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***Abstract*— A time-domain equalizer (TEQ) is inserted in discrete multitone (DMT) receivers to impose channel shortening and hence overcome the need for a too long cyclic prefix. Many algorithms have been developed to initialize this TEQ, but none of them really optimizes the bitrate. In this paper, we present a new *bitrate maximizing* TEQ (BM-TEQ) cost function that results in a TEQ design/initialization that outperforms any other TEQ design. In the derivation, we exploit the fact that the frequency-domain equalizers (FEQ) do not alter the SNR on the individual tones. The performance of this BM-TEQ comes close to the performance of the per-tone equalizer (PTEQ), an alternative DMT equalization structure that ensures consistently better performance than a TEQ. Finally, the presented BM-TEQ design is also used in a "per group" equalization scheme (PGEQ), which is intermediate (in terms of memory requirement and performance) between TEQ and PTEQ. The PGEQ design then encompasses BM-TEQ and PTEQ design procedures as extreme cases.**

I. Introduction

Multicarrier modulation has regained interest over the last decade. Several all-digital variants have been proposed: discrete multitone (DMT) is adopted as transmission format for asymmetric digital subscriber line (ADSL) and presented as a candidate for very high bit rate digital subscriber line (VDSL); orthogonal frequency division multiplexing (OFDM) is proposed for wireless local area applications, e.g. HiperLAN.

DMT schemes divide the bandwidth into parallel subbands or tones. The incoming bitstream is split into parallel streams that are used to QAM-modulate the different tones. The modulation is done by means of an inverse fast Fourier transform (IFFT). Before transmission of a DMT symbol, a cyclic prefix of ν samples is added. If the channel impulse response order is less than or equal to the cyclic prefix length ν, demodulation can be implemented by means of an FFT, followed by a (complex) 1-tap frequency domain equalizer (FEQ) per tone to compensate for channel amplitude and phase effects.

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Practical channel impulse responses can be very long, hence a large prefix would be required. However, a long prefix results in a large overhead with respect to the data rate. An existing solution for this problem is to insert a (real) T-tap time domain equalizer (TEQ) before demodulation that shortens the channel impulse response to the cyclic prefix length. Many algorithms have been developed to initialize this TEQ using training sequences, but none of them truly optimizes bitrate. They can be roughly divided into three classes.

1) Minimum mean-square error (MMSE) based methods, such as in [1], [2], minimize the time-domain error between the TEQ output and a desired output, i.e. the output of an FIR filter of order ν, called target impulse response (TIR), fed with the transmitted time-domain sequence. The TIR is part of the parameter vector. A non-triviality constraint is needed to avoid the trivial all-zero solution. However, minimizing the *time*-domain error energy does not optimize the bitrate, which is based on signal-to-noise ratios (SNR) in the *frequency* domain.
2) In [3], the authors attempt to maximize bitrate by maximizing an (approximate) geometric SNR. However, several assumptions render the method suboptimal: e.g. each subchannel SNR takes neither intercarrier interference (ICI) and intersymbol interference (ISI) of the DMT signal nor DFT leakage of the noise into account. In [4] and [5], ISI/ICI and/or DFT leakage are only partially taken into account.
3) A computationally efficient generalization of the "maximum shortening-SNR" (MSSNR) method is presented in [5], called "minimum-ISI method". The original MSSNR method is described in [6] and aimed at maximizing the ratio of the energy of the channel impulse response inside a target window of length ν to the energy outside the target window. None of these methods truly optimizes capacity.

In [7], an alternative equalization scheme is presented that always performs better than a TEQ based receiver while keeping complexity during data transmission at the same level. Equalization is done for each tone separately after the FFT-demodulation, hence the term "per-tone equalization" (PTEQ). The drawback is its memory requirement: $N_u T$ taps (with N_u the number of used tones, e.g. ca. 220 tones in downstream ADSL) need to be stored, instead of the T taps of a TEQ.

