KATHOLIEKE UNIVERSITEIT LEUVEN

SISTA Seminar 21/08/08

A First-Order Frequency-Warped Sigma Delta Modulator with Improved Signal-to-Noise Ratio*

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* This is joint work with Ludovico Ausiello (ARCES, University of Bologna, Italy)



Outline

- Introduction: Sigma Delta Modulation (SDM)
  - A/D conversion
  - SDM structure & mathematical model
  - SDM properties
- Frequency-warped SDM design
  - motivation
  - design procedure
- First-order frequency-warped SDM
  - first-order frequency-warped SDM design
  - SNR analysis
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- Conclusion & Ongoing work



Introduction: SDM (1/3)

- Analog-to-digital (A/D) conversion

traditional A/D conversion = Nyquist sampling + multi-bit quantization

oversampling: spreading the quantization noise power in a larger frequency range

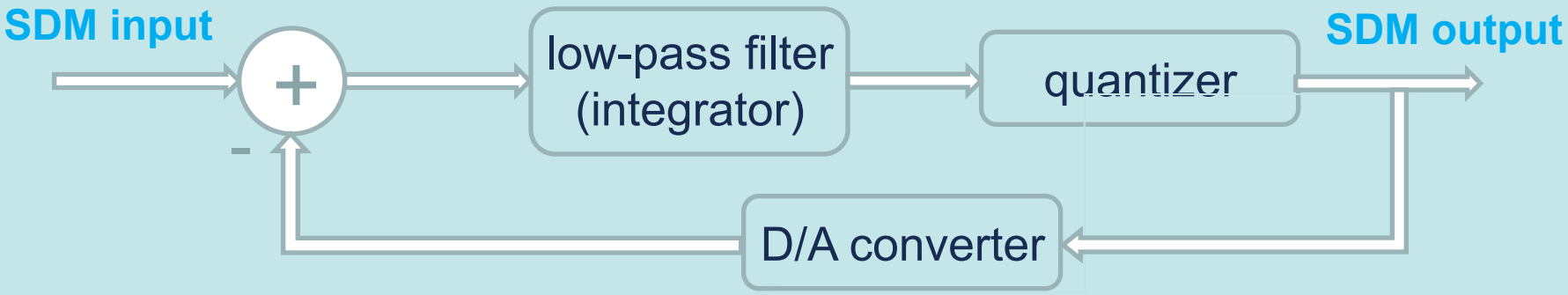
noise shaping: pushing the quantization noise into high-frequency range

SDM A/D conversion = oversampling + noise shaping + 1-bit/multi-bit quantization

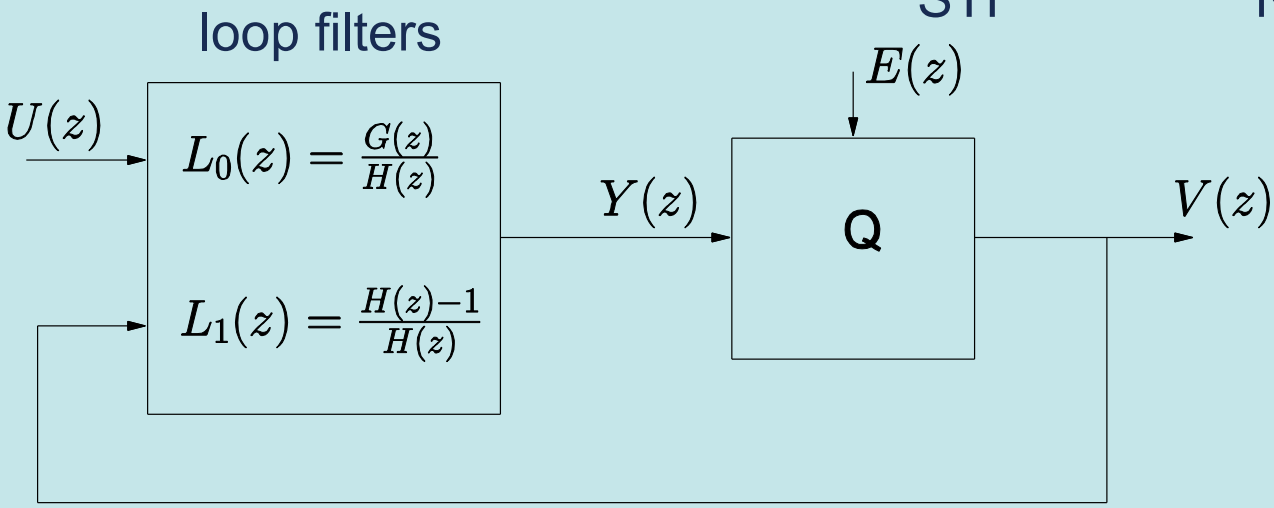
- Key SDM advantages
  - high resolution (quantified as signal-to-quantization noise ratio)
  - relatively low-precision hardware required

