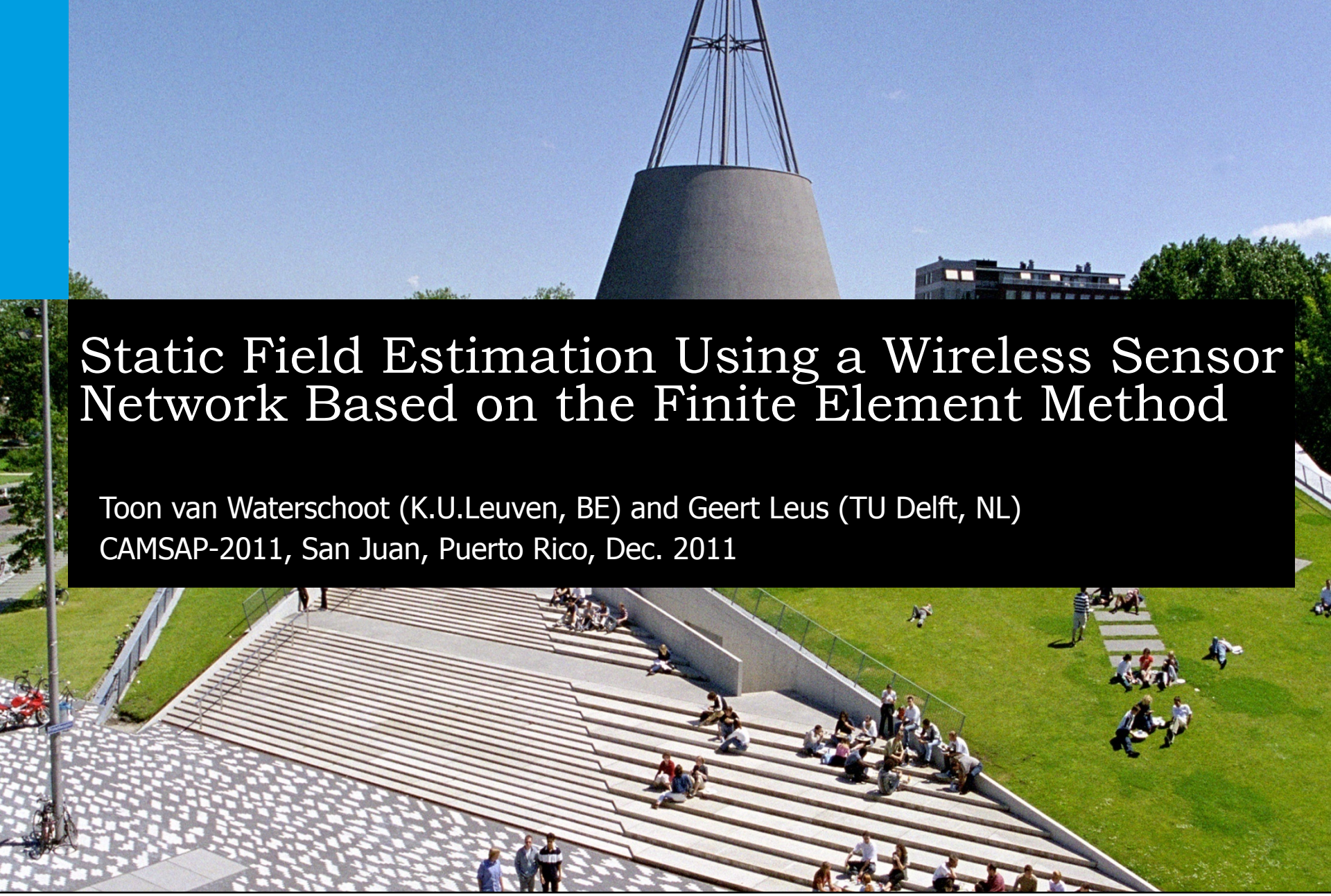

Static Field Estimation Using a Wireless Sensor Network Based on the Finite Element Method

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CAMSAP-2011, San Juan, Puerto Rico, Dec. 2011

Outline

- Introduction
  - Problem statement
  - Toy example
- Cooperative field estimation
  - Finite element method (FEM)
  - Centralized estimation approach
  - Distributed estimation approach
- Simulation results
- Conclusion & future work

1.

