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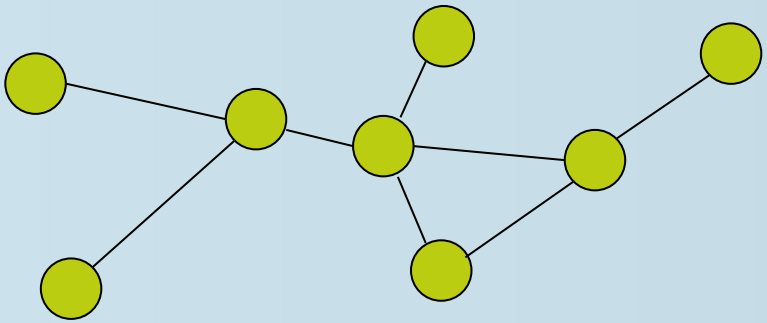
Distributed Optimization Problems in Signal Processing

Toon van Waterschoot (KU Leuven, ESAT-SCD)

OPTEC MANET Workshop on Tensors and Large Scale Optimization
KU Leuven – Feb. 7, 2012

Wireless Sensor Networks

- Wireless Sensor Networks (WSNs) require advanced signal processing algo's based on distributed optimization
- WSN = network of spatially distributed nodes with local sensing, processing, and communication capabilities

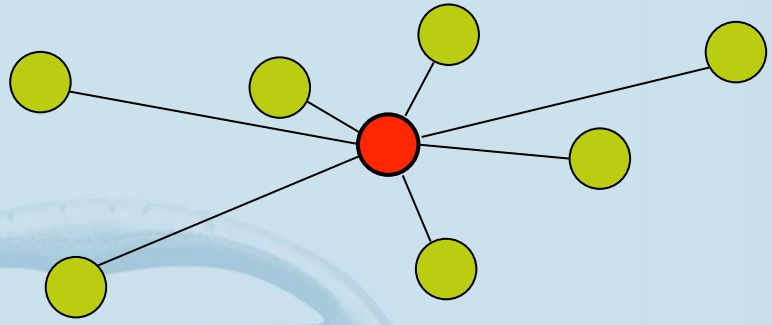


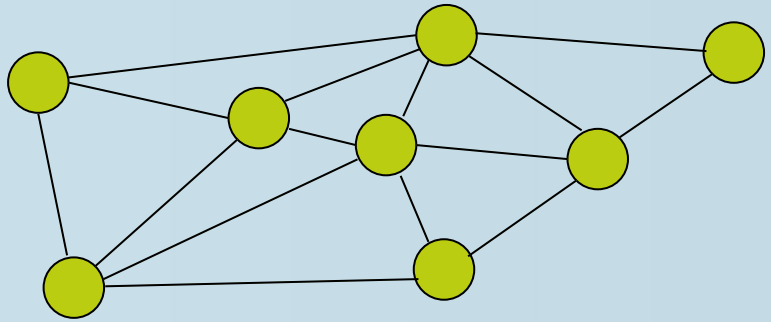
- WSN applications:
  - cognitive radio: distributed spectrum estimation, distributed detection of licensed users, …
  - speech enhancement: distributed estimation of beamformers & Wiener filters, distributed adaptive filtering, …
  - environmental monitoring: distributed estimation of dynamic fields, distributed source detection and localization, …
  - ...

WSN Topologies

- Centralized vs. Distributed Processing:
  - centralized processing: WSN nodes communicate measurements to fusion center, where global optimization problem is solved
  - distributed processing: WSN nodes solve local optimization problems and mutually exchange measurements/parameters/…

    = preferred processing mode (autonomy, bandwidth, power, …)





- WSN Connectivity:
  - fully connected: all WSN nodes can communicate with one another
  - partially connected: some missing connections, no isolated nodes
  - ad-hoc: partially connected without any topology assumptions

Connected Estimation Problems

- Connected Estimation Problems:
  - each WSN node has local estimation problem, with objective and/or constraints that depend on parameter vector of other (neighboring) problems, e.g.,

$$\min_{\mathbf{x}_j} \|\mathbf{A}_{jj}\mathbf{x}_j - \mathbf{b}_j(\mathbf{x}_i)\|, \;\; j = 1, ..., J, \;\; i \in \mathcal{N}_j$$