Introduction: SDM (2/3)

- SDM structure:



- SDM mathematical model: $V(z) = G(z)U(z) + H(z)E(z)$
  STF NTF





Introduction: SDM (3/3)

- SDM properties
  - first-order vs. high-order modulator filters
    - first-order filters tend to have better **stability** properties
    - first-order filters are very easy to **design**
    - high-order filters allow for a more selective **noise shaping**
    - high-order filters can achieve similar SNR at reduced OSR, hence paving the way for **wideband** applications (e.g., video coding)
  - 1-bit vs. multi-bit
    - feedback loop DAC is perfectly **linear** in 1-bit case
    - SDM filter **hardware** is simpler in 1-bit case
    - multi-bit SDM can achieve better **resolution** at same OSR
    - linearized SDM **model** is more accurate in multi-bit case

We will only consider first-order 1-bit oversampling SDM



Frequency-warped SDM (1/2)

- Motivation
  - incorporate psychoacoustics in audio conversion SDM design
    - if the SDM low-pass filter is preceded by a frequency-warping operation, and the warping parameter is properly chosen (>0), then a better **frequency resolution** in the audio signal band is obtained
    - previous approaches to psychoacoustic SDM design are based on fixed/adaptive **psychoacoustic models** with high-order SDM filters, see [Dunn and Sandler, 1997] and [De Koning and Verhelst, 2003]
  - increase SNR performance while avoiding stability issues typical to high-order SDM
    - frequency-warped SDM can be designed to provide a **boost in the signal band** without amplifying the quantization noise
    - frequency-warped SDM tend to have a **lower out-of-band gain** (= NTF magnitude response at Nyquist frequency) for positive warping parameters, which is beneficial to SDM stability



Frequency-warped SDM (2/2)

- Design procedure:

Step 1: design a non-warped NTF using any traditional method

↓

Step 2: calculate the warped NTF using the bilinear all-pass transform

↓

Step 3: scale the warped NTF such that it has a monic impulse response

↓

Step 4: decide in which way the warping should affect the STF

↓

Step 5: evaluate the warped NTF and STF & choose the warping parameter



First-order warped SDM (1/8)

- Design procedure:
  - Step 1: design a non-warped NTF using any traditional method

$$H(z) = 1 - z^{-1}$$ (pure differentiator)

  - Step 2: calculate warped NTF using bilinear all-pass transform

$$z^{-1} \mapsto \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}} \Rightarrow H(z,\lambda) = \frac{(1+\lambda)(1 - z^{-1})}{1 - \lambda z^{-1}}$$

  - Step 3: scale warped NTF to have monic impulse response

$$\widetilde{H}(z,\lambda) = \frac{1 - z^{-1}}{1 - \lambda z^{-1}}$$

The corresponding warped L1 loop filter is

$$\widetilde{L}_1(z,\lambda) = \frac{\widetilde{H}(z,\lambda) - 1}{\widetilde{H}(z,\lambda)} = -(1-\lambda)\frac{z^{-1}}{1 - \lambda z^{-1}}$$