In this paper, we present a *bitrate maximizing TEQ* (BM-TEQ) that achieves the maximum bitrate possible for a given number of TEQ taps T. The derivation in Section II starts from

the same bitrate expression as in [3]. As the FEQ does not alter the SNR on the individual tones, we consider the SNR at the FEQ *output* instead of its *input*, as is done in e.g. [3], [5]. By taking the dependence of the FEQs on the TEQ into account, we eliminate the FEQs. The result is an unconstrained nonlinear cost function in the TEQ coefficients. In Section III, we show how to use the presented BM-TEQ design in a "per group" equalization scheme (PGEQ). The PGEQ design then encompasses the TEQ and PTEQ design procedures as extreme cases. The simulations in Section IV show that the BM-TEQ always performs better than TEQs obtained with any other design procedure and approaches the performance of the PTEQ with T taps per tone (which is always better) very closely.

Notation. We summarize the most important notational conventions here; they are mostly adopted from [7].

- N is the (I)FFT size; N_u is the number of tones of interest, e.g. tones 38 to 256 for downstream ADSL; n is a tone index; k is the DMT symbol index.
- $\mathcal{F}$ is a DFT matrix of size N; its n-th row is $\mathcal{F}_n$.
- $\mathbf{w}$ is the time-domain equalizer (T taps).
- The time-domain vector of length N at time k at the TEQ output that is fed to the FFT is the result of the matrix-vector product $\mathbf{Y}_k\mathbf{w}$; $\mathbf{Y}_k$ is a Toeplitz matrix (of size $N \times T$) of received samples $y_k[l]$:

$$\mathbf{Y}_k = \begin{bmatrix} y_k[0] & \cdots & y_k[-T+1] \\ \vdots & \ddots & \vdots \\ y_k[N-1] & \cdots & y_k[N-T] \end{bmatrix} \quad (1)$$

  hence the matrix-vector product $\mathbf{Y}_k\mathbf{w}$ calculates the convolution of the k-th received DMT symbol and the TEQ; it is tacitly assumed that $\mathbf{Y}_k$ depends on the synchronization delay Δ, a design parameter that estimates the channel group delay [1].
- The sliding DFT $\mathcal{F}\mathbf{Y}_k$ (i.e. the DFT of the T columns of $\mathbf{Y}_k$) is denoted as $\mathbf{Y}_{k,sl}$ ($N \times T$ matrix); the n-th sliding DFT output is a $1 \times T$ row vector $\mathbf{Y}_{k,sl}[n] = \mathcal{F}_n\mathbf{Y}_k$.
- The $N \times 1$ FFT output vector *after* TEQ is given by $Y_{k,w} = \mathcal{F}\mathbf{Y}_k\mathbf{w} = \mathbf{Y}_{k,sl}\mathbf{w}$; the n-th FFT output (FEQ input) is $Y_{k,w}[n] = \mathcal{F}_n\mathbf{Y}_k\mathbf{w} = \mathbf{Y}_{k,sl}[n]\mathbf{w}$.
- The $N \times 1$ FFT output vector *without* TEQ is given by $Y_k = \mathcal{F}\begin{bmatrix} y_k[0] & \cdots & y_k[N-1] \end{bmatrix}^T$; the n-th entry of Y_k is denoted $Y_k[n]$.
- The transmitted frequency-domain symbol vector at time k is X_k; its n-th entry is $X_k[n]$.

II. An optimal bitrate maximizing TEQ

In the derivation towards an optimal TEQ cost function, we start from the same bitrate expression[1] as in [3]. The total number of bits transmitted in one DMT symbol is given by

$$b_{DMT} = \sum_{n=1}^{N_u} \log_2\left(1 + \frac{SNR_n}{\Gamma_n}\right) \quad (2)$$

where SNR_n represents the SNR on tone n and Γ_n is the SNR gap between SNR_n and the SNR required to achieve Shannon capacity. Γ_n is a function of the desired probability of error, coding gain and system margin. In [3], an approximate geometric SNR

$$SNR_{geom} = \left(\prod_{n=1}^{N_u} SNR_n\right)^{\frac{1}{N_u}} \quad (3)$$

with *suboptimal* subchannel SNR expression is optimized instead of bitrate (2). The derivation below for bitrate optimization (2) with an *exact* subchannel SNR expression could be equally applied to the approximate geometric SNR (3); the resulting cost function will be given in (16). This geometric SNR approximation is close to bitrate optimization as long as a bandwidth optimization scheme as in [8] is used such that only tones with high SNR are incorporated.