Introduction

Introduction (1)

Problem statement (1)

- Goal: field estimation in wireless sensor networks using a physical model
- **Field:** physical phenomenon that varies over space/time
- **Wireless Sensor Network (WSN):** collection of spatially distributed sensor nodes capable of measuring, sampling, processing, and communicating
- **Physical model:** partial differential equation (PDE) subject to boundary/initial conditions
  - First-order PDE
  - Second-order PDE: elliptic/parabolic/hyperbolic

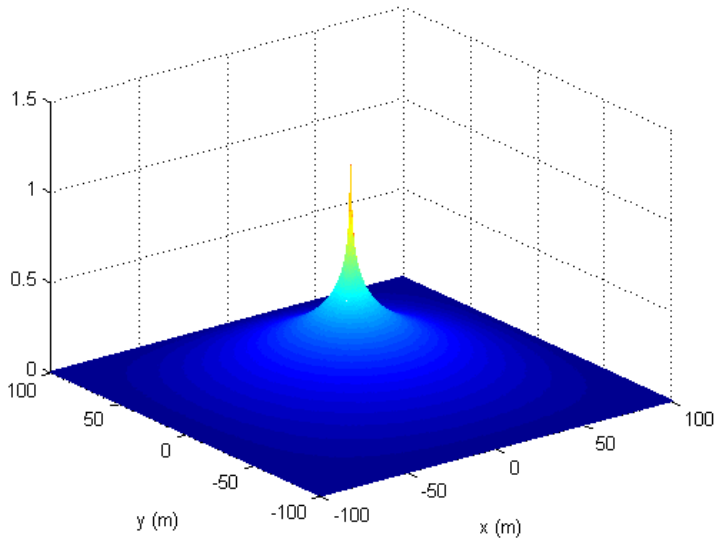
Introduction (2)

Problem statement (2)

- Goal: field estimation in wireless sensor networks using a physical model
- Motivation: to combine the strengths of data-driven and model-based approach
- **Data-driven:** WSN = spatiotemporal sampling device, but subject to aliasing, measurement noise, …
- **Model-based:** PDE = spatiotemporal "glue" between samples, but subject to modeling errors and uncertainty

Introduction (3)

Toy example



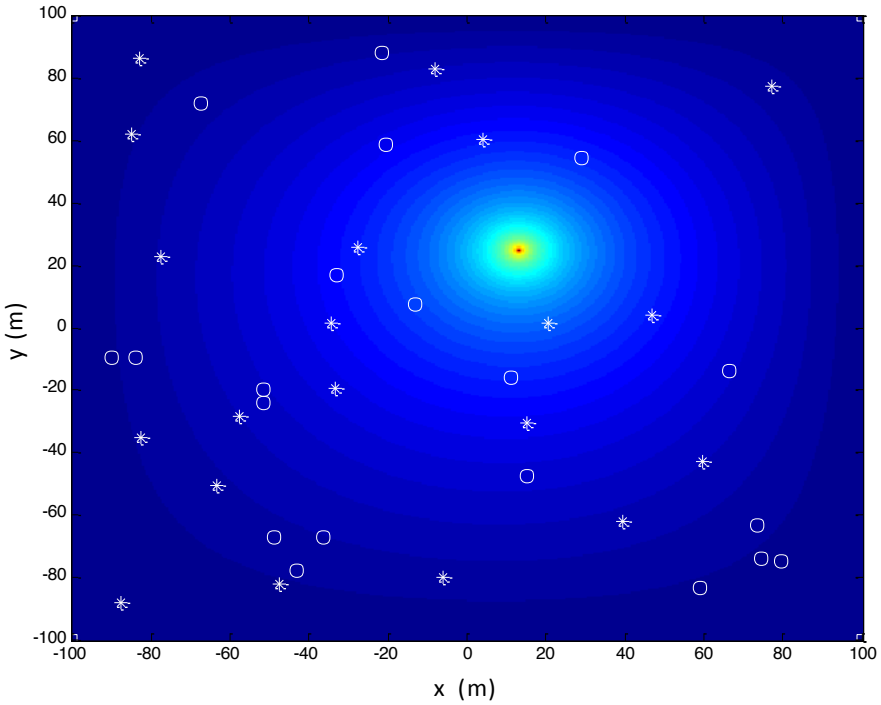
- **Field:** 0-BC static 2-D Poisson PDE

