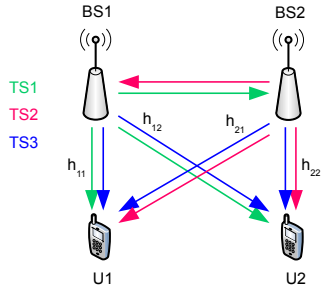

Fig. 1. System model: the parameter h_{lm} defines the channel gain of Um from BSl and TSt defines the t-th time-slot.

where

$$S_1^{\mathrm{INTF}} = \log_2\left(1+\frac{\gamma_{11}^{(1)}}{1+\gamma_{21}^{(1)}}\right)$$
$$S_2^{\mathrm{INTF}} = \log_2\left(1+\frac{\gamma_{22}^{(1)}}{1+\gamma_{12}^{(1)}}\right). \quad (7)$$

Since equation (6) is strictly decreasing with both $P_1^{(1)}$ and $P_2^{(1)}$, the maximization of the energy efficiency corresponds to using the minimum transmit power for both base stations as in [10].

D. Double Relay Communication Protocol (DRCP)

In [1], DRCP was shown to increase the spectral efficiency by finding a balance between spatial diversity and transmission time. It is described as follows.

In the first time-slot, BS1 transmits s_1 to U1, U2, and BS2. In the second time-slot, BS2 transmits s_2 to U1, U2, and BS1. In the third time-slot, each base station acts as relay to transmit the received symbol (s_2 for BS1 and s_1 for BS2) to U1 and U2. This is shown in Fig. 1.

Assuming a channel coherence time larger than 3 time-slots, the received signals for U1 in the three time-slots can be expressed as:

$$y_{\mathrm{U1}}^{(1)} = \sqrt{P_1^{(1)}}h_{11}s_1 + n_{\mathrm{U1}}^{(1)}$$
$$y_{\mathrm{U1}}^{(2)} = \sqrt{P_2^{(2)}}h_{21}s_2 + n_{\mathrm{U1}}^{(2)} \quad (8)$$
$$y_{\mathrm{U1}}^{(3)} = \sqrt{P_1^{(3)}}h_{11}s_2 + \sqrt{P_2^{(3)}}h_{21}s_1 + n_{\mathrm{U1}}^{(3)},$$

where $y_{\mathrm{U1}}^{(t)}$ is the received signal for U1 in time-slot t, and $n_{\mathrm{U1}}^{(t)}$ is the received noise for U1 in time-slot t. A similar set of equations describes the received signals for U2.

Using the results from [1], we can calculate the signal-to-noise ratio (SNR) of U1 and U2 as

$$\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{DRCP}} = \gamma_{11}^{(1)} + \gamma_{21}^{(3)}\left(\frac{1+\gamma_{21}^{(2)}}{1+\gamma_{11}^{(3)}+\gamma_{21}^{(2)}}\right)$$
$$\mathrm{SNR}_{\mathrm{U2}}^{\mathrm{DRCP}} = \gamma_{22}^{(2)} + \gamma_{12}^{(3)}\left(\frac{1+\gamma_{12}^{(1)}}{1+\gamma_{22}^{(3)}+\gamma_{12}^{(1)}}\right). \quad (9)$$

The energy efficiency can then be expressed as

$$\epsilon^{\mathrm{DRCP}} = \frac{S_1^{\mathrm{DRCP}}}{2P_1^c + P_1^{(1)} + P_1^{(3)}} + \frac{S_2^{\mathrm{DRCP}}}{2P_2^c + P_2^{(2)} + P_2^{(3)}}, \quad (10)$$

where

$$S_1^{\mathrm{DRCP}} = \log_2\left(1+\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{DRCP}}\right)$$
$$S_2^{\mathrm{DRCP}} = \log_2\left(1+\mathrm{SNR}_{\mathrm{U2}}^{\mathrm{DRCP}}\right). \quad (11)$$

The maximization of the energy efficiency of DRCP will be analyzed in the next section.

IV. Energy Efficiency of DRCP

Finding the optimal solution for $P_1^{(t)}$ and $P_2^{(t)}$ $\forall t$ that maximizes the energy efficiency of DRCP remains an analytically intractable problem since it requires the optimization over different time-slots and users. However, it is well known that in the case where each base station transmits separately without interference from the other base station, the optimal solution consists in using the minimum transmit power for both base stations [10]. Therefore, in this paper we assume for DRCP that we use the minimum transmit power for $P_1^{(1)}$ and $P_2^{(2)}$ and we optimize only for $P_1^{(3)}$ and $P_2^{(3)}$ over the interval between the minimum and maximum transmit power, i.e. $[P^{\min}, P^{\max}]$. We will later compare our solution with the global optimum that considers the optimization over all the time-slots.