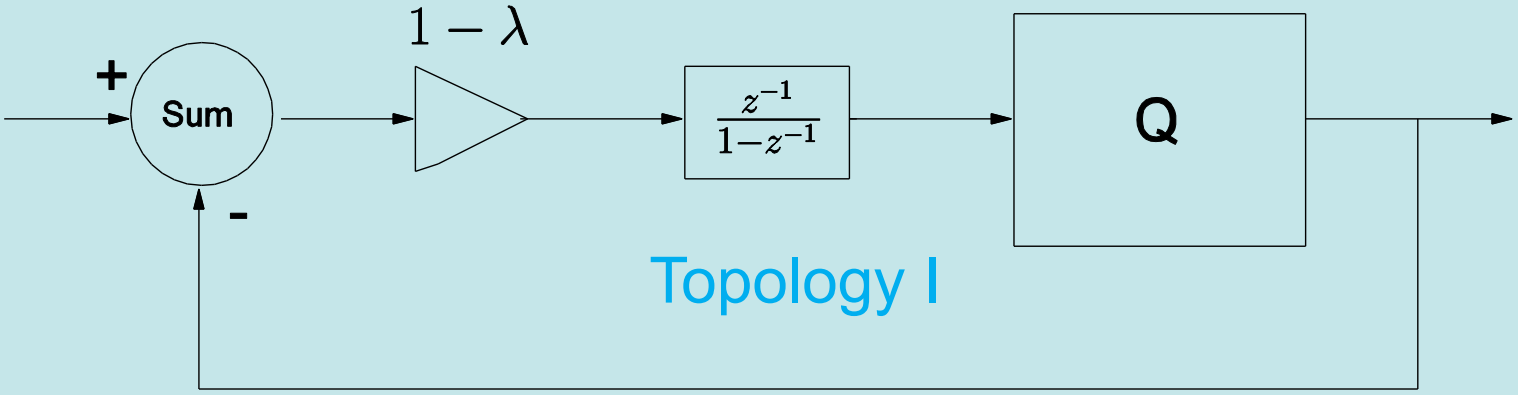


First-order warped SDM (2/8)

- Design procedure:
  - Step 4: decide in which way the warping should affect the STF
    - constrain warped loop filters to obey the same relationship as in the non-warped case:

$$L_0(z) = -L_1(z) \Rightarrow \tilde{L}_0(z,\lambda) = -\tilde{L}_1(z,\lambda) = (1-\lambda)\frac{z^{-1}}{1-z^{-1}}$$

$$\Leftrightarrow \tilde{G}(z,\lambda) = (1-\lambda)\frac{z^{-1}}{1-\lambda z^{-1}}$$



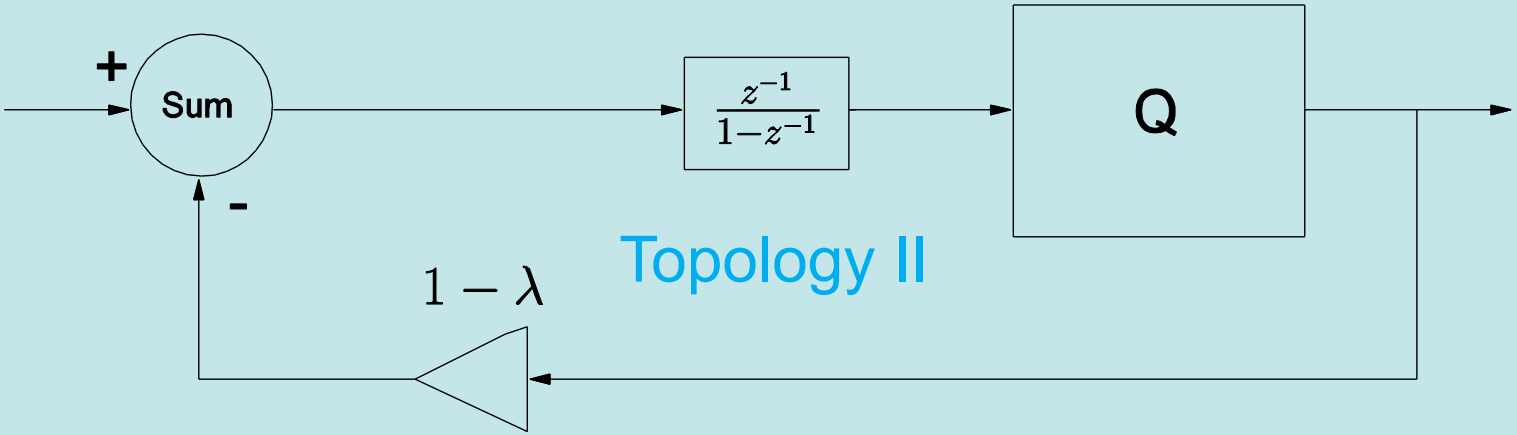
Topology I



First-order warped SDM (3/8)

- Design procedure:
  - Step 4: decide in which way the warping should affect the STF
    - constrain warped L0 loop filter to be equal to non-warped L0 loop filter

$$\tilde{L}_0(z,\lambda) = L_0(z) = \frac{z^{-1}}{1-z^{-1}} \Leftrightarrow \tilde{G}(z,\lambda) = \frac{z^{-1}}{1-\lambda z^{-1}}$$

