A Cell Breathing Approach in Green Heterogeneous Networks

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***Abstract*—Femto base stations constitute a promising solution to cope with the dramatic increase in mobile data traffic, but they also contribute to increase the network power consumption. Traditionally, full power transmissions are used to satisfy the users' data rate demands, causing interference to neighboring cells. Cell breathing can reduce the total transmit power by adjusting the cell size to the traffic load, hence limiting interference. However, it requires full knowledge of the channel conditions and power allocation strategies of neighboring cells. Its implementation becomes more challenging in broadband heterogeneous networks due to the channel frequency selectivity, the power difference between base station types, and the required network coordination. Therefore, we propose a cell breathing approach for green heterogeneous networks, referred to as path loss-based cell breathing (PL-CB), which uses only path loss information to limit the interference caused to neighboring cells. It achieves a higher energy efficiency and a larger achievable rate region than other state-of-the-art techniques with negligible degradation with respect to the approach with full knowledge of the channel and transmit powers of neighboring cells.**

***Index Terms*—Interference management, cell breathing, energy efficiency, victim users**

I. Introduction

Femto base stations (FBSs), or femtocells, constitute a promising solution to cope with the tremendous increase in mobile data traffic. However, the massive deployment of FBSs in an area already covered by high-power macro base stations (MBSs) requires the use of energy-efficient, or green, resource allocation algorithms.

During peak traffic periods, the network load obliges to use full power transmissions of FBSs and MBSs in the downlink to satisfy the users' data rate demands. This is especially severe with closed-access FBSs, which restrict an energy-efficient load balancing. This approach not only wastes a large amount of power, but it also affects the data rate of neighboring cells by creating inter-cell interference. Therefore, the implementation of inter-cell interference coordination (ICIC) techniques is of paramount importance, not only to maximize the network data rates, but also to make networks "greener".

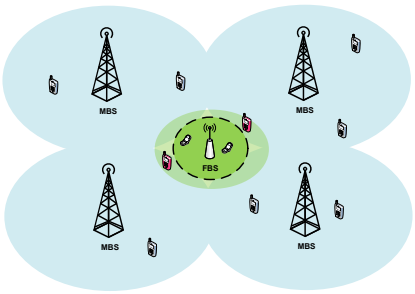

Fig. 1. Cell breathing. The FBS reduces the cell size to reduce the interference to victim users connected to neighboring MBSs.

Most ICIC techniques in wireless heterogeneous networks focus on interference coordination or cancellation to maximize the network data rates [1], [2], [3], but rarely on energy efficiency [4]. In any case, an energy-efficient approach must consider not only the cell power consumption, but also the impact of the power allocation strategy on neighboring cells.

A practical approach to reduce transmit power and, hence, avoid interference is cell breathing (or cell zooming) [5] shown in Fig. 1. The FBS gives access to the femto users within its coverage, but it also interferes with macro users within the neighborhood (shown in red). Through cell breathing, the FBS reduces the cell size to limit the damage to macro users and to save energy. Although MBSs can also use cell breathing, this approach is crucial for FBSs since they are likely to be deployed in an unplanned manner by end-users.

In the case of OFDM heterogeneous networks, the implementation of cell breathing becomes more challenging. First, because the transmit powers can be allocated independently on each subcarrier. Second, because the power levels can greatly vary according to the base station type. And third, because a certain coordination between base stations is required to adapt the power levels in neighboring cells to avoid coverage holes.

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A well-known approach for power allocation in OFDM networks is iterative waterfilling (IWF) [6], where each base station maximizes its own data rate in a greedy way by allocating power to those subcarriers with the best channel-to-interference-and-noise ratio (CINR). This is done without considering the interference caused to victim users from neighboring cells.

Non-greedy power allocation techniques that take into account the damage caused to victim users have been proposed in [7] and [8] in the context of digital subscriber lines (DSL) networks. In these papers, the concept of a protected reference line (or reference user) is introduced as a statistical average of all victim lines suffering interference. In [9] this concept has been applied to a wireless network, where the user suffering the strongest interference from the neighboring cells is selected as the reference user. In [10] this approach has been made fully autonomous and extended for heterogeneous networks.

