Comments on "Confidence Images for MR Spectroscopic Imaging" by Karl Young, Dennis Khetselius, Brian J. Soher and Andrew A. Maudsley (Magnetic Resonance in Medicine 44:537-545, 2000)

Leentje Vanhamme, Philippe Lemmerling, Sabine Van Huffel
Katholieke Universiteit Leuven, Department of Electrical Engineering (SISTA-COSIC),
Kasteelpark Arenberg 10, 3001 Leuven-Heverlee, Belgium

The paper "Confidence Images for MR Spectroscopic Imaging" by Karl Young, Dennis Khetselius, Brian J. Soher and Andrew A. Maudsley (Magnetic Resonance in Medicine 44:537-545, 2000) deals with the important issue of obtaining confidence intervals for parameter estimates. However, the reasoning that leads to Equations [12], [13] and [14], Figures 2, 3 and 4 and the conclusions derived from them appear to be incorrect or misstated. The authors state "Note the perhaps nonintuitive result that Fig. 2 demonstrates, that the size of the confidence interval increases (i.e., decreased confidence and higher uncertainty) with decreasing linewidth." The conclusions drawn by the authors reflect the statistical properties of the assessment of the height of the real part of a Lorentzian in the frequency domain and not the resonance area as stated in the text. In the following we derive the CR bounds on the amplitude and the area (related to the amplitude by a factor of two) when the model consists of one Lorentzian line. We derive these CR bounds in two ways by carrying out the calculation in the time domain and in the frequency domain, giving of course identical results. Before starting, we define our notation:

- the frequency is represented by ν (and not by ω since this can be confused with the pulsation $2\pi\nu$).
- A: the time-domain amplitude.
- *area*: the total integrated area under the real part of a Lorentzian.

- d: the time-domain damping.
- L: the linewidth (the width of the Lorentzian in the frequency domain at half the peak height, $L = d/\pi$).
- σ: standard deviation of the real and imaginary part of circular complex white Gaussian noise.

Time-domain derivation. The expression for the time-domain representation of a single (continuous) Lorentzian line, of frequency 0, time-domain amplitude A and time-domain damping d is:

$$s(t) = Ae^{-dt}, \tag{1}$$

where $s(t) = 0$ for $t < 0$. The damping is assumed to be kwown and only A is estimated. To compute the CR bound of the amplitude A, the Fisher information is computed first (by taking the derivative with respect to A, squaring and integrating):

$$I = \frac{1}{\sigma^2}\int_0^\infty \left[e^{-dt}\right]^2 dt = \frac{1}{\sigma^2}\left.\frac{e^{-2dt}}{-2d}\right|_0^\infty = \frac{1}{\sigma^2 2d}$$

In terms of the linewidth L, this result becomes:

$$I = \frac{1}{\sigma^2 2L\pi}$$

Consequently the CR bound on the amplitude becomes:

$$CR(\text{amplitude}) = I^{-1} = \sigma^2 2L\pi \tag{2}$$

Since the amplitude A and the area of a Lorentzian relate as $A = 2\,area$, the CR bound on the area becomes:

$$CR(\text{area}) = \frac{\sigma^2 L\pi}{2} \tag{3}$$

Frequency-domain derivation. After Fourier Transformation of Eq. (1), the expression of the real part of the frequency-domain representation of the Lorentzian becomes:

$$s(\nu) = \frac{A}{d}\frac{1}{1+(\frac{2\pi\nu}{d})^2}$$

Again the Fisher information is computed by taking the derivative of $s(\nu)$ with respect to A, squaring and integrating:

$$\begin{aligned} I &= \frac{1}{\sigma^2 d^2}\int_{-\infty}^{\infty}\frac{1}{\left(1+(\frac{2\pi\nu}{d})^2\right)^2}d\nu \qquad \text{Put: } \frac{2\pi\nu}{d}=k \\ &= \frac{1}{\sigma^2 2\pi d}\int_{-\infty}^{\infty}\frac{dk}{(1+k^2)^2} = \frac{1}{\sigma^2 2\pi d}\left[\frac{k}{2(k^2+1)}\Big|_{-\infty}^{\infty} + \frac{1}{2}bgtg(k)\Big|_{-\infty}^{\infty}\right] \\ &= \frac{1}{\sigma^2 2\pi d}\frac{\pi}{2} = \frac{1}{\sigma^2 4d} \end{aligned}$$

Consequently the CR bound on the amplitude (assuming d is known) becomes:

$$CR(\text{amplitude}) = \sigma^2 4d = \sigma^2 4\pi L \tag{4}$$

Since the amplitude and the area of a Lorentzian relate as $A = 2\ area$, the CR bound on the area becomes:

$$CR(\text{area}) = \sigma^2 d = \sigma^2 \pi L \tag{5}$$

Conclusion. Eq. (4) and Eq. (2) (and Eq. (3) and Eq. (5)) differ by a factor 2: this is due to the fact that in the derivation of Eq. (4) only the real part of the frequency-domain representation is taken into account, as a result the CR bound is a factor of 2 larger since only half the available information is used.

It is however clear that there exists a linear relation between the linewidth and the CR bound, contradicting the results of the paper. In our opinion there is an accumulation of errors in Eqs. [12] and [13] of the paper, leading to the wrong end result:

- The authors claim that Eq. [12] in their paper is the expression for the absorption part of a Lorentzian line, of frequency ω, time domain amplitude A and linewidth L. In reality,

Eq. [12] in their paper gives the expression for a Lorentzian line with frequency 0 (and not ω) and furthermore the expression is not completely correct.

- The authors claim that Eq. [13] in their paper is obtained by taking the derivative with respect to the area but the derivative is taken with respect to A/L (L is assumed to be constant) which is certainly not the area.

We want to conclude by pointing out that the CR bound of the maximum value of the real part of the Lorentzian in the frequency domain (the height = A/d) is proportional to $1/d$ (or $1/L$). The CR bound on the peak height, assuming that only the parameter A is estimated in the time domain, is given by:

$$CR(\text{peak height}) = \frac{CR(\text{amplitude})}{d^2} = \frac{2\sigma^2}{d} = \frac{2\sigma^2}{\pi L}$$