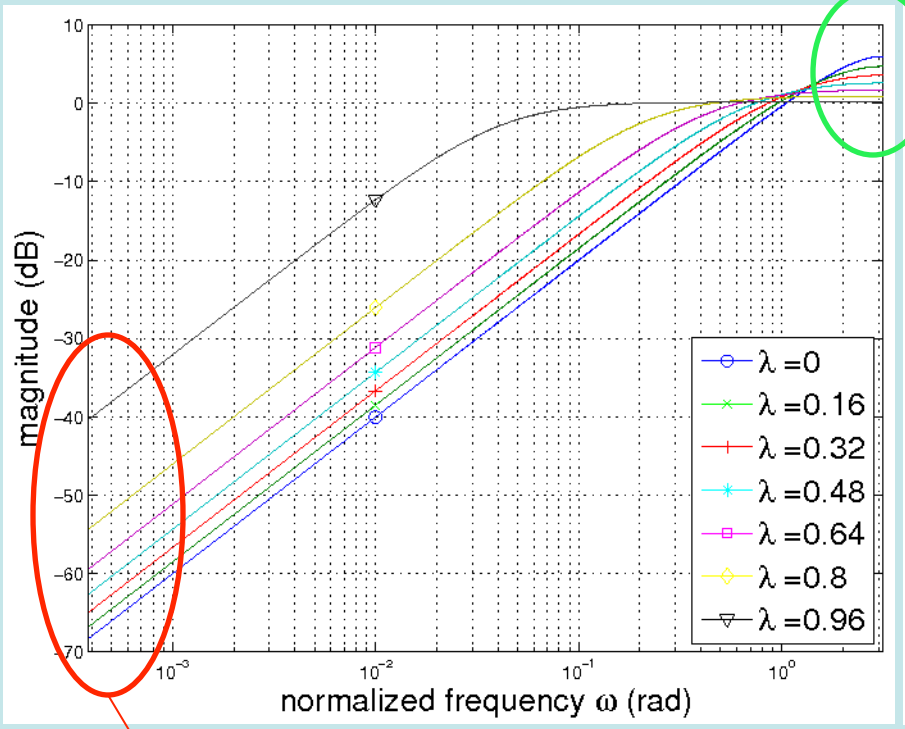


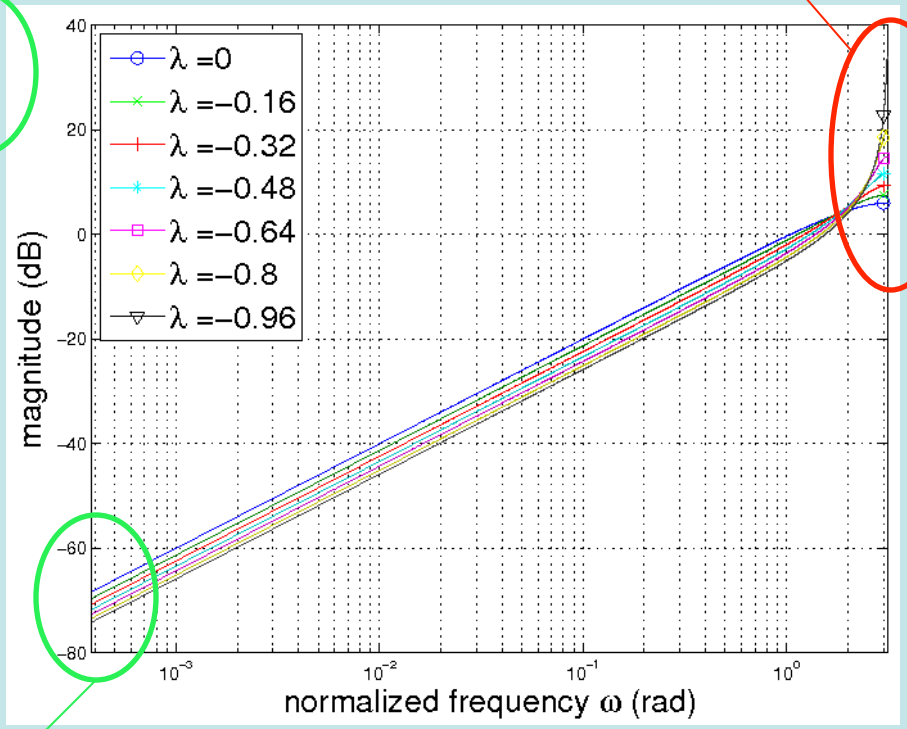
First-order warped SDM (4/8)