Let us start by assuming that we have a fixed $P_2^{(3)}$. This can be the case where only one base station can vary its transmit power. The optimal solution for the energy efficiency of equation (10) can then be obtained by setting the derivative $\frac{\partial \epsilon^{\mathrm{DRCP}}}{\partial P_1^{(3)}}$ equal to zero and using the Lambert function $W(\cdot)$ as shown in the appendix. The optimal power value for $P_1^{(3)}$ can be found by solving the following equation:

$$\alpha = W\left(\frac{\alpha \exp\left[\beta(P_1^{(1)}+P_1^{(3)})^2\right]}{1+\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{DRCP}}}\right), \quad (12)$$

where

$$\alpha = \frac{|h_{11}|^2\gamma_{21}^{(3)}(1+\gamma_{21}^{(2)})(P_1^{(1)}+P_1^{(3)})}{(1+\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{DRCP}})\left(1+\gamma_{11}^{(3)}+\gamma_{21}^{(2)}\right)^2} \quad (13)$$

and

$$\beta = \frac{|h_{12}|^2(1+\gamma_{12}^{(1)})}{(P_2^{(2)}+P_2^{(3)})(1+\mathrm{SNR}_{\mathrm{U2}}^{\mathrm{DRCP}})(1+\gamma_{22}^{(3)}+\gamma_{12}^{(1)})} \quad (14)$$

and $\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{DRCP}}$ and $\mathrm{SNR}_{\mathrm{U2}}^{\mathrm{DRCP}}$ are defined in equation (9).

Analogously, for a fixed $P_1^{(3)}$, the maximization of the energy efficiency is achieved by the power value $P_2^{(3)}$ that solves the following equation:

$$\delta = W\left(\frac{\delta \exp\left[\eta(P_2^{(2)}+P_2^{(3)})^2\right]}{1+\mathrm{SNR}_{\mathrm{U2}}^{\mathrm{DRCP}}}\right), \quad (15)$$

where

$$\delta = \frac{|h_{22}|^2 \gamma_{12}^{(3)} (1+\gamma_{12}^{(1)})(P_2^{(2)}+P_2^{(3)})}{(1+\mathrm{SNR}_{\mathrm{U2}}^{\mathrm{DRCP}})\left(1+\gamma_{22}^{(3)}+\gamma_{12}^{(1)}\right)^2} \quad (16)$$

and

$$\eta = \frac{|h_{21}|^2(1+\gamma_{21}^{(2)})}{(P_1^{(1)}+P_1^{(3)})(1+\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{DRCP}})(1+\gamma_{11}^{(3)}+\gamma_{21}^{(2)})}. \quad (17)$$

Note that equations (12) and (15) do not always have a solution within the interval between the minimum and maximum transmit powers. Also, the solution can represent a minimum instead of a maximum. In these cases the optimal power control is binary (using minimum or maximum transmit power). This can be summarized with the following lemma.

Lemma 1: The optimal $P_1^{(3)}$ that maximizes the energy efficiency of DRCP with fixed values of $P_1^{(1)}$, $P_2^{(2)}$, and $P_2^{(3)}$ is found among the values $P_1^{(3)} = P^{\min}$, $P_1^{(3)} = P^{\max}$, or $P_1^{(3)}$ that solves equation (12). Similarly, the optimal $P_2^{(3)}$ that maximizes the energy efficiency of DRCP with fixed values of $P_1^{(1)}$, $P_2^{(2)}$, and $P_1^{(3)}$ is found among the values $P_2^{(3)} = P^{\min}$, $P_2^{(3)} = P^{\max}$, or $P_2^{(3)}$ that solves equation (15).

Proof. By examining the Hessian of equation (10) $\frac{\partial^2 \epsilon^{\mathrm{DRCP}}}{\partial \left(P_1^{(3)}\right)^2}$, we can see that ϵ^{DRCP} has only one maximum or one minimum. If the Hessian is negative semidefinite, the maximum is found according to equation (12) if it is found in the interval $[P^{\min}, P^{\max}]$. Else, the maximum is $P_1^{(3)} = P^{\min}$ or $P_1^{(3)} = P^{\max}$. An analogous situation holds for $P_2^{(3)}$. □

V. Performance Evaluation

In this section we compare the approaches analyzed in previous sections in terms of energy efficiency and total spectral efficiency per time-slot (i.e. the sum of the spectral efficiencies of both users divided by the number of time-slots per transmission round). Our evaluations consider a Rayleigh fading channel model and $\mathbb{E}\{|h_{11}|^2\} = \mathbb{E}\{|h_{21}|^2\} = \mathbb{E}\{|h_{22}|^2\} = \mathbb{E}\{|h_{12}|^2\} = 1$, $\sigma_{\mathrm{s}}^2 = \sigma_{\mathrm{n}}^2 = 1$, $P_1^c = P_2^c = -10$dBW, and $P^{\min} = 0$dBW.

