Neighbor-Friendly Autonomous Algorithm for Power Spectrum Allocation in OFDM Networks

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***Abstract*—To cope with the dramatic increase in mobile data traffic, the widespread deployment of base stations constitutes a promising solution. However, it also causes high levels of interference, especially at the cell-edges. Most interference management techniques assume coordination between base stations, which involves undesired overhead and delays. To tackle this problem, we propose a neighbor-friendly autonomous algorithm for power spectrum allocation in wireless OFDM networks that protects victim users from neighboring cells through a penalty factor in the power allocation level. We refer to it as the neighbor-friendly iterative waterfilling (NF-IWF) algorithm. In high interference scenarios, it can achieve a victim user data rate increase by a factor of 3.5 compared to IWF and 60 compared to equal power allocation with a marginal decrease of the primary user data rate.**

***Index Terms*—Interference management, autonomous power allocation, victim users**

I. Introduction

While physical layer techniques have shown a low potential to deal with the dramatic increase in mobile data traffic, the deployment of more base stations seems a promising solution. However, the addition of new cells creates serious interference problems in both uplink and downlink transmissions, especially for users located at the cell-edge. In this paper we focus on the downlink transmission.

To overcome these problems, it is crucial for base station manufacturers to implement inter-cell interference coordination (ICIC) techniques. Several ICIC solutions have been proposed in literature (see [1], [2], [3], [4], [5]). The common idea of most of them is to allocate separate resources to neighboring cells, i.e. they exploit resource orthogonalization. This is usually done in the frequency or in the time domain.

In addition, the resource orthogonalization is usually static, while channel conditions, the number of users attached to a cell, the user locations, and interference conditions are always dynamic. For an improved performance, the network can adapt to these varying conditions by sharing information between base stations. The main drawback is, however, that this requires a dedicated backhaul link to exchange this information and hence may result in large delays.

An alternative approach is that each base station optimizes its own power spectrum allocation without any run-time information exchange between base stations. We refer to this as an autonomous algorithm.

Autonomous power spectrum allocation algorithms have been studied in the context of multi-tone digital subscriber line (DSL) networks. A low-complexity set of autonomous power spectrum balancing algorithms (ASB and ASB-2) for DSL has been proposed in [6] and [7], which allows a non-orthogonalized sharing of resources. The concept of a protected reference line (or user) is used in these papers as a statistical average of all victim lines suffering interference. However, the implementation of these algorithms imposes a challenge for wireless networks given the multi-user scheduling and the non-stationatity of the wireless channel.

A first attempt was made in [8] to apply the concept

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of a protected reference user to a wireless orthogonal frequency division multiplexing (OFDM) network by selecting the reference user as the user suffering the strongest interference from the neighboring cells. However, this scheme is not autonomous since it needs periodical information exchange between base stations to adapt to the time-varying channel conditions of the reference user.

To tackle these problems, we propose a neighbor-friendly autonomous algorithm for power spectrum allocation that protects victim users within a certain distance from the base station through a penalty factor in the power allocation level. The protection level can be tuned with the weight given to the user. We refer to it as the neighbor-friendly iterative waterfilling (NF-IWF) algorithm. We also propose a low-complexity closed-form version that fixes the penalty factor by assuming an average bit rate for the victim user.

The paper is organized as follows. Section II describes the state of the art in autonomous power spectrum allocation algorithms. Section III presents the proposed approach. Section IV shows the performance evaluation of the proposed approach. Finally section V draws some conclusions.

II. Autonomous Power Spectrum Allocation

Autonomous algorithms do not rely on information exchange between base stations, but only exploit locally-available (and a-priori known) information about the environment such as direct channel gains, received interference, and noise power.