- Design procedure:
  - Step 5: evaluate warped NTF/STF & choose warping parameter

positive λ → out-of-band gain decreases

negative λ → out-of-band gain increases





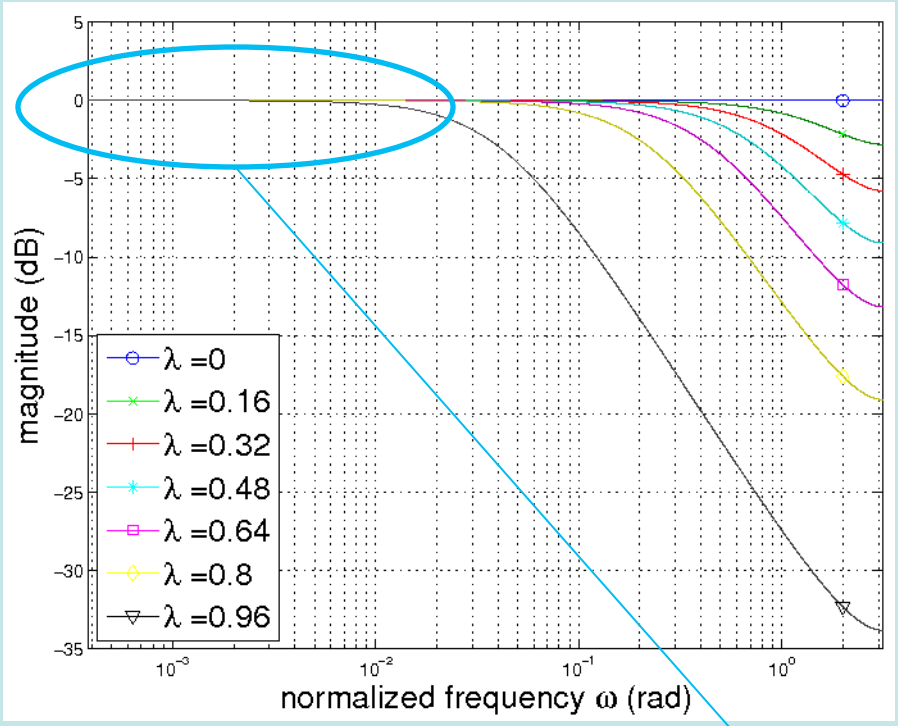
negative λ → noise power in signal band increases

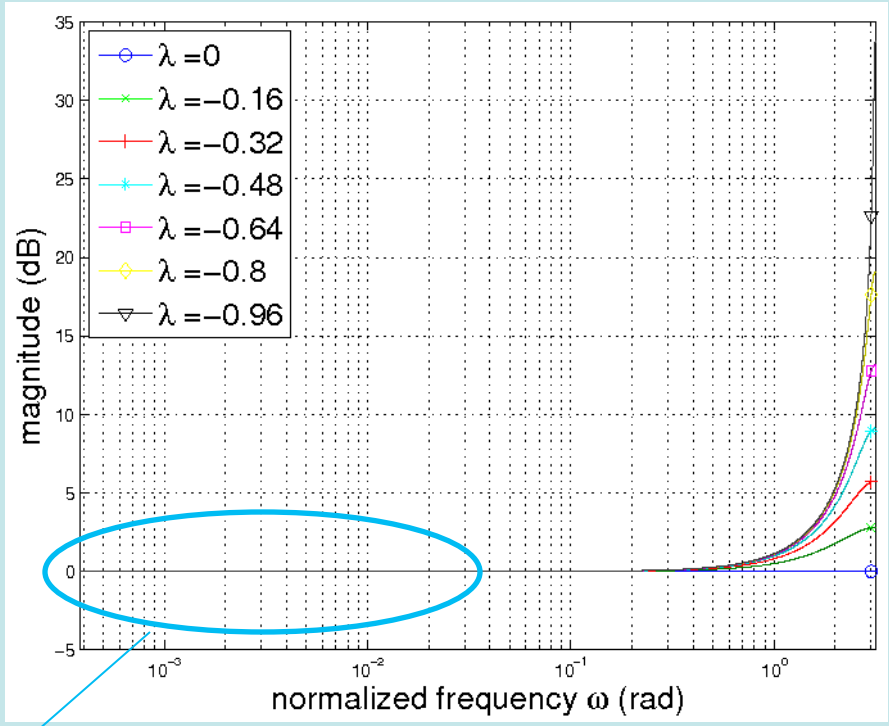
positive λ → noise power in signal band increases