$$-\nabla^2 u(x,y) = s(x,y)$$

- **Domain:** 200 m x 200 m
- **Source:** point source at (13,25)
- **WSN:** J=20 sensor nodes (o), N=20 measurements per node

$$\mathbf{v}_j = \begin{bmatrix} v_j^{(1)} \\ \vdots \\ v_j^{(N)} \end{bmatrix} = u(x_j, y_j)\mathbf{1}_{N\times 1} + \begin{bmatrix} w_j^{(1)} \\ \vdots \\ w_j^{(N)} \end{bmatrix}$$

- **Challenge:** field estimation at 20 sensor node positions (o) + 20 points of interest (*)



2.

Cooperative Field Estimation

Cooperative Field Estimation (1)

Finite Element Method (1)

- FEM: 4-step procedure to discretize boundary value problem

1. Weak formulation of boundary value problem

$$-\nabla^2 u(x,y) = s(x,y) \Rightarrow \int_\Omega \left[\nabla^2 u(x,y) + s(x,y)\right] g(x,y)dxdy = 0$$

2. Integration by parts to relax differentiability requirements

$$\int_\Omega \nabla u(x,y) \cdot \nabla g(x,y)dxdy = \int_\Omega s(x,y)g(x,y)dxdy$$

3. Subspace approximation of field and source functions

$$\tilde{u}(x,y) = \sum_{k=1}^{K_\Omega} u_k \phi_k(x,y), \quad \tilde{s}(x,y) = \sum_{k=1}^{K_\Omega} s_k \phi_k(x,y)$$

4. Enforce orthogonality of approximation error to subspace

$$\mathbf{Au} = \mathbf{Bs}$$

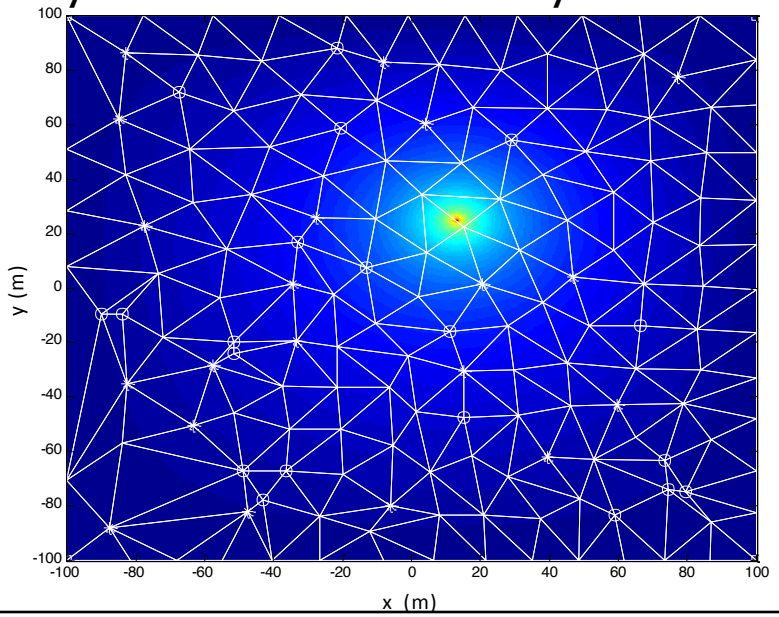
Stiffness matrix $[\mathbf{A}]_{ij} = \int_\Omega \nabla\phi_j(x,y) \cdot \nabla\phi_i(x,y)dxdy$