A well-known autonomous power spectrum allocation algorithm is IWF [9], which corresponds to maximizing the data rate in each cell:

$$\begin{aligned} \underset{s_k^c\ \forall k}{\text{maximize}} \quad & R^c \\ \text{s.t.} \quad & \sum_{k\in\mathcal{K}} s_k^c \le P^{c,\text{tot}} \\ & 0 \le s_k^c \le s_k^{c,\text{mask}}\ \forall k \in \mathcal{K} \end{aligned} \tag{1}$$

with

$$\begin{aligned} R^c &= f_s \sum_{k\in\mathcal{K}} b_k^c \\ &= f_s \sum_{k\in\mathcal{K}} \log_2\left(1+\frac{1}{\Gamma}\frac{|h_k^c|^2 s_k^c}{\sum_{\substack{\bar{c}\neq c\\ \bar{c}\in\mathcal{C}}} |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \sigma_k^c}\right) \end{aligned} \tag{2}$$

where R^c is the data rate in cell c, f_s is the symbol rate, b_k^c, h_k^c, σ_k^c, s_k^c, and $s_k^{c,mask}$ are the bit loading for a standard interference channel model, the channel transfer function, the noise power, the base station transmit power, and the spectral emission mask constraints on subcarrier k in cell c, respectively. $h_k^{\bar{c}}$ and $s_k^{\bar{c}}$ are the channel transfer function and transmit power on subcarrier k from the interfering cell $\bar{c}$, which are both assumed to be known as they affect users in cell c. We call h_k^c the direct channel and $h_k^{\bar{c}}$ the interference channel of the users attached to cell c. $\mathcal{C}$ and $\mathcal{K}$ are the set of available cells and subcarriers, respectively, and $P^{c,tot}$ is the total power budget in cell c. A given subcarrier can only be allocated to one user in each cell, but it can also be allocated to (or shared by) users attached to neighboring cells resulting in inter-cell interference. The allocation of subcarriers to users can be done prior to the power allocation strategies described in this paper based, for example, on instantaneous channel conditions. However, our focus is only on the power allocation. Γ denotes the signal-to-noise ratio (SNR) gap to capacity, which depends on the desired bit error rate (BER), the coding gain, and the noise margin. We will assume it to be equal to 1 without loss of generality.

Using the corresponding Karush-Kuhn-Tucker (KKT) conditions, it can be shown that the transmit powers have a closed-form solution as follows

$$s_k^c = \left[\frac{f_s}{\log(2)\lambda_c} - \frac{\sum_{\bar{c}\neq c}\Gamma|h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \Gamma\sigma_k^c}{|h_k^c|^2}\right]_0^{s_k^{c,\text{mask}}} \tag{3}$$

where $[x]_a^b = \max(a, \min(x, b))$ and λ_c is the Lagrange multiplier that has to be adjusted (e.g. with bisection) to satisfy the total power constraint $P^{c,tot}$.

The advantage of IWF is its simplicity, its closed-form solution, and the fact that it does not need coordination between cells. However, each cell maximizes its own data rate in a greedy fashion by allocating power to those subcarriers with the best channel-to-interference-and-noise ratio (CINR), without considering the interference caused to victim users from neighboring cells.

III. Neighbor-Friendly Autonomous Power Spectrum Allocation

A. Neighbor-Friendly IWF

Our goal is therefore to design a neighbor-friendly algorithm that, without any information exchange in the network, limits this damage. Following the idea of a protected reference user, we formulate the optimization problem as the maximization of the weighted sum of the data rate of users attached to cell c, or primary users, denoted as R^c and the data rate of victim users attached to neighboring cells, denoted as R^{vc}. Again, the subcarrier allocation is assumed to be done prior to the power allocation, therefore R^c refers to all the users in cell c and R^{vc} refers to all the users interfered

by cell c to be protected.

$$\begin{aligned} \underset{s_k^c\ \forall k}{\text{maximize}} \quad & w^c R^c + w^{vc} R^{vc} \\ \text{s.t.} \quad & \sum_k s_k^c \le P^{c,\text{tot}} \\ & 0 \le s_k^c \le s_k^{c,\text{mask}}\ \forall k \in \mathcal{K} \end{aligned} \tag{4}$$

with R^c defined in equation (2) and

$$\begin{aligned} R^{vc} &= f_s \sum_{k\in\mathcal{K}} b_k^{vc} \\ &= f_s \sum_{k\in\mathcal{K}} \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{vc}|^2 s_k^{vc}}{|h_k^{vc,c}|^2 s_k^c + \sigma_k^{vc}}\right) \end{aligned} \tag{5}$$