First-order warped SDM (5/8)

- Design procedure:
  - Step 5: evaluate warped NTF/STF & choose warping parameter

Topology I



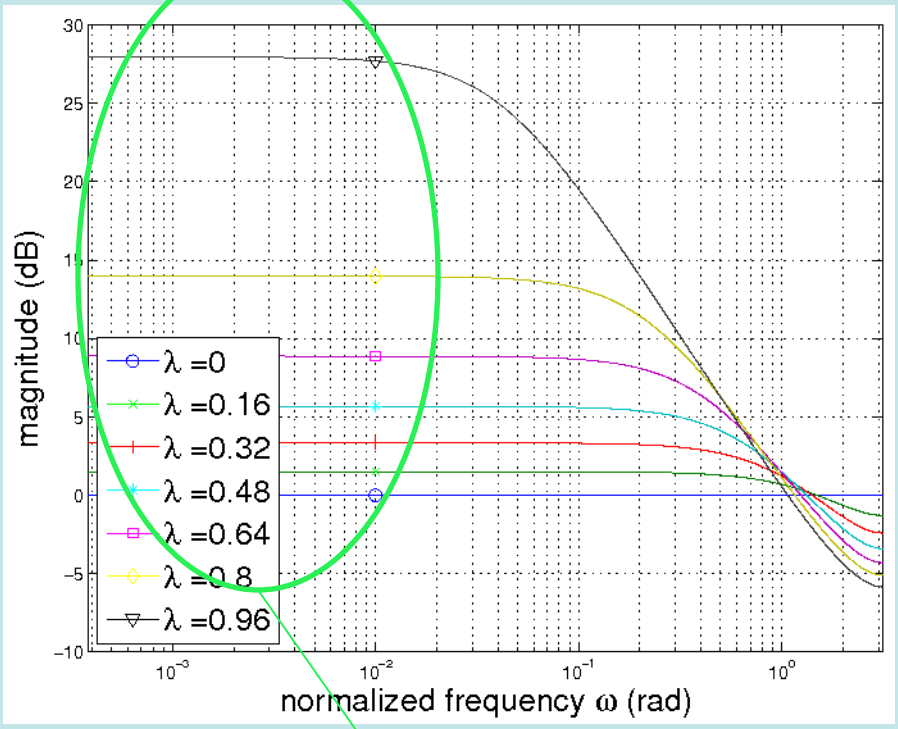


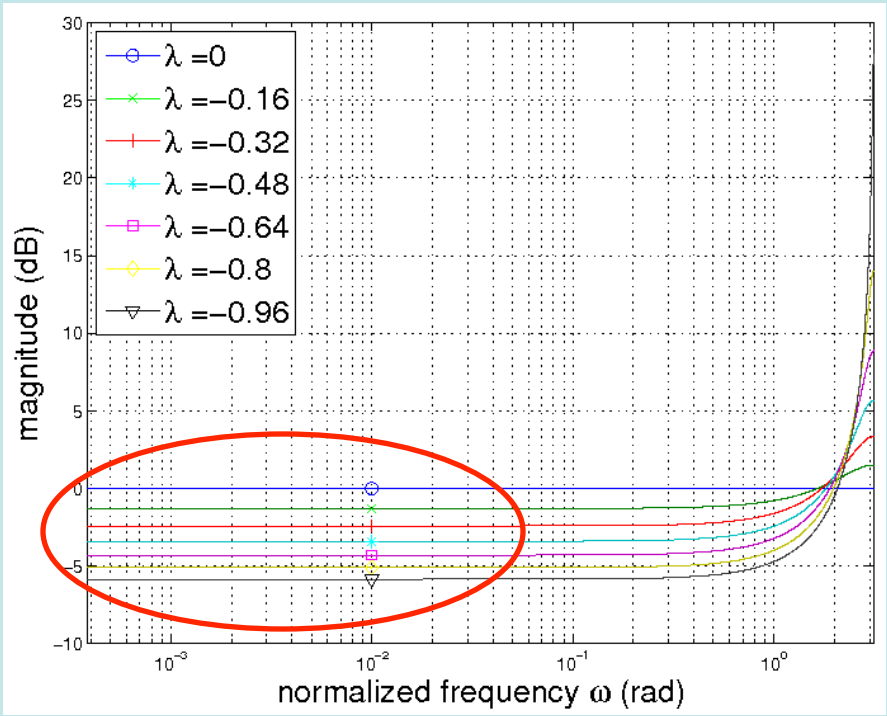
varying λ has no influence in signal band