The SNR on tone n can be written as:

$$SNR_n = \frac{\text{desired received signal energy}_n}{\text{energy in (received signal}_n - \text{desired received signal}_n)} \quad (4)$$

As the 1-tap FEQ does not alter this ratio, one can determine the signal energy portions in (4) at the input or output of the FEQs. Whereas in [3], [5] the signal energies are determined at the FEQ input, we consider the energies at the FEQ output, in this way making the FEQs part of the optimization criterion. These FEQs depend on the TEQ, hence the FEQs can be written as a function of the TEQ.

When designing the FEQs, typically a choice is made between MMSE and zero forcing (ZF) equalizers. In [9], it is explained that the traditional *unconstrained MMSE equalizer* is *biased*, hence it is preferable to turn it into an *unbiased MMSE equalizer.* An unbiased MMSE equalizer gives an unbiased desired signal part at the equalizer output, resulting in minimum probability of error. In general, a ZF equalizer is also unbiased, but has a lower SNR because of noise enhancement. However, in case of a one-tap equalizer, such as the FEQs in a DMT system, there is no noise enhancement, hence the ZF equalizer and unbiased MMSE equalizer are equal[2] ([1] p. 320) and so will be our FEQ of choice. The FEQ output can be written as:

$$D_n \underbrace{\mathcal{F}_n\mathbf{Y}_k\mathbf{w}}_{\text{FEQ input } Y_{k,w}[n]} = \alpha_n X_k[n] + E_k[n] \quad (5)$$

where $X_k[n]$ is the transmitted frequency-domain symbol on tone n at time k and $E_k[n]$ is the residual ISI/ICI and noise at the n-th FEQ output. The factor α_n in (5) is 1, i.e. the desired signal component at the FEQ output is unbiased, in case of ZF FEQs D_n:

$$D_n = \frac{\mathcal{E}\left\{|X_k[n]|^2\right\}}{\mathcal{E}\left\{Y_{k,w}[n]X_k^*[n]\right\}} \quad (6)$$

In case of unconstrained, hence biased MMSE FEQs

$$D_n^{MMSE} = \frac{\mathcal{E}\left\{Y_{k,w}^*[n]X_k[n]\right\}}{\mathcal{E}\left\{|Y_{k,w}[n]|^2\right\}} \quad (7)$$

[1] We do not consider the mathematically much more difficult case of integer bit loading.

[2] If SNR_n is large, the unconstrained, biased MMSE FEQ and unbiased ZF FEQ will not differ much.

α_n is equal to a squared normalized correlation function $\rho_w^2[n]$ ($0 \leq \rho_w^2[n] \leq 1$) that will show up again further on:

$$\alpha_n = \frac{\left|\mathcal{E}\left\{X_k^*[n]Y_{k,w}[n]\right\}\right|^2}{\mathcal{E}\left\{\left|Y_{k,w}[n]\right|^2\right\}\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}} = \rho_w^2[n] \quad (8)$$

With ZF FEQs, the numerator in (4), i.e. the desired received signal energy on tone n, is independent of TEQ and FEQ and equal to $\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}$. The denominator in (4) is the MSE $\mathcal{E}\left\{\left|E_k[n]\right|^2\right\}$ at the FEQ output and takes *all* noise sources and residual ISI/ICI of the DMT signal into account. Moreover, (4) is suited for an arbitrary transmit power allocation scheme. The SNR_n in (4) can thus be written as:

$$SNR_n = \frac{\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}}{\mathcal{E}\left\{\left|E_k[n]\right|^2\right\}} = \frac{\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}}{\mathcal{E}\left\{\left|D_nY_{k,w}[n] - X_k[n]\right|^2\right\}} \quad (9)$$