where b_k^{vc}, h_k^{vc}, s_k^{vc}, and σ_k^{vc} are the bit loading, the direct channel, the transmit power, and the noise power on subcarrier k of the victim users, respectively, and $h_k^{vc,c}$ is the interference channel on subcarrier k from cell c to victim users. w^c and w^{vc} are the weights of the primary users and the victim users, repectively. We assume w^c equal for all primary users and w^{vc} equal for all victim users. We consider that $w^c = 1 - w^{vc}$, which represents a tradeoff between protecting the victim users from neighboring cells at the cost of degrading the data rate of the primary users. In practice, these weights can be chosen based on, for example, quality of service requirements.

Applying the KKT stationarity condition to problem (4) leads to

$$\begin{aligned} \forall k: \frac{\frac{1}{\log(2)} w^c f_s |h_k^c|^2}{\left(|h_k^c|^2 s_k^c + \sum_{\bar{c}\neq c} \Gamma |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \Gamma\sigma_k^c\right)} \\ - \frac{\frac{1}{\log(2)} w^{vc} f_s |h_k^{vc}|^2 s_k^{vc} |h_k^{vc,c}|^2}{\left(\Gamma|h_k^{vc,c}|^2 s_k^c + \Gamma\sigma_k^{vc}\right)\left(|h_k^{vc}|^2 s_k^{vc} + \Gamma|h_k^{vc,c}|^2 s_k^c + \Gamma\sigma_k^{vc}\right)} \\ - \lambda_c = 0 \end{aligned} \tag{6}$$

By taking into account the KKT complementarity conditions of (4), s_k^c from the first term of equation (6) can be isolated:

$$s_k^c = \left[\frac{\frac{w^c f_s}{\log(2)}}{\lambda_c + P_k^{VC,c}} - \frac{\sum_{\bar{c}\neq c} \Gamma |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \Gamma\sigma_k^c}{|h_k^c|^2}\right]_0^{s_k^{c,\text{mask}}} \tag{7}$$

where $P_k^{VC,c}$ is called the penalty factor, defined as

$$P_k^{VC,c} = \frac{\frac{1}{\log(2)} w^{vc} f_s |h_k^{vc}|^2 s_k^{vc} \Gamma |h_k^{vc,c}|^2}{\left(\Gamma|h_k^{vc,c}|^2 s_k^c + \Gamma\sigma_k^{vc}\right)\left(|h_k^{vc}|^2 s_k^{vc} + \Gamma|h_k^{vc,c}|^2 s_k^c + \Gamma\sigma_k^{vc}\right)} \tag{8}$$

resulting in a fixed point equation as $P_k^{VC,c}$ depends on s_k^c. Note that the first term in equation (7) corresponds to a power level with per-subcarrier offset $P_k^{VC,c}$, which reduces the interference to victim users from neighboring cells. By setting $P_k^{VC,c}$ to zero, equation (7) is reduced to the IWF solution (3). Problem (4) is a nonconvex function for which the duality gap between the primal and dual formulation goes to zero as the number of subcarriers increases [10]. By adding to equation (7) a bisection search on the Lagrange multiplier to satisfy the total cell power constraint, we obtain Algorithm (1), which we refer to as the neighbor-friendly IWF (NF-IWF). δ indicates the accuracy of the total power constraint, γ indicates the stopping criterion of the bisection search on λ_c in the case of an inactive total power constraint, and $\Lambda^{\max}$ is the maximum value for λ_c. The transmit powers of the neighboring cells (s_k^{vc}) are assumed as an equal power allocation (EPA) without performance degradation as observed in later sections.

