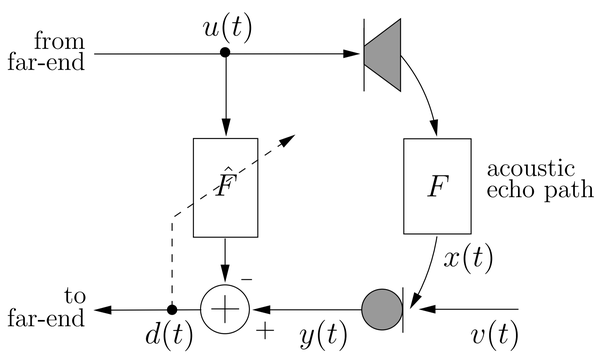
MSE Optimal Regularization of APA and NLMS Algorithms in Room Acoustic Applications

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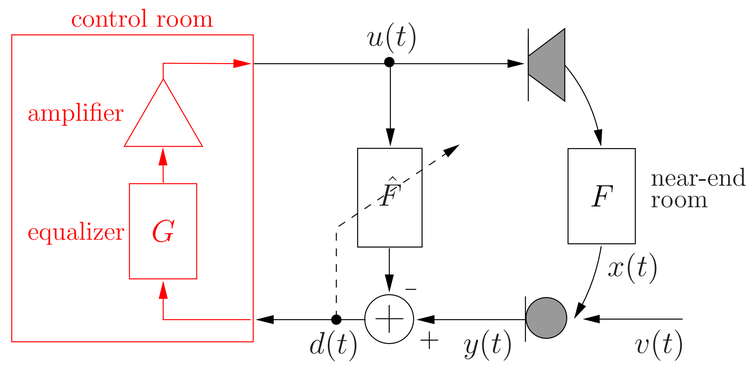
1 Introduction

Acoustic Echo Cancellation



- True room impulse response (RIR) coefficient vector $\mathbf{f} = \begin{bmatrix} f_0 & f_1 & \dots & f_{n_F} \end{bmatrix}^T$
- Estimated RIR coefficient vector $\hat{\mathbf{f}}(t) = \begin{bmatrix} \hat{f}_0(t) & \hat{f}_1(t) & \dots & \hat{f}_{n_F}(t) \end{bmatrix}^T$
- Loudspeaker signal vector $\mathbf{u}(t) = \begin{bmatrix} u(t) & u(t-1) & \dots & u(t-n_F) \end{bmatrix}^T$
- Echo- or feedback-compensated signal $d(t) = y(t) - \mathbf{u}^T(t)\hat{\mathbf{f}}(t)$
- APA loudspeaker signal matrix $\mathbf{U}(t) = \begin{bmatrix} \mathbf{u}(t) & \dots & \mathbf{u}(t-M+1) \end{bmatrix}$
- APA microphone signal vector $\mathbf{y}(t) = \begin{bmatrix} y(t) & \dots & y(t-M+1) \end{bmatrix}^T$

Adaptive Feedback Cancellation



2 MSE Optimal APA and NLMS Algorithms

Mean Square Error (MSE) criterion

$$\min_{\hat{\mathbf{f}}(t)} E\Big\{ [\hat{\mathbf{f}}(t) - \mathbf{f}]^T [\hat{\mathbf{f}}(t) - \mathbf{f}] \Big\}$$

Traditional framework | **Bayesian framework**

Statistical assumptions

- true RIR **deterministic**
- near-end signal drawn from a **stochastic** process with $\begin{cases} E\{\mathbf{v}\} = \mathbf{0} \\ \text{cov}\{\mathbf{v}\} = E\{\mathbf{v}\mathbf{v}^T\} = \alpha\mathbf{I} \end{cases}$

- true RIR drawn from a **stochastic** process with $\begin{cases} E\{\mathbf{f}\} = \hat{\mathbf{f}}(t-1) \\ \text{cov}\{\mathbf{f}\} = E\{(\mathbf{f} - E\{\mathbf{f}\})(\mathbf{f} - E\{\mathbf{f}\})^T\} = \mathbf{R_f} \end{cases}$
- near-end signal drawn from a **stochastic** process with $\begin{cases} E\{\mathbf{v}\} = \mathbf{0} \\ \text{cov}\{\mathbf{v}\} = E\{\mathbf{v}\mathbf{v}^T\} = \mathbf{R_v} \end{cases}$