However, these algorithms use detailed channel and power information in order to maximize the network data rates (and not the energy efficiency). In a wireless network this information needs to be updated periodically resulting in delays, information overhead over the backhaul link, and hence additional power consumption. This is especially severe in OFDM networks, where the channel characteristics vary significantly over time and frequency. Path loss information, on the other hand, can be easily estimated.

Therefore, we propose in this paper a cell breathing approach for green heterogeneous networks that uses only the path loss information to limit the interference damage caused to neighboring cells. We refer to the algorithm as the path loss-based cell breathing (PL-CB) algorithm.

II. Transmit Power Minimization

In an OFDM network, the total available transmission bandwidth is divided in subcarriers. Assuming a cyclic prefix longer than the channel length and perfect synchronization, each subcarrier transmits data independently and can be given a different transmit power level.

Our goal is to minimize the total transmit power in each cell subject to data rate constraints, formulated as:

$$\begin{aligned} \underset{s_k^c\ \forall k}{\text{minimize}} \quad & P^{c,\text{tot}} = \sum_{k\in\mathcal{K}} s_k^c \\ \text{s.t.} \quad & R^c \geq R^{c,\text{target}} \\ & 0 \leq s_k^c \leq s_k^{c,\text{mask}} \quad \forall k \in \mathcal{K} \end{aligned} \tag{1}$$

with

$$R^c = f_s \sum_{k\in\mathcal{K}} \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^c|^2 s_k^c}{\sum_{\substack{\bar{c}\neq c \\ \bar{c}\in\mathcal{C}}} |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \sigma_k^c}\right) \tag{2}$$

where $P^{c,tot}$ is the total transmit power in cell c, R^c and $R^{c,\text{target}}$ are the data rate and the target data rate in cell c, f_s is the symbol rate, h_k^c, σ_k^c, s_k^c, and $s_k^{c,mask}$ are the channel transfer function, the noise power, the base station transmit power, and the spectral emission mask constraints on subcarrier k in cell c, respectively. $h_k^{\bar{c}}$ and $s_k^{\bar{c}}$ are the channel transfer function and transmit power on subcarrier k from the interfering base station, which are assumed to be known as they refer to the users in cell c. We call h_k^c the direct channel and $h_k^{\bar{c}}$ the interference channel. $\mathcal{C}$ and $\mathcal{K}$ are the set of available cells and subcarriers, respectively. A given subcarrier can only be allocated to one user in each cell, but it can also be allocated to users attached to neighboring cells resulting in inter-cell interference. The allocation of subcarriers to users can be done prior to the power allocation strategies described in this paper. However, our focus is only on the power allocation. Γ denotes the signal-to-noise ratio (SNR) gap to capacity, which depends on the desired bit error rate (BER), the coding gain, and the noise margin. We will assume it to be equal to 1 without loss of generality.

Using the Karush-Kuhn-Tucker (KKT) conditions, the transmit powers have a closed-form solution as follows

$$s_k^c = \left[\frac{\lambda_c f_s}{\log(2)} - \frac{\sum_{\bar{c}\neq c} \Gamma |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \Gamma\sigma_k^c}{|h_k^c|^2}\right]_0^{s_k^{c,\text{mask}}} \tag{3}$$

where $[x]_a^b = \max(a, \min(x, b))$ and λ_c is the Lagrange multiplier that should be updated (e.g. with bisection) to satisfy the corresponding data rate constraint $R^{c,\text{target}}$. Differently from conventional IWF that maximizes the data rate given total power constraints, equation (3) provides a solution for the transmit power minimization given data rate constraints. We refer to this algorithm as green IWF (GIWF).

With GIWF, each cell minimizes its transmit power by allocating it to those subcarriers with the best CINR without taking into account the impact on users from neighboring cells. The advantage of GIWF is its simplicity, its closed-form solution, and the fact that it does not need coordination between base stations. Actually, GIWF can be viewed as a particular case of cell breathing if the total transmit power is varied to improve the energy efficiency of the network.