Algorithm 1 NF-IWF

For each cell c:
Initialize w^c and w^{vc} according to the protection level assigned to users
Initialize h^{vc} according to the victim user path loss of section III-B
Initialize $s_k^c = 0$ and s_k^{vc} =EPA
repeat
  $\lambda_c^{\min} = 0$; $\lambda_c^{\max} = \Lambda^{\max}$
  $\lambda_c = (\lambda_c^{\max} + \lambda_c^{\min})/2$
  while $|\sum_k s_k^c - P^{c,\text{tot}}| > \delta$ and $\lambda_c > \gamma$ **do**
    $\lambda_c = (\lambda_c^{\max} + \lambda_c^{\min})/2$
    for $k = 1 : \mathcal{K}$ **do**
      repeat
        Update s_k^c in (7)
      until convergence
    end for
    if $\sum_k s_k^c > P^{c,\text{tot}}$ **then**
      $\lambda_c^{\min} = \lambda_c$
    else
      $\lambda_c^{\max} = \lambda_c$
    end if
  end while
until network convergence

Contrary to most ICIC techniques that orthogonalize resources, NF-IWF allows sharing of subcarriers between users of different cells as long as adequate power levels are used. This can be observed in Figure 1. Interestingly, NF-IWF tends to allocate transmit power to those subcarriers less used for transmission by the interfering base station, which uses simple IWF. Still, some subcarriers are shared by both base stations.

B. Estimation of the victim users channel

In a wireless network, $h_k^{vc,c}$ can be obtained from the users channel feedback when scanning the neigh-

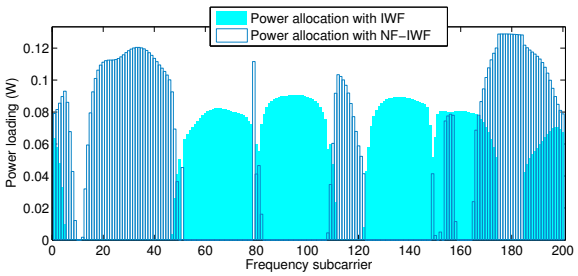
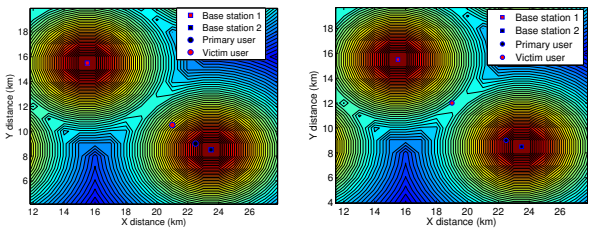


Fig. 1. Transmit power allocation of 2 neighboring base stations, one uses NF-IWF and the interfering base station uses IWF.



(a) Case 1: high interference. (b) Case 2: low interference.

Fig. 2. Network with 2 base stations, 1 primary user and 1 victim user.

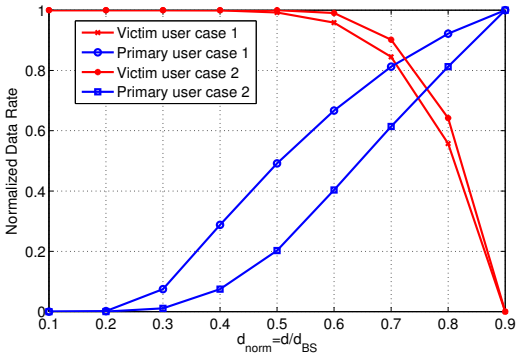


Fig. 3. Normalized rates for different values of path loss distance d_{norm} when applying NF-IWF with $w^c = w^{vc} = 0.5$.

boring cell for a handover [11], [12]. However, the direct channel of the victim users h_k^{vc} in equation (8) can only be known from the information received from other base stations. Therefore we propose a novel approach to estimate h_k^{vc} based on the distance from the base station to the cell-edge.

Full knowledge of h_k^{vc} is unfeasible in an autonomous approach. However, a user's path loss, i.e. the average channel gain over all the subcarriers, is easier to obtain and only dependent on the distance from the user to the base station. Since the signal strength coming from 2 neighboring base stations can be considered equal at the cell-edge (this is how the cell-edge is typically defined), the path loss from each base station to the cell-edge can be known. This can be exploited to approximate the direct channel of the victim users from neighboring cells (h_k^{vc}) by the path loss from the base station to the cell-edge (defined by the radial distance d). This results in a constant value along all the subcarriers, i.e. $h_k^{vc} = \tilde{h}^{vc}\ \forall k \in \mathcal{K}$ where $\tilde{h}^{vc} = \frac{1}{\mathcal{K}} \sum_{k=1}^{\mathcal{K}} h_k^{vc}$. Even though an irregular propagation channel (i.e. with shadowing) might affect each victim user differently, we will see later that $\tilde{h}^{vc}$ is a good approximation for the direct channel of all potential victim users if $h_k^{vc,c}$ of each user is known.