Mass matrix $[\mathbf{B}]_{ij} = \int_\Omega \phi_j(x,y)\phi_i(x,y)dxdy$

Cooperative Field Estimation (2)

Finite Element Method (2)

- Choice of nodes, elements, and basis functions: discretization must be simple, accurate, well-conditioned, and sparse
  - Choose a "good" mesh (high resolution, well-shaped elements)
  - Choose piecewise linear basis functions with small spatial support
  - Omit boundary elements from FEM system of equations



Cooperative Field Estimation (3)

Centralized estimation approach

- WSN measurements + FEM: underdetermined problem

$$\min_{\mathbf{u},\mathbf{s}} \quad \sum_{j=1}^{J} \|\mathbf{v}_j - u_j \mathbf{1}_{N\times 1}\|_2^2 \quad \text{s. t.} \quad \mathbf{A}\mathbf{u} = \mathbf{B}\mathbf{s}$$

- WSN measurements + FEM + prior knowledge: convex problem

$$\min_{\mathbf{u},\mathbf{s}} \quad \sum_{j=1}^{J} \|\mathbf{v}_j - u_j \mathbf{1}_{N\times 1}\|_2^2 + \lambda\|\mathbf{s}\|_1$$

$$\text{s. t.} \quad \mathbf{A}\mathbf{u} = \mathbf{B}\mathbf{s}, \ \mathbf{u} \geq \mathbf{0}_{K\times 1}, \ \mathbf{s} \geq \mathbf{0}_{K\times 1}$$

→ Sparsity of point source

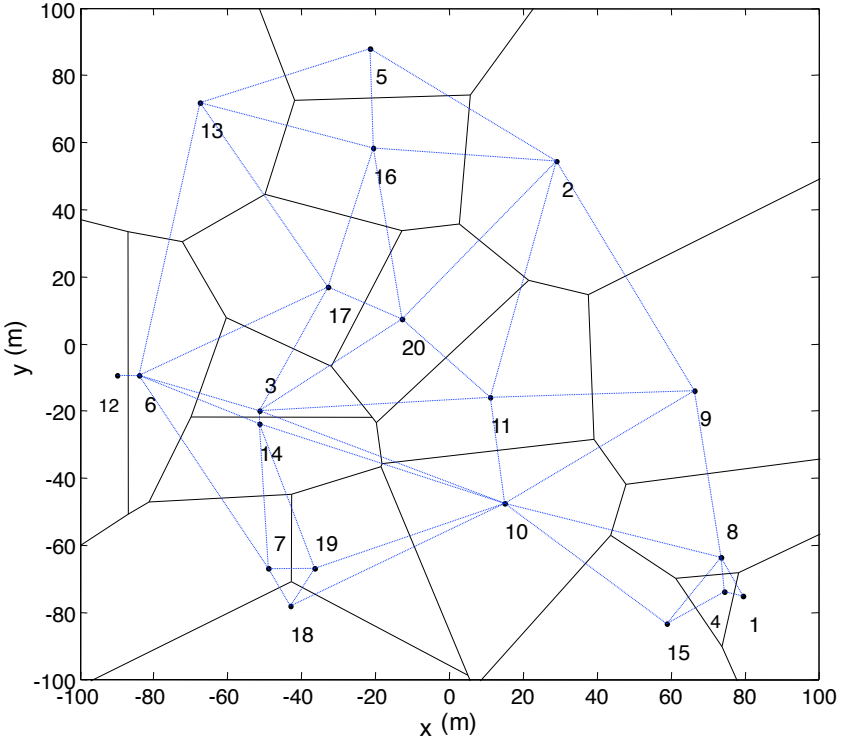
→ Nonnegativity of field/ source

- **Procedure:** fusion center collects all WSN measurements and performs centralized estimation