III. Cell Breathing Power Control

A. Victim User Protection

Following the idea of a protected reference user, we formulate the optimization problem as the minimization of an objective function comprising the total transmit power and the weighted data rate of victim users from neighboring cells (R^{vc}), subject to the data rate constraints of primary users connected to cell c (R^c):

$$\begin{aligned} \underset{s_k^c\ \forall k}{\text{minimize}} \quad & P^{c,\text{tot}} - \mu R^{vc} \\ \text{s.t.} \quad & R^c \geq R^{c,\text{target}} \\ & 0 \leq s_k^c \leq s_k^{c,\text{mask}} \quad \forall k \in \mathcal{K} \end{aligned} \tag{4}$$

with

$$R^{vc} = f_s \sum_{k\in\mathcal{K}} \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{vc}|^2 s_k^{vc}}{|h_k^{vc,c}|^2 s_k^c + \sigma_k^{vc}}\right) \tag{5}$$

where h_k^{vc}, s_k^{vc}, and σ_k^{vc} are the direct channel, the transmit power, and the noise power on subcarrier k of the victim user, respectively, and $h_k^{vc,c}$ is the interference channel on subcarrier k from cell c to the victim users. We distinguish between primary and victim users as those suffering from low or high interference, respectively. The parameter μ serves as

weighting factor to protect the victim users' data rate. Again, the subcarrier allocation is assumed to be done prior to the power allocation, therefore R^c and R^{vc} refer to all the users in a cell.

Applying the KKT stationarity condition to problem (4) leads to

$$\forall k: 1 - \frac{\lambda_c f_s |h_k^c|^2}{\log(2)\left(|h_k^c|^2 s_k^c + \sum_{\bar{c}\neq c} \Gamma |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \Gamma \sigma_k^c\right)} + \frac{\mu_c f_s |h_k^{vc}|^2 s_k^{vc} |h_k^{vc,c}|^2}{\log(2)\left(\Gamma |h_k^{vc,c}|^2 s_k^c + \Gamma \sigma_k^{vc}\right)\left(|h_k^{vc}|^2 s_k^{vc} + \Gamma |h_k^{vc,c}|^2 s_k^c + \Gamma \sigma_k^{vc}\right)} = 0. \tag{6}$$

By taking into account the KKT complementarity conditions of (4), s_k^c from the first term of equation (6) can be isolated:

$$s_k^c = \left[\frac{\frac{\lambda_c f_s}{\log(2)}}{P_k^{VC,c} + 1} - \frac{\sum_{\bar{c}\neq c} \Gamma |h_k^{\bar{c}}|^2 s_k^{\bar{c}} + \Gamma \sigma_k^c}{|h_k^c|^2} \right]_0^{s_k^{c,\text{mask}}} \tag{7}$$

where $P_k^{VC,c}$ is called the penalty factor, defined as

$$P_k^{VC,c} = \frac{\mu_c f_s |h_k^{vc}|^2 s_k^{vc} \Gamma |h_k^{vc,c}|^2}{\log(2)\left(\Gamma |h_k^{vc,c}|^2 s_k^c + \Gamma \sigma_k^{vc}\right)\left(|h_k^{vc}|^2 s_k^{vc} + \Gamma |h_k^{vc,c}|^2 s_k^c + \Gamma \sigma_k^{vc}\right)} \tag{8}$$

resulting in a fixed point equation as $P_k^{VC,c}$ depends on s_k^c. Note that the first term in equation (7) corresponds to a power level with per-subcarrier offset $P_k^{VC,c}$, which reduces the damage to victim users from neighboring cells. Setting $P_k^{VC,c}$ to zero will result in the GIWF algorithm.

Problem (4) is a nonconvex optimization problem for which a duality gap exists between the solution of (7) and the optimal solution. However, as the number of subcarriers increases, this duality gap goes to zero [11]. By adding to equation (7) a bisection search on the Lagrange multiplier to satisfy the data rate constraints, we obtain Algorithm (1) with line (3) omitted. Since the computation of $P_k^{VC,c}$ requires full information of the channel and transmit powers of neighboring cells, we refer to the algorithm as full cell breathing (full-CB). δ indicates the accuracy of the data rate constraint, γ indicates the stopping criterion of the bisection search on λ_c in the case of an inactive data rate constraint, and $\Lambda^{\max}$ is the maximum value for λ_c.

B. Reduction of Information Exchange Overhead

To reduce the overhead over the backhaul link and, hence, complexity and extra power consumption, the periodical information exchange between base stations needs to be minimized or, preferably, avoided. However, equation (8) requires full knowledge of $h_k^{vc,c}$ and h_k^{vc}, which can only be known from the information received from other base stations or from the users channel feedback during a handover.