Equivalent least-squares criterion

$$\min_{\hat{\mathbf{f}}(t)} \Big\{ [\mathbf{y}(t) - \mathbf{U}^T(t)\hat{\mathbf{f}}(t)]^T [\mathbf{y}(t) - \mathbf{U}^T(t)\hat{\mathbf{f}}(t)] + \alpha \|\hat{\mathbf{f}}(t)\|_2^2 \Big\}$$

$$\min_{\hat{\mathbf{f}}(t)} \Big\{ [\mathbf{y}(t) - \mathbf{U}^T(t)\hat{\mathbf{f}}(t)]^T \mathbf{R_v}^{-1}(t) [\mathbf{y}(t) - \mathbf{U}^T(t)\hat{\mathbf{f}}(t)] + [\hat{\mathbf{f}}(t) - \hat{\mathbf{f}}(t-1)]^T \mathbf{R_f}^{-1} [\hat{\mathbf{f}}(t) - \hat{\mathbf{f}}(t-1)] \Big\}$$

Resulting APA and NLMS algorithms

APA - Affine Projection Algorithm

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mu \mathbf{U}(t) [\mathbf{U}^T(t)\mathbf{U}(t) + \alpha\mathbf{I}]^{-1} \boldsymbol{\varepsilon}(t),$$

$$\boldsymbol{\varepsilon}(t) = \mathbf{y}(t) - \mathbf{U}^T(t)\hat{\mathbf{f}}(t-1).$$

NLMS - Normalized Least Mean Squares

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mu \frac{\mathbf{u}(t)\varepsilon(t)}{\mathbf{u}^T(t)\mathbf{u}(t) + \alpha},$$

$$\varepsilon(t) = y(t) - \mathbf{u}^T(t)\hat{\mathbf{f}}(t-1).$$

LMR-APA - Levenberg-Marquardt Regularized APA

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mu \mathbf{R_f} \mathbf{U}(t) [\mathbf{U}^T(t)\mathbf{R_f}\mathbf{U}(t) + \mathbf{R_v}(t)]^{-1} \boldsymbol{\varepsilon}(t),$$

$$\boldsymbol{\varepsilon}(t) = \mathbf{y}(t) - \mathbf{U}^T(t)\hat{\mathbf{f}}(t-1).$$

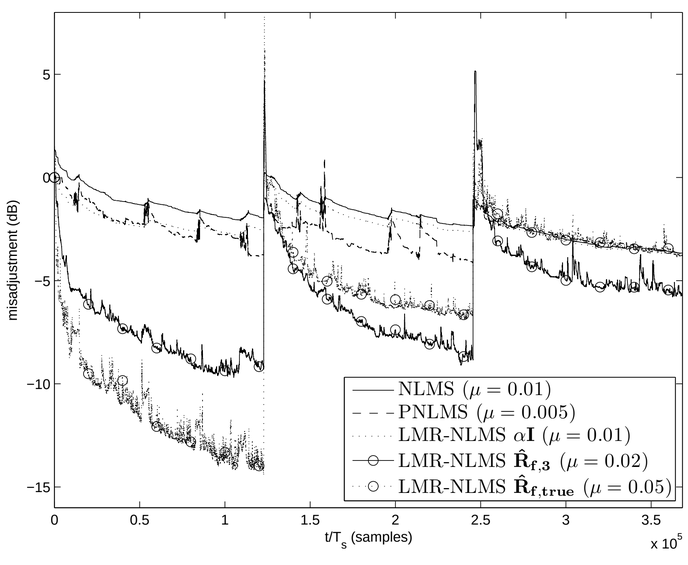
LMR-NLMS - Levenberg-Marquardt Regularized NLMS

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mu \frac{\mathbf{R_f}\mathbf{u}(t)\varepsilon(t)}{\mathbf{u}^T(t)\mathbf{R_f}\mathbf{u}(t) + \sigma_v^2(t)},$$

$$\varepsilon(t) = y(t) - \mathbf{u}^T(t)\hat{\mathbf{f}}(t-1).$$

3 Simulation Results

Acoustic Echo Cancellation



1st RIR change = mic position change (maintaining distance to speaker)
2nd RIR change = RT60 change
FE signal = speech, NE signal = GWN
PNLMS = Proportionate NLMS [Duttweiler, 2000]

LMR $\alpha\mathbf{I}$ = scaled identity regularization matrix, α = NE signal power
LMR $\hat{\mathbf{R}}_{\mathbf{f},3}$ = diagonal estimate of $\mathbf{R_f}$ based on 3-parameter RIR model
LMR $\hat{\mathbf{R}}_{\mathbf{f},\mathbf{true}}$ = diagonal estimate of $\mathbf{R_f}$ based on true RIR

LMR-NLMS includes prefiltering with AR(12) prediction error filter
NE signal = speech

Adaptive Feedback Cancellation