  - often results from spatial separation of global estimation problem, where global parameter vector represents spatially sampled signal

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\| \qquad \mathbf{b}_j(\mathbf{x}_i) = \sum_{i \in \mathcal{N}_j} \mathbf{A}_{ji}\mathbf{x}_i + \mathbf{b}_j$$

- Solution Strategy: Block Coordinate Descent
  - iterative procedure with sequential updating and forwarding of local parameter vector estimates
  - convergence to solution of global problem under certain conditions

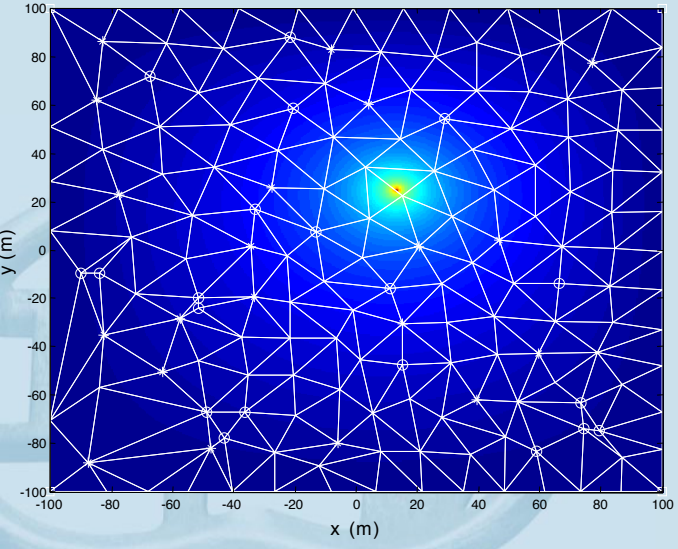
[P. Tseng, "Convergence of a block coordinate descent method for nondifferentiable minimization," *J. Optimiz. Theory App.*, vol. 109, no. 3, pp. 475–494, June 2001.]

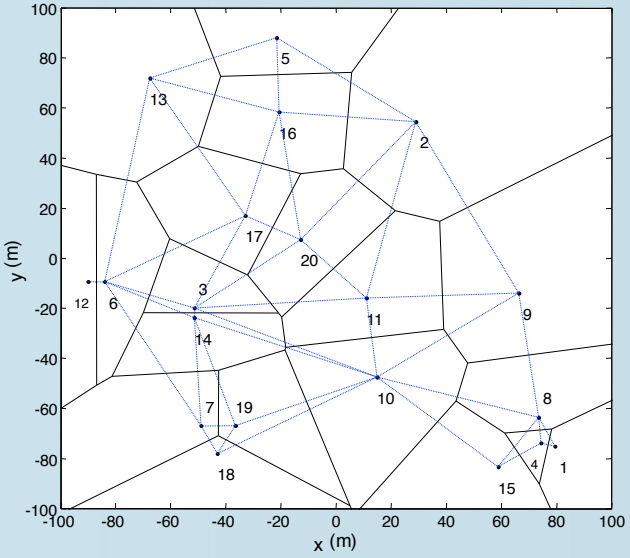
Connected Estimation Problems

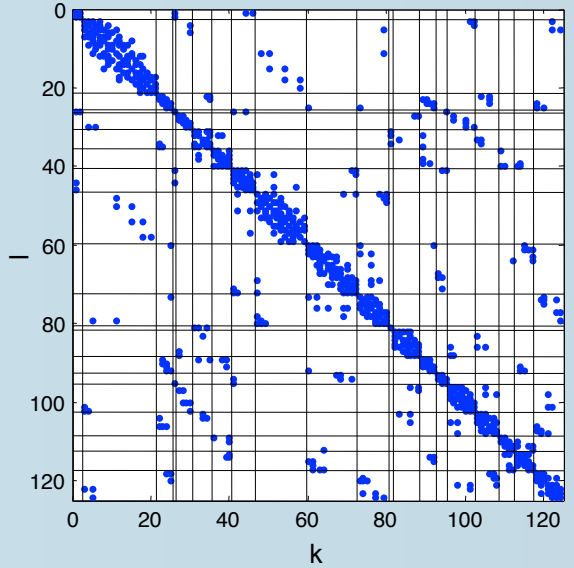
- Connected Estimation Problems: Example
  - distributed estimation of wave fields on finite element (FE) grid
  - WSN measurements available in (small) subset of FE grid points
  - global optimization problem:

$$\min_{\mathbf{u},\mathbf{s}} \sum_{j=1}^{J} \|\mathbf{v}_j - u_j \mathbf{1}_{N\times 1}\|_2^2 + \lambda \|\mathbf{s}\|_1 \text{ s. t. } \boxed{\mathbf{A}\mathbf{u} = \mathbf{B}\mathbf{s},} \ \mathbf{u} \geq \mathbf{0}_{K\times 1}, \ \mathbf{s} \geq \mathbf{0}_{K\times 1}$$

Discretized PDE connects local subproblems







[T. van Waterschoot and G. Leus, "Distributed estimation of static fields in wireless sensor networks using the finite element method", to appear in *Proc. 2012 IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP '12)*, Kyoto, Japan, Mar. 2012.]

Consensus Problems

- Consensus Problems:
  - each WSN node solves local estimation problem w.r.t. global parameter vector, and should reach consensus with other nodes

$$\min_{\mathbf{x}} \|\mathbf{A}_j\mathbf{x} - \mathbf{b}_j\|, \ j = 1, ..., J$$

  - consensus requirement can be formulated using equality constraints on the local parameter vector estimates

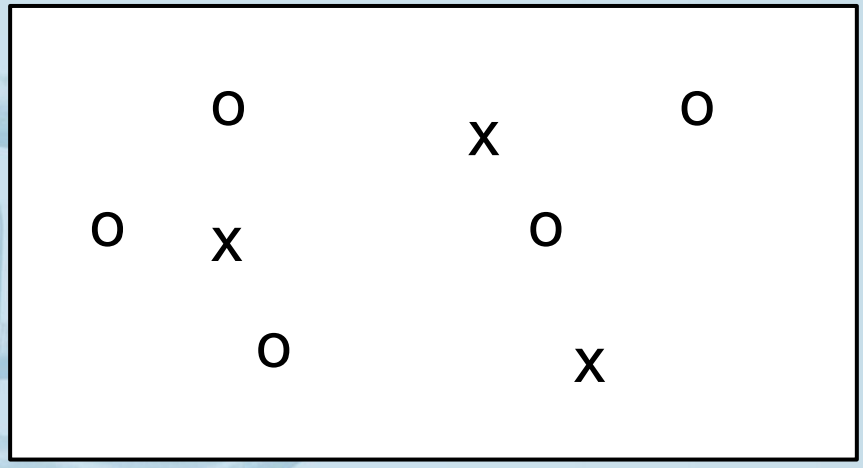
$$\min_{\mathbf{x}_j} \|\mathbf{A}_j\mathbf{x}_j - \mathbf{b}_j\| \text{ s.t. } \mathbf{x}_j = \mathbf{x}, \ j = 1, ..., J$$

- Solution Strategy: ADMM
  - Alternating-Direction Method of Multipliers (ADM(o)M): iterative procedure with sequential updating and forwarding of estimates of local and global parameter vectors and Lagrange multipliers
  - requires fusion center or "bridge" nodes to update and communicate global parameter vector estimate

[I. D. Schizas, A. Ribeiro, and G. B. Giannakis, "Consensus in Ad Hoc WSNs with Noisy Links - Part I: Distributed Estimation of Deterministic Signals," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 350-364, Jan. 2008.]

Consensus Problems

- Consensus Problems: Example
  - black-box identification of MIMO wave propagation model
  - all-pole model captures resonant behavior common to all pairs of input-output signals: consensus problem
  - pole-zero model captures resonant behavior common to all pairs of input-output signals and anti-resonances specific to each input-output pair: combined consensus and connected estimation problem



x = sources (input)

o = WSN nodes (output)

— = reflective boundaries

[T. van Waterschoot, M. Diehl, M. Moonen, and G. Leus, "Identification of black-box wave propagation models using large-scale convex optimization", to appear in *Proc. 16th IFAC Symp. System Identification (SYSID '12),* Brussels, Belgium, Jul. 2012.]