On the other hand, the users' path loss, i.e. the average channel gain over all the allocated subcarriers, is easier to obtain and only dependent on the distance from the base station to the user. For example, the distance to the cell-edge can be known when users scan the cell for a handover [12].

In an interference dominant scenario, the interference channel ($h_k^{vc,c}$) is stronger than the direct channel of the neighboring cell (h_k^{vc}). This is the case, for example, of a closed-access base station. As h_k^{vc} is to represent the weaker channel we propose to fix it to the worst case path loss from the base station of cell c to the cell-edge. As $h_k^{vc,c}$ is to represent the stronger channel, we propose to set it to the actual path loss from the base station of cell c to an existing user from a neighboring cell. In this way h_k^{vc} does not need to be updated regularly, while $h_k^{vc,c}$ is only updated with the users mobility. Therefore, instead of exchanging detailed channel information between base stations and the victim users, we rely on only 2 path loss values. The transmit power of the neighboring cell (s_k^{vc}) can be assumed as an equal power allocation (EPA), but s_k^c and $s_k^{\bar{c}}$ have a frequency variation due to the power allocation strategies of neighboring cells that is updated between iterations. We refer to this approach as PL-CB and it corresponds to Algorithm (1) with the additional initialization step of line (3). As this algorithm depends on the path loss of each user, the best performance is achieved if the subcarriers allocated to each user are contiguous.

Algorithm 1 full-CB / PL-CB

1: For each user in cell c:
2: Initialize $s_k^c = 0$
3: Only for PL-CB: Initialize h^{vc} and $h^{vc,c}$ according to the victim user path loss of section III-B and assume s_k^{vc} =EPA
4: **repeat**
5: $\lambda_c^{\min} = 0; \lambda_c^{\max} = \Lambda^{\max}$
6: $\lambda_c = (\lambda_c^{\max} + \lambda_c^{\min})/2$
7: **while** $|R^c - R^{c,\text{target}}| > \delta$ and $\lambda_c > \gamma$ **do**
8: $\lambda_c = (\lambda_c^{\max} + \lambda_c^{\min})/2$
9: **for** $k = 1 : \mathcal{K}$ **do**
10: **repeat**
11: Update s_k^c in (7)
12: **until** convergence
13: **end for**
14: **if** $R^c > R^{c,\text{target}}$ **then**
15: $\lambda_c^{\max} = \lambda_c$
16: **else**
17: $\lambda_c^{\min} = \lambda_c$
18: **end if**
19: **end while**
20: **until** network convergence

The protection level of PL-CB can be observed in Fig. 2. Interestingly, both PL-CB and full-CB allocate transmit power to those subcarriers less used for transmission by the neighboring base station (BS1), which uses simple IWF. This result suggests an implicit coordination between cells through their power allocation strategy. With PL-CB the transmit power allocation has a smoother frequency variation than with full-CB due to the lack of detailed channel information. Still, as seen in section IV, the performance difference between PL-CB and full-CB is minimal.

C. Complexity Reduction

The path loss assumption has an impact not only on avoiding information overhead, but also on the algorithmic complexity of PL-CB. For example, full-CB has 2 levels of iterations: one for the bisection search of λ_c until the data rate constraints are satisfied and another for $P_k^{VC,c}$ to update s_k^c. We denote the number of iterations as N_λ for the first level and N_P for the second. A low-complexity approach as proposed

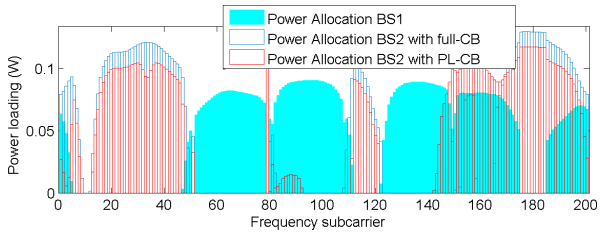
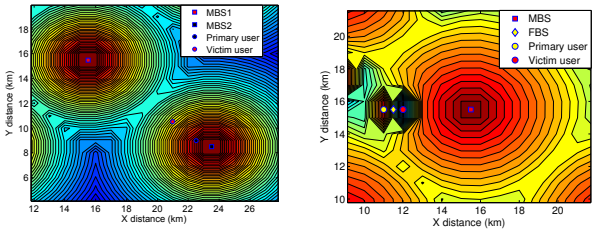