For illustration purposes, we consider that BS2 has a maximum transmit power of 10dBW and we will only vary the maximum transmit power of BS1. Two cases were analyzed: the first one (maximum spectral efficiency) considers that all the approaches use maximum transmit power, i.e. $P_1^{(t)} = P^{\max}$ $\forall t$ and $P_2^{(t)} = 10$dBW $\forall t$. The second case considers that all the approaches use the transmit powers that maximize their energy efficiency. This means that TDMA, DIV, and INTF transmit with $P_1^{(t)} = P_2^{(t)} = P^{\min} = 0$dBW $\forall t$, and DRCP transmits with $P_1^{(1)} = P_2^{(2)} = 0$dBW, $P_2^{(3)} = 10$dBW, and $P_1^{(3)}$ according to Lemma 1.

When using maximum transmit power, the studied approaches show a similar energy efficiency. This can be seen in Fig. 2, where DRCP and TDMA show a slightly larger energy efficiency than the other the approaches. However, the advantage of transmitting with maximum power is that all of the approaches can achieve a high spectral efficiency (see Fig. 3).

When using the transmit powers that optimize the energy efficiency, all of the approaches can achieve a larger energy efficiency than when using maximum transmit power (see Fig. 4). In the case of DRCP this happens when using minimum transmit power in the first two time-slots and the transmit power defined by Lemma 1 in the third time-slot. For the rest of the approaches this happens when using the minimum transmit power in all the time-slots. Nevertheless, DRCP has the highest energy efficiency of all. In fact, the proposed solution for DRCP is close to the global optimum, which is obtained through an exhaustive search over all the possible transmit powers in *all* the time-slots. As expected, this results in a lower spectral efficiency of all the approaches compared to the previous case (see Fig. 5). Still, it is remarkable to observe that in this case DRCP is able to achieve both the largest energy efficiency and the largest spectral efficiency.

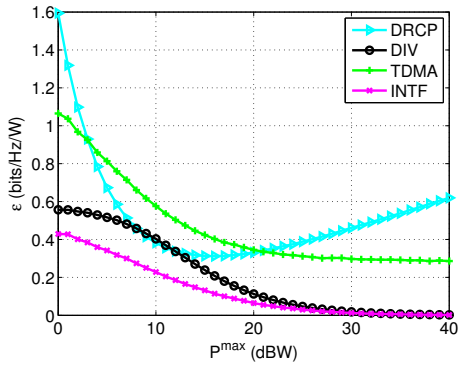

Fig. 2. Energy efficiency of all the studied approaches with maximum transmit power, i.e. $P_1^{(t)} = P^{\max}$ $\forall t$ and $P_2^{(t)} = 10$dBW $\forall t$.

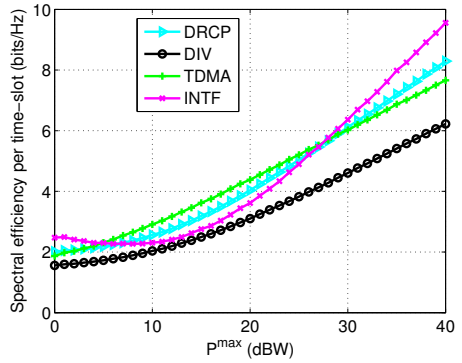

Fig. 3. Total spectral efficiency per time-slot of all the studied approaches with maximum transmit power, i.e. $P_1^{(t)} = P^{\max}$ $\forall t$ and $P_2^{(t)} = 10$dBW $\forall t$.

VI. Conclusions

In this paper, we have proposed the use of power control to optimize the energy efficiency of DRCP. Although finding the optimal solution for variable transmit powers in all the time-slots remains analytically intractable, we provide the optimal

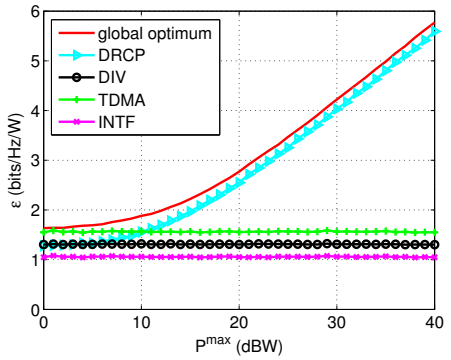

Fig. 4. Energy efficiency of all the studied approaches with maximum energy efficiency. TDMA, DIV, and INTF transmit with $P_1^{(t)} = P_2^{(t)} = 0$dBW $\forall t$ (since using minimum power maximizes the energy efficiency). DRCP transmits with $P_1^{(1)} = P_2^{(2)} = 0$dBW and $P_2^{(3)} = 10$dBW, while $P_1^{(3)}$ is set according to Lemma 1.