After substitution of (6) into (9) and some rearranging, we obtain a very compact expression:

$$SNR_n = \frac{1}{\rho_w^{-2}[n] - 1} \quad (10)$$

with the squared normalized correlation function $\rho_w^2[n]$ as defined in (8). $\rho_w^2[n]$ can also be written as

$$\rho_w^2[n] = \frac{\left|\mathcal{E}\left\{X_k^*[n]\mathbf{Y}_{k,sl}[n]\right\}\mathbf{w}\right|^2}{\left(\mathbf{w}^T\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]\mathbf{Y}_{k,sl}[n]\right\}\mathbf{w}\right)\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}} \quad (11)$$

The bitrate maximizing TEQ (BM-TEQ) cost function (2) is then given by:

$$b_{DMT} = \sum_{n=1}^{N_u}\log_2\left(\frac{\Gamma_n + (1-\Gamma_n)\rho_w^2[n]}{\Gamma_n(1-\rho_w^2[n])}\right) \quad (12)$$

$$b_{DMT} = \sum_{n=1}^{N_u}\log_2\left(\frac{\mathbf{w}^T\mathbf{A}_n\mathbf{w}}{\mathbf{w}^T\mathbf{B}_n\mathbf{w}}\right) \quad (13)$$

where $\mathbf{A}_n$ and $\mathbf{B}_n$ are independent of $\mathbf{w}$ and given by

$$\begin{aligned}\mathbf{A}_n = {} & \Gamma_n\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]\mathbf{Y}_{k,sl}[n]\right\} \quad (14)\\ & +(1-\Gamma_n)\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]X_k[n]\right\}\mathcal{E}\left\{X_k^*[n]\mathbf{Y}_{k,sl}[n]\right\}\\ \mathbf{B}_n = {} & \Gamma_n\left(\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]\mathbf{Y}_{k,sl}[n]\right\}\right. \quad (15)\\ & \left.-\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]X_k[n]\right\}\mathcal{E}\left\{X_k^*[n]\mathbf{Y}_{k,sl}[n]\right\}\right)\end{aligned}$$

Some remarks:

- Maximizing SNR_{geom} in (3) using the SNR expression (10) is equivalent with minimizing its denominator

$$\prod_{n=1}^{N_u}\mathcal{E}\left\{\left|E_k[n]\right|^2\right\} = \prod_{n=1}^{N_u}\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}\left(\rho_w^{-2}[n]-1\right) \quad (16)$$

  i.e. the product of mean-square errors $\mathcal{E}\left\{\left|E_k[n]\right|^2\right\}$ at the FEQ outputs.
- Although channel shortening is the original underlying idea for using a TEQ, the BM-TEQ criterion (13) does not explicitly impose channel shortening any more. Hence, one could also use a complex valued TEQ, which performs slightly better than a real valued TEQ at the cost of extra complexity (e.g. a more complex FFT).
- With $N_u = 1$ and a complex $\mathbf{w}$ per tone, (13) reduces to the design of a linear unbiased MMSE PTEQ [7].

Maximizing (13) is an unconstrained nonlinear optimization problem. To avoid a parameter ambiguity, a constraint on $\mathbf{w}$, e.g. $\|\mathbf{w}\| = 1$, is needed, but contrary to MMSE-based TEQ design, this constraint does not influence the optimum bitrate. The problem can be solved using nonlinear optimization methods such as iterative (quasi-)Newton algorithms, simplex algorithms, (recursive) stochastic approximation algorithms, etc. Therefore, one needs an estimate of the autocorrelation matrix $\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]\mathbf{Y}_{k,sl}[n]\right\}$ for all sliding DFTs $\mathbf{Y}_{k,sl}[n]$, as well as the crosscorrelation vectors $\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]X_k[n]\right\}$ of sliding DFT $\mathbf{Y}_{k,sl}[n]$ and transmitted symbols $X_k[n]$. These estimates can be obtained accurately with the data model of [7] based on an estimate of channel impulse response and noise characteristics. Alternatively, one can estimate the statistics using the training sequence (with cyclic prefix) that is transmitted during the connection set-up. In the simulations of Section IV, we use a recursive strategy based on a stochastic Newton algorithm with Gauss-Newton-like search direction [10]. The algorithm uses recursive estimates of all matrices $\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]\mathbf{Y}_{k,sl}[n]\right\}$ and vectors $\mathcal{E}\left\{\mathbf{Y}_{k,sl}^H[n]X_k[n]\right\}$ to calculate the gradient and an approximate positive-semidefinite Hessian. It converges very fast (a good estimate for $\mathbf{w}$ is obtained after less than 100 updates) and does not seem to get stuck in possible local optima, independent of the initialization of $\mathbf{w}$. Moreover, it allows for further adaptation and tracking during data transmission. Current research focuses on cheaper adaptive implementations.