Fig. 2. Transmit power allocation of two neighboring base stations (BS1 and BS2) using PL-CB and full-CB.

in [10] needs only one level of iterations on λ_c. However, within each iteration level, the transmit power computation is done on a per-subcarrier basis, i.e. $\mathcal{K}$ transmit power computations are necessary. This brings the total complexity in the number of transmit power computations to $N_\lambda N_P K$ for full-CB and $N_\lambda K$ for the approach of [10]. Since PL-CB assumes one path loss value for h_k^{vc} and $h_k^{vc,c}$, this brings the total complexity to $N_\lambda N_P$, which represents a large complexity reduction, especially in broadband systems.



(a) Case 1. Macro-macro interference.

(b) Case 2. Femto-macro interference.

Fig. 3. The user color indicates the base station to which it is attached. In case 1, interference comes from MBS2, while in case 2 it comes from a FBS. The color regions indicate the signal strength from the closest base station.

IV. Performance Evaluation

We consider an OFDM heterogeneous network with parameters from Table I. We consider two users connected to neighboring cells. These results, however, can be extended for any number of users once their channels and path loss values are known. In the first case the victim user is within the coverage of an interfering MBS as seen in seen in Fig. 3(a). In the second case the victim user is within the coverage of an interfering FBS as seen in seen in Fig. 3(b). The latter is a common scenario of a closed-access base station.

As performance metric we use the energy efficiency of the victim user, defined as $R^{vc}/BW/P^{vc,\text{total}}$ as suggested in [4], where BW is the available bandwidth and $P^{vc,\text{total}}$ is the total transmit power for the victim user. For PL-CB and full-CB, we vary the target data rate constraints of the interfering base station ($R^{c,\text{target}}$), which results in different total transmit power values (MBS Transmit Power for case 1 and FBS Transmit Power for case 2). The transmit power minimization of the interfering base station impacts the energy efficiency of the victim user by limiting the interference, and hence, reducing the power consumption of the neighboring cell. For GIWF, we simply vary the target data rate constraints without protection to the victim user to obtain different transmit power operating points.

TABLE I
Simulation Parameters

Parameter	Value
System Bandwidth	5 MHz
Number of data subcarriers	200
Γ	1
δ	10^{-6}
γ	10^{-6}
Λ^{max}	10^8
f_s	2.8 Gsymbols/s
Channel profile	3GPP SCM suburban macro [13]
MBS maximum transmit power	45 dBm
FBS maximum transmit power	19 dBm
Number of transmitting antennas	1
Number of receiving antennas	1

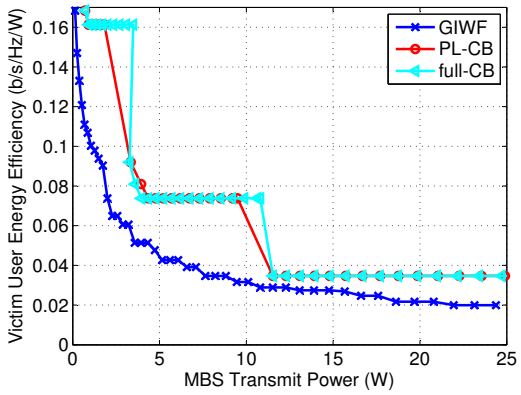


Fig. 4. Energy efficiency of the victim user of Fig. 3(a).

A. Macro-Macro Interference

Fig. 4 shows the impact of the MBS transmit power on the energy efficiency of the victim user of Fig. 3(a). A decrease in the MBS transmit power results in an exponential increase of the energy efficiency of the victim user. However, by providing protection to the victim user, PL-CB can increase the energy efficiency compared to GIWF. Also, we can observe that the proposed approach lies close to the upper-bound imposed by full-CB. The step behavior of PL-CB and full-CB comes from the selection of different local optimization points.