To estimate d in practice, we can use either the information on the channel feedback of users entering the cell after a handover procedure or the cell size predefined by the manufacturer. This information does not need to be updated regularly (since the cell-edge is only modified when a new base station is deployed in the neighborhood).

Despite its simplicity, this model provides an accurate estimation of the victim users direct channel. To analyze the sensitivity of this model, we consider a high interference case (where the victim user is within the coverage of the neighboring cell) and a low interference case (where the victim user is at the cell-edge) as shown in Figures 2a and 2b. We assume that there is no handover like in a closed-access base station. The color regions indicate the signal strength in the direct channel to the closest base station and the user color indicates the base station to which the user is attached.

We then vary the path loss distance d of $\tilde{h}^{vc}$ between 0 and the actual distance between base stations and we compute the normalized data rates of both cases when applying NF-IWF, resulting in Figure 3. The value of d is normalized such that $d_{norm} = 0$ means a radius of zero and $d_{norm} = 1$ means a radius equal to the distance between both base stations (d_{BS}). The smaller the path loss distance, the more protection to the victim user at the cost of the primary user data rate. At $d_{norm} = 0.5$, i.e. between the two base stations, we have the highest normalized data rate of the victim user for a primary user data rate of 20% in case 1 and 50% in case 2. This is because in case 1 the base station needs to significantly lower its transmit power when protecting a more vulnerable user located within its coverage. We assume from here on a victim user path loss at a distance $d_{norm} = 0.5$.

C. Low-complexity version of NF-IWF

Additionally, we propose a low complexity version of NF-IWF by fixing s_k^c in equation (8) to a constant value $\tilde{s}^c$ for all subcarriers. The intuition behind this approach is to protect the victim users through the average bit loading value $\tilde{b}^{vc}$. From equation (5) we

Parameter	Value
System Bandwidth	5 MHz
Number of data subcarriers	200
Γ	1
δ	10^{-6}
γ	10^{-6}
Λ^{max}	10^{8}
f_s	2.8 Gsymbols/s
Channel profile	SCM suburban macro [13]
Base station total transmit power	43 dBm

TABLE I
SIMULATION PARAMETERS.

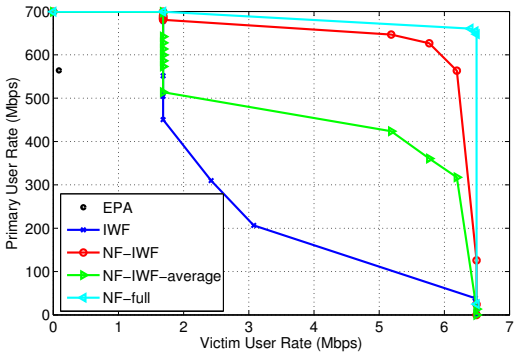


Fig. 4. Rate region of the center and victim users for the high interference scenario of Figure 2a.

have:

$$\tilde{b}^{vc} = \log_2\left(1 + \frac{1}{\Gamma}\frac{|\tilde{h}^{vc}|^2 s_k^{vc}}{|h_k^{vc,c}|^2 \tilde{s}^c + \sigma_k^{vc}}\right) \quad (9)$$

which leads to the fixed power $\tilde{s}^c$ as follows

$$\tilde{s}^c = \frac{1}{|h_k^{vc,c}|^2}\left(\frac{|\tilde{h}^{vc}|^2 s_k^{vc}}{\Gamma(2^{\tilde{b}^{vc}} - 1)} - \sigma_k^{vc}\right). \quad (10)$$

This leads to a closed-form formula with complexity similar to IWF (and lower than NF-IWF), but improved performance. The reduction in complexity results in discarding the loop of line 11 in Algorithm (1). We refer to this algorigthm as NF-IWF-average. We use an average bit value of 5 bits per subcarrier in our simulations.