Cooperative Field Estimation (4)

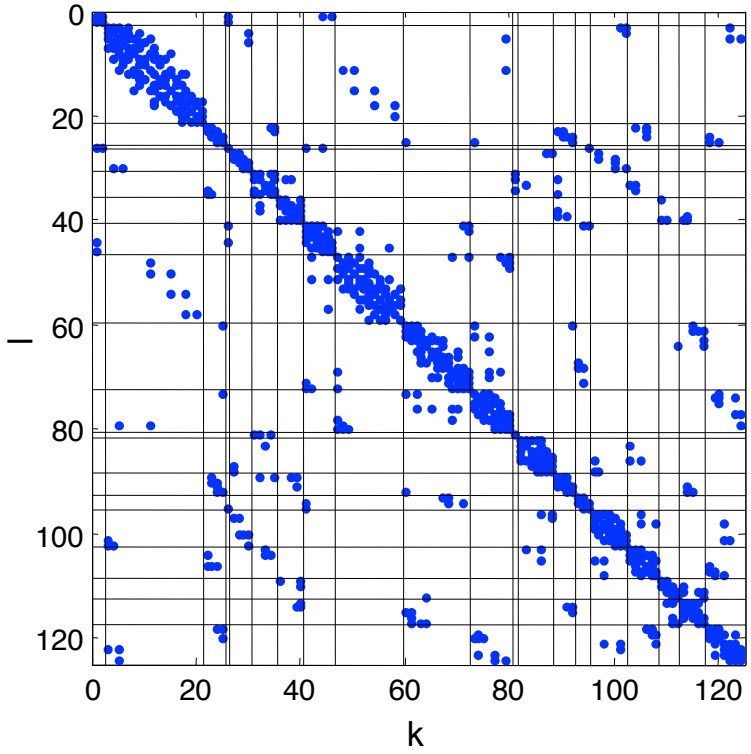
Distributed estimation approach (1)

- Clustering of FEM nodes = partitioning of field and source vector, and of stiffness and mass matrices



$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_J \end{bmatrix}$$

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_J \end{bmatrix}$$



Cooperative Field Estimation (5)

Distributed estimation approach (2)

- Separation of optimization problem into J subproblems

Fully separable

Partially separable

$$\min_{\mathbf{u},\mathbf{s}} \quad \sum_{j=1}^{J} \|\mathbf{v}_j - u_j \mathbf{1}_{N\times 1}\|_2^2 + \lambda \|\mathbf{s}\|_1$$

$$\text{s. t.} \quad \mathbf{A}\mathbf{u} = \mathbf{B}\mathbf{s}, \quad \mathbf{u} \geq \mathbf{0}_{K\times 1}, \quad \mathbf{s} \geq \mathbf{0}_{K\times 1}$$

- **Exact separation** of equality constraints:
  - in j-th subproblem, consider all equality constraints where j-th block column of A,B contains non-zero elements
  - requires excessive communication between WSN nodes
- **Approximate separation** of equality constraints:
  - in j-th subproblem, consider only equality constraints corresponding to j-th block row of A,B
  - sparsity of A,B can be exploited to reduce communication

3.