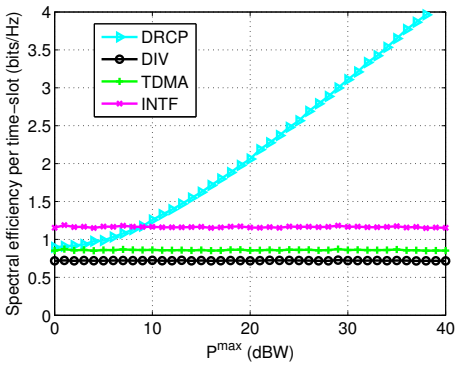

Fig. 5. Total spectral efficiency per time-slot of all the studied approaches with maximum energy efficiency. TDMA, DIV, and INTF transmit with $P_1^{(t)} = P_2^{(t)} = 0$dBW $\forall t$ (since using minimum power maximizes the energy efficiency). DRCP transmits with $P_1^{(1)} = P_2^{(2)} = 0$dBW and $P_2^{(3)} = 10$dBW, while $P_1^{(3)}$ is set according to Lemma 1.

solution for the first two time-slots and for the case of one variable transmit power in the third time-slot. This corresponds to using minimum transmit power in the first two time-slots and in the third time-slot it corresponds to using maximum or minimum transmit power or the power value defined by Lemma 1. Using this strategy, DRCP is able to achieve both the largest energy efficiency and the largest spectral efficiency compared to the studied approaches. Furthermore, our results show our solution is close to the global optimum that considers variable transmit powers in all the time-slots.

APPENDIX

The maximization of energy efficiency of DRCP when $P_1^{(1)}$ and $P_2^{(2)}$ are fixed can be expressed as the following optimization problem:

$$\begin{aligned} \underset{P_1^{(3)},P_2^{(3)}}{\text{maximize}} \quad & \epsilon^{\text{DRCP}} = \frac{\log_2(1+\text{SNR}_{\text{U1}}^{\text{DRCP}})}{2P_1^c+P_1^{(1)}+P_1^{(3)}} + \frac{\log_2(1+\text{SNR}_{\text{U2}}^{\text{DRCP}})}{2P_2^c+P_2^{(2)}+P_2^{(3)}} \\ \text{s.t.} \quad & P^{\min} \leq P_1^{(3)} \leq P^{\max} \\ & P^{\min} \leq P_2^{(3)} \leq P^{\max}, \end{aligned} \tag{18}$$

where $\text{SNR}_{\text{U1}}^{\text{DRCP}}$ and $\text{SNR}_{\text{U2}}^{\text{DRCP}}$ are given in equation (9). We assume that $P_1^c \ll P_1^{(t)}$ and $P_2^c \ll P_2^{(t)}$,

For a given $P_2^{(3)}$, derivating with respect to $P_1^{(3)}$ we obtain

$$\frac{\partial \epsilon^{\text{DRCP}}}{\partial P_1^{(3)}} = \frac{\alpha + \ln(1+\text{SNR}_{\text{U1}}^{\text{DRCP}})}{(P_1^{(1)}+P_1^{(3)})^2} - \beta = 0 \tag{19}$$

where

$$\alpha = \frac{|h_{11}|^2 \gamma_{21}^{(3)}(1+\gamma_{21}^{(2)})(P_1^{(1)}+P_1^{(3)})}{(1+\text{SNR}_{\text{U1}}^{\text{DRCP}})\left(1+\gamma_{11}^{(3)}+\gamma_{21}^{(2)}\right)^2} \tag{20}$$

and

$$\beta = \frac{|h_{12}|^2(1+\gamma_{12}^{(1)})}{(P_2^{(2)}+P_2^{(3)})(1+\text{SNR}_{\text{U2}}^{\text{DRCP}})(1+\gamma_{22}^{(3)}+\gamma_{12}^{(1)})}. \tag{21}$$

This results in

$$(1+\text{SINR}_{\text{U1}}^{\text{DRCP}})\exp[\alpha] = \exp\left[\beta(P_1^{(1)}+P_1^{(3)})^2\right]. \tag{22}$$

Finally, using the Lambert function $W(\cdot)$, we can express equation (22) as

$$\alpha = W\left(\frac{\alpha \exp\left[\beta(P_1^{(1)}+P_1^{(3)})^2\right]}{1+\text{SNR}_{\text{U1}}^{\text{DRCP}}}\right). \tag{23}$$

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