IV. PERFORMANCE EVALUATION

We consider a wireless OFDM network with parameters from Table I. We first focus on a two-user case with high and low interference as shown in Figures 2a and 2b, respectively. We evaluate the proposed algorithms and compare them against EPA and IWF. Changing the weights w^c and w^{vc} allows us to reach different full-power operating points to form a rate region. This is not possible with IWF, for which the total power budget has to be tuned between 0 and $P^{c,tot}$ to obtain different operating points. This tunability can be seen as an important advantage of NF-IWF.

As an upper bound, we simulate the case in which the base station has full knowledge of the interference and direct channels and the power allocation of the neighboring cells, i.e. $h_k^{vc,c}$, h_k^{vc}, and s_k^{vc} are perfectly known. We call this approach NF-full, which is equivalent to the distributed algorithm (DSB) of [7]. Evidently, this is not anymore an autonomous approach.

Figure 4 shows the achievable gains of NF-IWF in a two-user high interference scenario. With a primary user data rate of 600 Mbps, NF-IWF can achieve 6 Mbps for the victim user instead of 1.7 Mbps for IWF and 0.1 Mbps for EPA, i.e. an increase by a factor of 3.5 compared to IWF and 60 compared to EPA. More importantly, the difference between NF-full and NF-IWF is marginal. We can also observe that NF-IWF-average can achieve similar gains with reduced complexity when the data rate of the primary user is lower than 500 Mbps. The underperformance and lack of tunability of EPA is evident.

The two-user low interference scenario shows that all of the algorithms, except for EPA, have a similar performance since the direct channels are strong compared to the interference channels. The rate region is omitted for conciseness.

We now consider a multi-user multi-cell wireless OFDM network with 5 base stations and 500 users (randomly distributed) as shown in Figure 5. The user color indicates the base station to which each user is attached. Users within the coverage of their attached base station are classified as primary users, while users farther than the base station coverage (but still attached to it) are classified as victim users. For a fair comparison all the base stations use the same power allocation algorithm and we use the best rate region operating point. For IWF, this point corresponds to a full-power scheme, while for NF-IWF, NF-IWF average, and NF-full we consider a point on the rate region boundary with a tangent with slope equal to -1.

The cumulative distribution function (CDF) of the upper graph of Figure 6 shows the achievable gains of the proposed approach on the victim users' data rates. As expected, NF-full achieves the best data rate performance for the victim users, followed closely by NF-IWF and NF-IWF-average. EPA and IWF present a clear underperformance due to their lack of protection towards victim users. For example, with the NF algorithms, victim users have a 10% probability of obtaining a normalized data rate less than 95% (i.e. 90% probability of a higher data rate); while with EPA

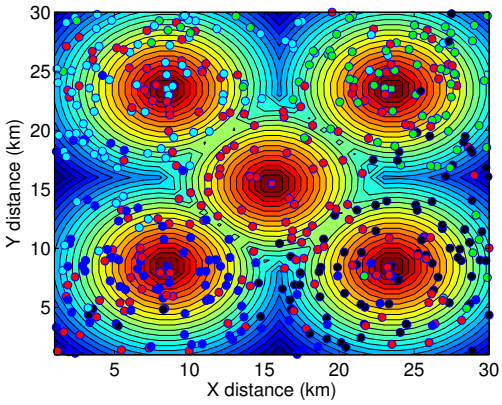


Fig. 5. Network with 5 base stations and 500 users randomly distributed. The user color indicates the base station to which the users are attached.

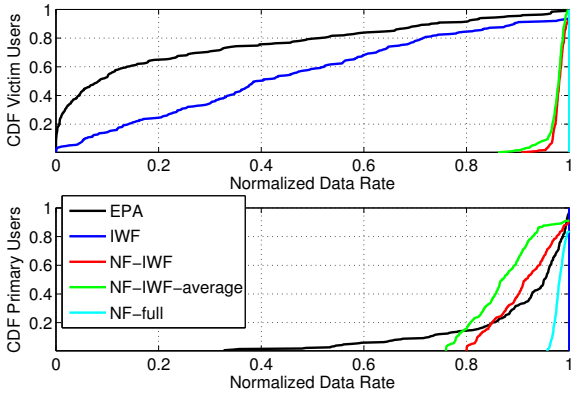


Fig. 6. CDF of primary and victim users of Figure 5.

and IWF they have 50% probability of a normalized data rate less than 10% and 40%, respectively.