Simulation Results

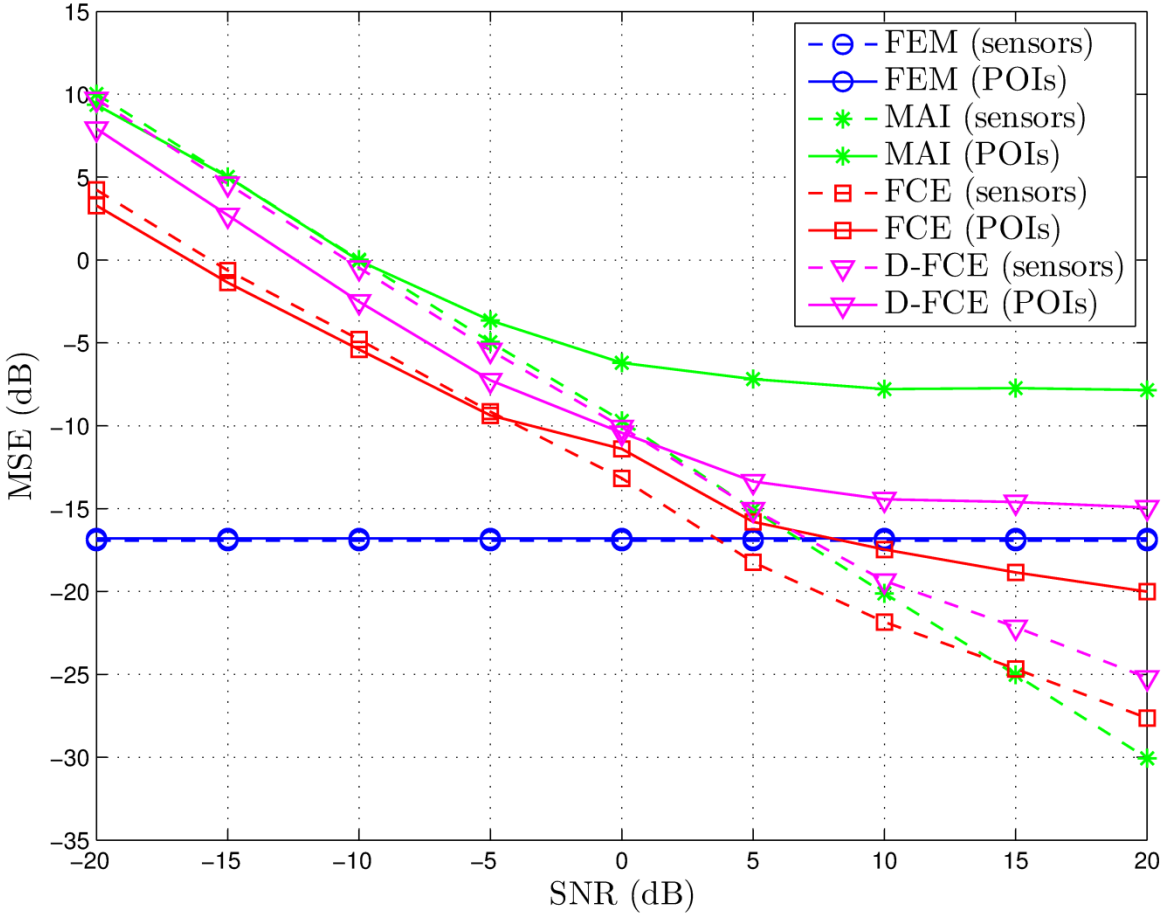
Simulation Results (1)

Simulation setup

- Toy example with N=10, SNR = 0 dB, mesh resolution = 20 m
- Benchmark algorithms:
  - Model-based: FEM with known source vector
  - Data-driven: Measurement averaging + interpolation (MAI)
- Proposed algorithms: model-based + data-driven
  - FCE: FEM-constrained cooperative estimation with sparsity and nonnegativity prior
  - D-FCE: FEM-constrained distributed estimation with sparsity and nonnegativity prior + approximately separated equality constraints
- Performance measure: mean squared relative field estimation error (MSE) at sensor node (--) and POI (–) locations