The solution of (13) gives as a by-product the ZF FEQs:

$$D_n = \frac{\mathcal{E}\left\{\left|X_k[n]\right|^2\right\}}{\mathcal{E}\left\{X_k^*[n]\mathbf{Y}_{k,sl}[n]\right\}\mathbf{w}} \quad (17)$$

Alternatively, one can use a cheap zero forcing LMS adaptive algorithm as presented in [1]:

$$D_n \leftarrow D_n + \mu X_k^*[n](X_k[n] - D_nY_{k,w}[n]) \quad (18)$$

III. Per group equalization (PGEQ)

In [11], a complexity reduced scheme for the PTEQ is introduced. Tones are grouped and only the PTEQ of the center tone of each group is optimized; the other tones in the group get a PTEQ, corresponding to the equivalent TEQ of the center tone. This scheme allows a considerable reduction in initialization complexity and memory requirement, but it does not

guarantee better performance than a TEQ, especially for large tone groups.

The BM-TEQ criterion (13) can be used in a new "per group" equalization scheme (PGEQ), which is intermediate (in terms of memory requirement and performance) between TEQ and PTEQ. The new scheme comes down to the calculation of multiple TEQs, one for each tone group. As the TEQ for each tone group optimizes the bitrate for this group, the PGEQ scheme will perform better than the original PTEQ structure with tone grouping (for the same number of tones per group) and obviously better than a single TEQ. The PGEQ design encompasses the TEQ and PTEQ design procedure as extreme cases.

Assume we use N_G tone groups and a real TEQ $\mathbf{w}_g$ ($g = 1, \cdots, N_G$) of T taps per group, $N_G T$ taps need to be stored. The computational complexity during data transmission can be kept almost as low as for a single TEQ. One could naively calculate the $N \times 1$ TEQ output $\mathbf{Y}_k \mathbf{w}_g$ for all groups and then apply a (partial) FFT $\mathcal{F}\mathbf{Y}_k\mathbf{w}_g$ for each group. However, it is often computationally cheaper to exploit the structure in the sliding DFT $\mathcal{F}\mathbf{Y}_k$. As shown in [7], the sliding DFT on tone n, $\mathbf{Y}_{k,sl}[n] = \mathcal{F}_n\mathbf{Y}_k$, can be written as

$$\mathbf{Y}_{k,sl}[n] = \begin{bmatrix} Y_k[n] & \Delta\mathbf{y}_k \end{bmatrix} \begin{bmatrix} 1 & \alpha_n & \cdots & \alpha_n^{T-1} \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha_n \\ 0 & \cdots & 0 & 1 \end{bmatrix} \quad (19)$$

where $\alpha_n = e^{-\frac{j2\pi(n-1)}{N}}$, i.e. $\mathbf{Y}_{k,sl}[n]$ is a linear combination of $T-1$ difference terms

$$\Delta\mathbf{y}_k = [(y_k[-1] - y_k[N-1]) \ \cdots \ (y_k[-T+1] - y_k[-T+N+1])]$$

and one FFT output

$$Y_k[n] = \mathcal{F}_n \begin{bmatrix} y_k[0] & \cdots & y_k[N-1] \end{bmatrix}^T$$

The n-th FFT output with TEQ $\mathbf{w}_g$, $Y_{k,w_g}[n] = \mathcal{F}_n\mathbf{Y}_k\mathbf{w}_g$, can then be obtained efficiently by:

1) recursively calculating the T entries $\mathbf{Y}_{k,sl}[n,t]$ of the n-th sliding DFT $\mathbf{Y}_{k,sl}[n]$ using (19):

$$\begin{aligned} \mathbf{Y}_{k,sl}[n,t] &= \alpha_n \mathbf{Y}_{k,sl}[n,t-1] + (y_k[-t+1] \\ &\quad - y_k[-t+N+1]),\ 2 \le t \le T \\ &= Y_k[n],\ t = 1 \end{aligned}$$

2) calculating the inner product:

$$Y_{k,w_g}[n] = \mathbf{Y}_{k,sl}[n]\mathbf{w}_g$$

The computational complexity per DMT symbol is then roughly $6N_uT$ real multiplications and 1 FFT operation, whereas a single TEQ requires NT real multiplications and 1 FFT operation and a full PTEQ needs $2N_uT$ real multiplications and 1 FFT. Note that $N \ge 2N_u$.

IV. Simulations

We compare the performance of the BM-TEQ with the PTEQ and with other TEQ designs such as the MMSE-based design with unit norm and unit tap constraint on the TIR $\mathbf{b}$ [1], [2], the maximum bitrate (MBR) and minimum-ISI approach [5] and the approximate geometric SNR (AGSNR) design (with suboptimal subchannel SNR) in [3]. For each synchronization delay Δ, the MBR optimization is initialized with the minimum-ISI TEQ for that Δ. The AGSNR design starts from the MMSE-based TEQ with norm constraint on $\mathbf{b}$; the TEQ is optimized over a number of possible settings of the parameter MSE_{max} [3], [8].

For all methods, an equalizer is designed first and then used to estimate the noise energy $\mathcal{E}\left\{|E_k[n]|^2\right\}$ on each tone by means of a 1024-point average of the square of the residual noise $E_k[n]$. The ratio of signal energy $\mathcal{E}\left\{|X_k[n]|^2\right\}$ over estimated residual noise energy yields an estimate of SNR_n needed in the bitrate calculation. The bitrates depicted below are based on constellations with an integer number of bits.

Two frequency-division duplexing ADSL simulation scenarios are presented: an upstream and downstream simulation, both with CSA4 loop. They are representative of simulations with other loops. The channel noise consists of crosstalk (near-end and far-end) and AWGN background noise. Analog and digital front-end filters to separate the upstream tones 8 to 32 and the downstream tones 38 to 256, as well as the POTS filter, have been included. These filters lengthen the overall impulse response (which can become longer than 512 samples at $f_s = 2.208$ MHz) and introduce a considerable amount of ISI/ICI near the band edges. E.g. for the CSA4 loop *without* front-end filters, there is only a 5% difference in bitrate between a "poorly" performing TEQ with 2 taps and a PTEQ with 32 taps; for the same loop *with* front-end filters, a larger number of taps is needed and the performance difference between the equalizer designs is clearer.

Although the BM-TEQ cost function (13) is nonlinear and non-convex, simulations with random initialization always converge to the same TEQ. For the studied scenarios, our approximate geometric SNR cost function (16) and bitrate optimization (13) give the same performance because of high SNR, hence only the BM-TEQ curves are shown.

Figure 1 shows the bitrate as a function of synchronization delay Δ for a TEQ of 44 taps in the *upstream* simulation. The transmit IFFT size is 512 and the receiver FFT size is 128. The real and complex BM-TEQs outperform all other TEQ algorithms[3]. Its bitrate is smooth as a function of Δ and approaches the PTEQ performance. The AGSNR approach outperforms the MMSE-based TEQ designs for most delays but, due to its suboptimal subchannel SNR, it stays away from the BM-TEQ.

[3]The minimum-ISI TEQ method is not applicable as the number of taps ($T = 44$), which is larger than the number of upstream tones ($N_u = 25$) and the cyclic prefix length ($\nu = 8$), renders matrices in the cost function rank deficient.