Additionally, the gains from the proposed approach come with a marginal decrease in the primary users' data rates as seen in the lower graph of Figure 6. Since we consider the full-power operating point for IWF, it achieves the highest data rate for the primary users (as seen in Figure 4), followed closely by NF-full, NF-IWF, and NF-IWF-average. EPA can sometimes present higher primary users' rates than the NF algorithms. This is because, depending on the channel conditions, the EPA operating point can have higher primary users' data rates than the NF operating point. However, the NF algorithms never achieve a data rate less than 75% for primary users.

V. Conclusion

In this paper we have proposed NF-IWF, a neighbor-friendly autonomous algorithm for power spectrum allocation in wireless OFDM networks. In contrast to greedy algorithms like EPA and IWF, NF-IWF protects the data rate of victim users located within a certain distance from the base station. This is done through a per-subcarrier penalty factor on the power allocation level. Furthermore, we have designed a low-complexity version, NF-IWF-average, which reduces the complexity by fixing the penalty factor to the average bit rate of the victim user, resulting in a closed-form formula. The proposed approach offers a higher tunability and much better data rate performance, especially in high interference conditions. We have shown an increase by a factor of 3.5 compared to IWF and 60 compared to EPA in the victim users data rate with a marginal decrease of the primary users data rate. Also, the rate region of our approach lies close to the upper bound with full knowledge of the interference channel and the power allocation of the neighboring cells.

References

[1] C. Kosta, B. Hunt, A. UI Quddus, and R. Tafazolli, "On interference avoidance through inter-cell interference coordination (ICIC) based on OFDMA mobile systems", *IEEE Communications Surveys & Tutorials*, pp. 1-23, 2013.

[2] D. Lopez-Perez, I. Guvenc, G. de la Roche, M. Kountouris, T. Q. S. Quek, and J. Zhang, "Enhanced inter-cell interference coordination challenges in heterogeneous networks", *IEEE Wireless Communications*, vol.18, no.3, pp.22-30, Jun 2011.

[3] R. Kwan and C. Leung, "A Survey of Scheduling and Interference Mitigation in LTE", *Journal of Electrical and Computer Engineering*, 2010.

[4] G. Boudreau, J. Panicker, N. Guo, R. Chang, N. Wang, and S. Vrzic, "Interference coordination and cancellation for 4G networks", *IEEE Communications Magazine*, vol.47, no.4, pp.74-81, Apr. 2009.

[5] M. Necker, "Interference Coordination in Cellular OFDMA Networks", *IEEE Network*, vol.22, no.6, pp.12-19, Dec. 2008.

[6] R. Cendrillon, J. Huang, M. Chiang, and M. Moonen, "Autonomous Spectrum Balancing for Digital Subscriber Lines", *IEEE Transactions on Signal Processing*, vol. 55, no. 8, Oct. 2007.

[7] P. Tsiaflakis, M. Diehl, and M. Moonen, "Distributed spectrum management algorithms for multiuser DSL networks", *IEEE Transactions on Signal Processing*, vol. 56, no. 2, Oct. 2008.

[8] K. Son, S. Lee, Y. Yi, and S. Chong, "REFIM: A practical interference management in heterogeneous wireless access networks", *IEEE Transactions on Selected Areas in Communications*, vol. 29, no. 6, Aug. 2011.

[9] W. Yu, G. Ginis, and J. Cioffi, "Distributed multiuser power control for digital subscriber lines", *IEEE Transactions on Selected Areas in Communications*, vol. 20, no. 5, Jun. 2002.

[10] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines", *IEEE Transactions on Communications*, vol. 54, no. 5, May. 2006.

[11] 3GPP, "Considerations on interference coordination in heterogeneous networks", R1-101369, San Francisco, CA, Feb. 2010.

[12] 3GPP, "Summary of the description of candidate eICIC solutions", R1-104968, Madrid, Spain, Aug. 2010.

[13] 3GPP, "Spatial channel model for Multiple Input Multiple Output (MIMO) simulations", TR 25.996, v11.0.0 Sep. 2012.