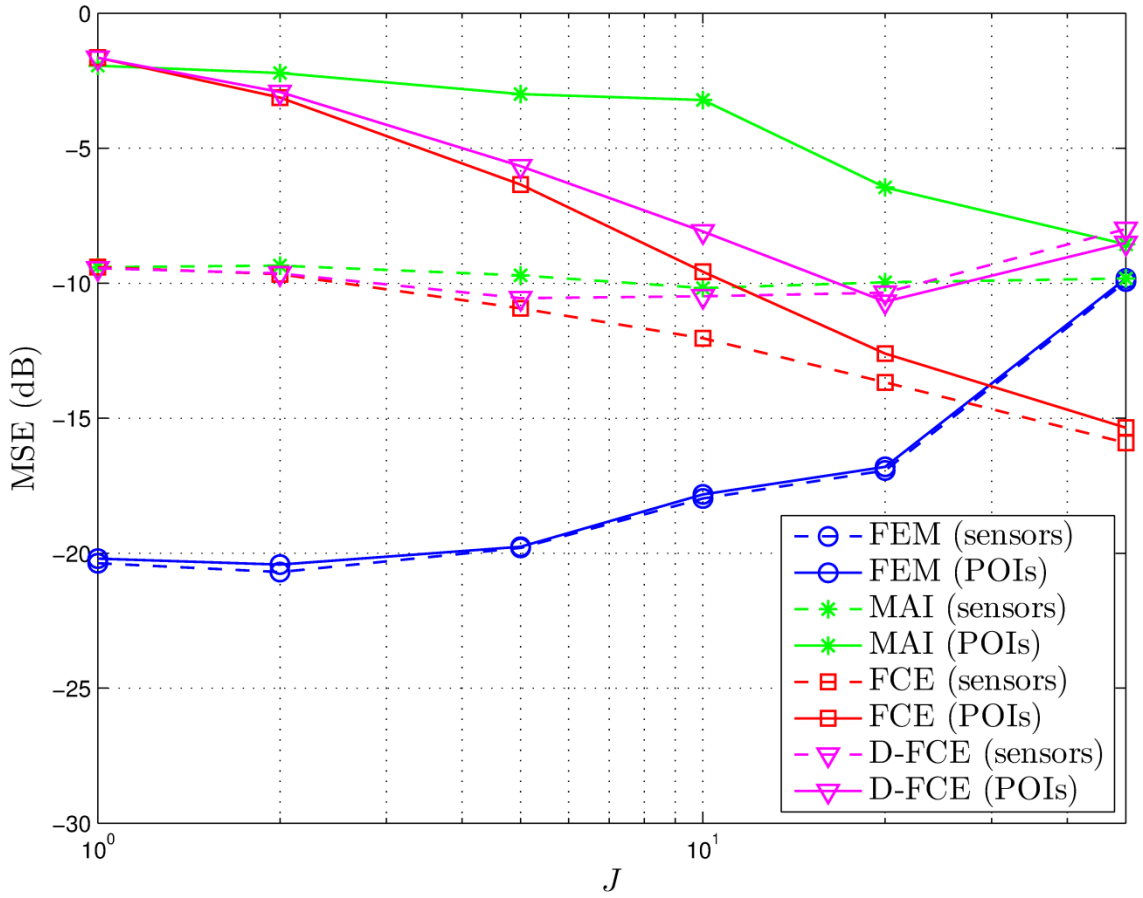
Simulation Results (2)

MSE vs. SNR



Simulation Results (3)

MSE vs. number of WSN nodes J



4.

Conclusion & Future Work

Conclusion & Future Work

Conclusion

- Novel framework for data-driven + model-based field estimation
- Cooperative field estimation algorithms: centralized/distributed approach
- Proposed algorithms consistently outperform data-driven algorithm, and in some scenarios even perform better than model-based approach

Future work

- Extension to 3-D and dynamic boundary value problems
- Inverse problems: estimation of source function based on field measurements
- Localization of WSN nodes and point sources