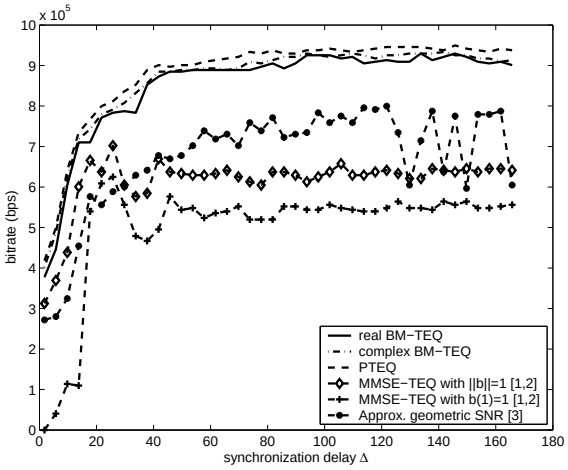

Fig. 1. Bitrate as a function of Δ for the upstream CSA4 loop.

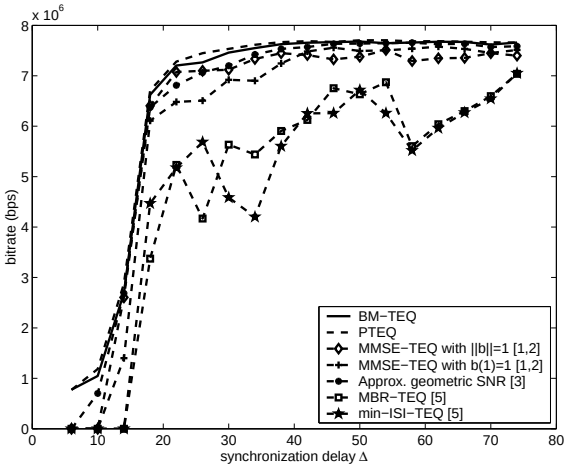

Fig. 2. Bitrate as a function of Δ for the downstream CSA4 loop.

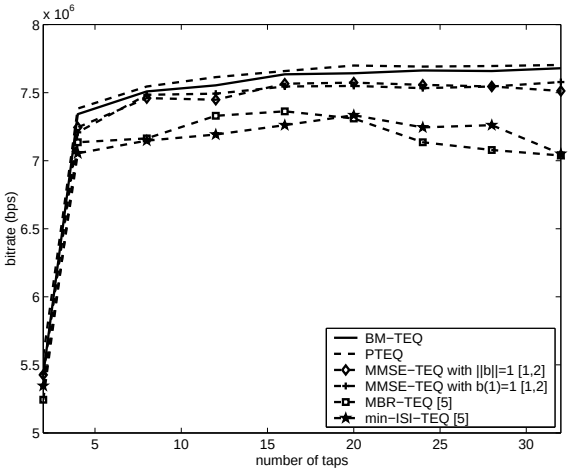

Fig. 3. Bitrate as a function of the number of TEQ taps for the downstream CSA4 loop.

Figure 2 shows the bitrate as a function of synchronization delay for a TEQ of 32 taps in the *downstream* simulation. Both IFFT and FFT have size 512. Again, the BM-TEQ outperforms the other TEQs and has almost the same performance as the PTEQ. Among the other TEQ designs, the MMSE-based TEQ and AGSNR designs perform well.

Figure 3 shows the bitrate as a function of the number of taps for the downstream scenario. The synchronization delay Δ has been optimized. Only the BM-TEQ and PTEQ perform consistently better for an increasing number of taps. The other designs do not, because they do not optimize the true bitrate.

V. Conclusions

We have presented a new nonlinear cost function for TEQ design that maximizes the bitrate (BM-TEQ). The cost function is based on the traditional bitrate expression. The SNR on each tone is calculated at the output of the FEQs. By imposing zero-forcing FEQs, we obtain a compact SNR and bitrate expression. A recursive stochastic Newton algorithm is suggested to initialize the TEQ as this typically results in fast convergence. Moreover, it allows for tracking during data transmission. The presented TEQ design is also used in a "per group" equalization scheme (PGEQ), which is intermediate (both complexity-wise and performance-wise) between TEQ and PTEQ. The PGEQ scheme designs a individual BM-TEQ for a number of tone groups. The PGEQ design then encompasses the TEQ and PTEQ design procedures as extreme cases. Simulations confirm the superior performance of the BM-TEQ, compared with other TEQs. Its performance comes close to the PTEQ performance while strongly reducing memory requirements.

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