B. Femto-Macro Interference

Fig. 5 shows the impact of the FBS transmit power on the energy efficiency of the victim user of Fig. 3(b). Again, PL-CB provides an increase in energy efficiency compared to GIWF.

However, the energy-efficiency increase of the victim users would be meaningless if it comes with a significant decrease in the primary users' data rates. Consequently, we need to consider the trade-off between the primary and victim users' data rates. This trade-off can be observed with the achievable rate regions in Fig. 6 and 7. The different operating points can be achieved by varying the weight μ and maximizing the weighted sum of data rates of each user subject to total transmit power constraints.

These rate regions are compared with additional transmit power allocation strategies. These include EPA, where the

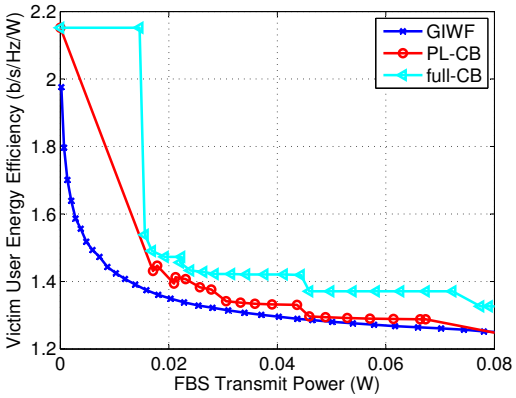


Fig. 5. Energy efficiency of the victim user of Fig. 3(b).

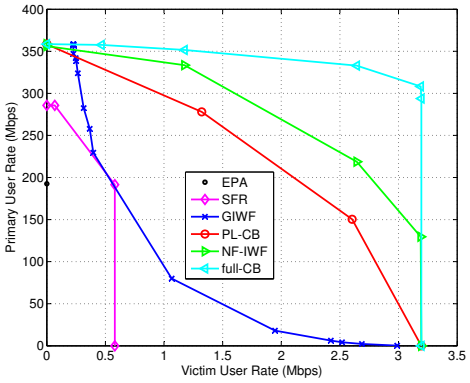


Fig. 6. Rate region of the primary and victim users of Fig. 3(a).

same transmit power is allocated to all subcarriers (hence only one point). We use as benchmark a practical approach used in LTE: soft frequency reuse (SFR), in which the total bandwidth of each cell is divided in two non-overlapping frequency bands with constant power level, one for center users (i.e. primary users in our scenario) and one for cell-edge users (i.e. victim users in our scenario). We generate a rate region by varying the total transmit power fraction and total bandwidth fraction of the center users' band [14]. However, this can only be achieved through network coordination. A final comparison is with NF-IWF [10], which maximizes network data rates with protection to victim users. This is done with knowledge of the interference channel of the victim user ($h_k^{vc,c}$).

Evidently not all these algorithms are simulated under an energy efficiency perspective since the target data rates need to be within the achievable rate region, which constitutes a feasibility problem. From the presented rate regions we see that PL-CB increases the energy efficiency while still achieving a larger rate region compared to most state-of-the-art approaches.

V. Conclusion

In this paper we have proposed PL-CB, an energy-efficient cell breathing approach for green OFDM heterogeneous networks. It minimizes the cell power consumption and contributes to the network energy efficiency by limiting the interference to victim users from neighboring cells. Compared to

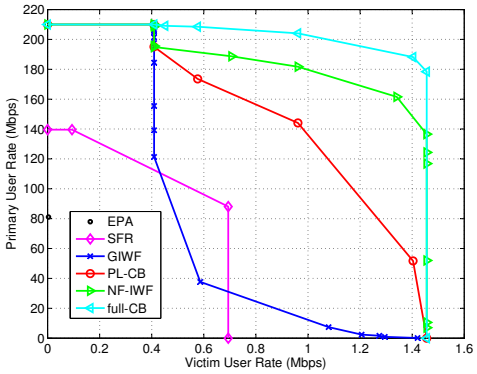


Fig. 7. Rate region of the primary and victim users of Fig. 3(b).

other approaches that require detailed and periodical information exchange between base stations, PL-CB uses only the path loss information of victim users to limit the interference. It achieves a higher energy efficiency and a larger achievable rate region than other state-of-the-art techniques with negligible degradation with respect to the approach with full knowledge of the channel and transmit powers of neighboring cells.

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