First-order warped SDM (6/8)

- Design procedure:
  - Step 5: evaluate warped NTF/STF & choose warping parameter

Topology II





positive λ → signal power increases to (over)compensate noise power increase



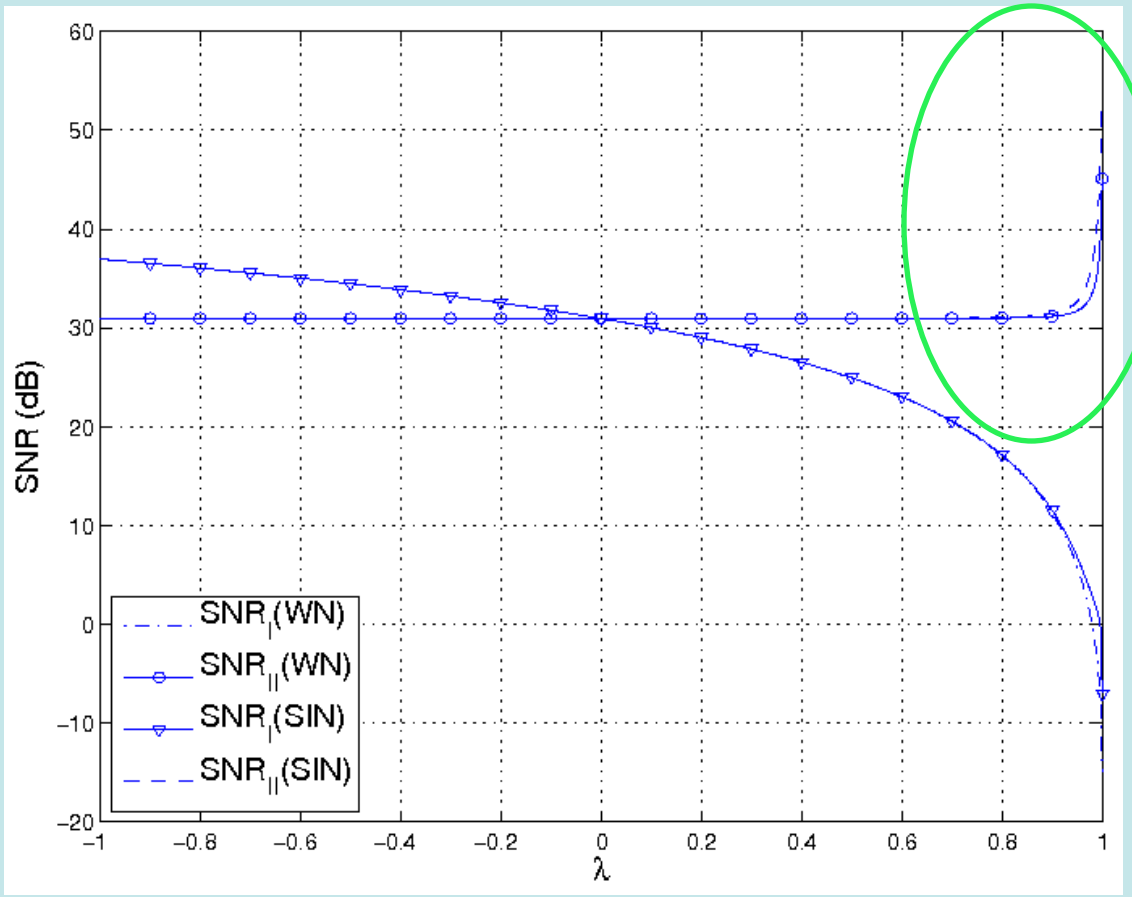
First-order warped SDM (7/8)

- SNR analysis:
  - predicted SNR

$$\mathrm{SNR} = 10\log_{10}\frac{\sigma_{xv}^2}{\sigma_{ev}^2}$$

for white noise (WN) and sinusoidal (SIN) input signals

  - most promising configuration:



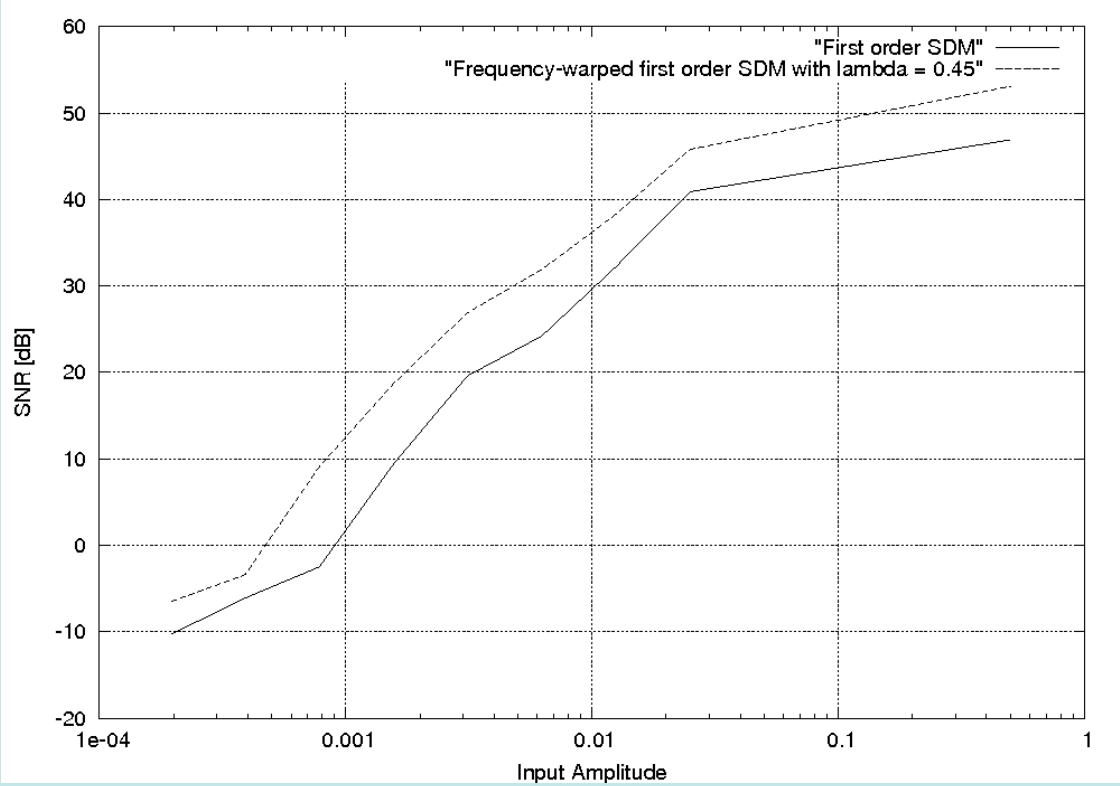
Topology II with λ→1

(also provides optimal out-of-band gain)



First-order warped SDM (8/8)

- Simulation results:
  - input = 1kHz sinusoid
  - signal band = 24kHz
  - OSR = 64
  - topology II
  - $0<\lambda<1$
  - optimal $\lambda=0.45$:
    - stable SDM
    - SNR increase = 6dB
  - Remarks:
    - not every positive value of λ results in a stable SDM
    - discrepancy between simulated SNR and predicted SNR is mainly due to white quantization noise assumption in SNR analysis





Conclusion & Ongoing work

- A procedure has been proposed for designing frequency-warped SDM, with the aim of
  - increasing the SDM resolution (in terms of SNR)
  - increasing the SDM stability (in terms of out-of-band gain)
- In the first-order case, the frequency-warped SDM design
  - requires little additional hardware
  - may lead to a stable 6dB SNR increase, provided the warping parameter is properly chosen
  - provides a framework that can be extended to high-order frequency-warped SDM design*
- SNR analysis and simulation results both confirm that combining a warping parameter $\lambda>0$ with a $(1-\lambda)$ scaling in the feedback loop is most promising in terms of SNR

*L. Ausiello, T. van Waterschoot, and M. Moonen, “Design and evaluation of frequency-warped sigma delta modulators”, IEEE Trans. Signal Process., submitted for publication, Apr. 2008. [Online]. Available: ftp://ftp.esat.kuleuven.be/pub/sista/vanwaterschoot/abstracts/07-190.html