KATHOLIEKE UNIVERSITEIT
LEUVEN



Identification of Black-Box Wave Propagation Models Using Large-Scale Convex Optimization

Toon van Waterschoot[1], Moritz Diehl[1], Marc Moonen[1], and Geert Leus[2]

[1]Dept. ESAT-SCD, KU Leuven, Belgium
[2]Faculty EEMCS, TU Delft, The Netherlands

SYSID 2012 – July 2012, Brussels, Belgium

Outline

- Introduction and Motivation
- Identification of Black-Box Wave Propagation Models
- Simulation Results
- Conclusion

Introduction and Motivation (1)

Problem under study

- Wave propagation in 3-D enclosures
- Input-output relation given by wave equation

$$\nabla^2 u(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} u(\mathbf{r},t) = s(\mathbf{r},t)$$

  + boundary conditions in spatiotemporal domain $\Omega \times T$

- Assumption 1: $s(\mathbf{r},t)$ = superposition of *M* point sources

  $\Rightarrow$ alternative input-output relation using Green's function

$$u(\mathbf{r},t) = \sum_{m=1}^{M} \left(\int_{-\infty}^{\infty} s_m(\tau) h(\mathbf{r}, \mathbf{r}_m, t-\tau) d\tau \right)$$

- Assumption 2: wave field measured at *J* observer positions

  $\Rightarrow$ wave propagation = MIMO-LTI system

Introduction and Motivation (2)

Parametric models for wave propagation

- Useful for prediction, simulation, deconvolution
- **Black-box model:** common-denominator pole-zero model (related to “assumed modes solution” of wave equation)

$$\mathbf{H}(z) = \frac{\sum_{n=0}^{Q} \mathbf{B}_n z^{-n}}{\sum_{n=0}^{P} a_n z^{-n}}$$

  + linear estimation problem (for ARX model)
  – structural information on wave equation hardly exploited
- **Grey-box model:** parametrization of pole-zero model in terms of resonance frequencies and damping factors
  + structural information on wave equation largely exploited
  – nonlinear estimation problem

Introduction and Motivation (3)

Main contribution

- **Goal:** incorporate structural information on wave equation while keeping estimation problem simple to solve (*convex*)
- **Key ingredients:**
  - representation of structural information on wave equation using numerical rather than analytical solution to wave equation

    $\Rightarrow$ finite element method (FEM)
  - exploitation of structure and sparsity in FEM representation for formulation of related estimation problem

    $\Rightarrow$ large-scale convex optimization
- **Result:** increased model accuracy at selected frequencies (e.g., resonance frequencies)

Identification of Black-Box Wave Propagation Models (1)

Problem statement

- *M* point sources (inputs) at positions $\mathbf{r}_m,\ m = 1, \ldots, M$
- *J* observers (outputs) at positions $\bar{\mathbf{r}}_j,\ j = 1, \ldots, J$
- data set: $Z^N = \{s_m(n), y_j(n)\}_{n=1}^{N},\ m = 1, \ldots, M,\ j = 1, \ldots, J$
- model: $\mathbf{H}(z) = \dfrac{\sum_{n=0}^{Q} \mathbf{B}_n z^{-n}}{\sum_{n=0}^{P} a_n z^{-n}}$
- parameter vector $[(J(Q+1)+P+1) \times 1]$:

$$\boldsymbol{\theta} = \begin{bmatrix} \mathbf{b}_1^T & \mathbf{b}_2^T & \ldots & \mathbf{b}_M^T & \mathbf{a}^T \end{bmatrix}^T$$

$$\mathbf{b}_m = \begin{bmatrix} b_0(\bar{\mathbf{r}}_1, \mathbf{r}_m) \ldots b_Q(\bar{\mathbf{r}}_1, \mathbf{r}_m) \ldots b_0(\bar{\mathbf{r}}_J, \mathbf{r}_m) \ldots b_Q(\bar{\mathbf{r}}_J, \mathbf{r}_m) \end{bmatrix}^T$$

$$\mathbf{a} = \begin{bmatrix} a_0 & \ldots & a_P \end{bmatrix}^T$$

Identification of Black-Box Wave Propagation Models (2)

Data-based identification [Verboven *et al.* '04]

- frequency domain data model: $\mathbf{Y}(\omega) = \frac{\boldsymbol{\mathcal{B}}(\omega)}{A(\omega)}\mathbf{S}(\omega) + \mathbf{V}(\omega)$

$$\begin{aligned} \mathbf{S}(\omega) &= \begin{bmatrix} S_1(\omega) & \dots S_M(\omega) \end{bmatrix}^T \\ \mathbf{V}(\omega) &= \begin{bmatrix} V_1(\omega) & \dots V_J(\omega) \end{bmatrix}^T \\ \mathbf{Y}(\omega) &= \begin{bmatrix} Y_1(\omega) & \dots Y_J(\omega) \end{bmatrix}^T \end{aligned} \qquad \boldsymbol{\mathcal{B}}(\omega) = \begin{bmatrix} B_{11}(\omega) & \dots & B_{1M}(\omega) \\ \vdots & \ddots & \vdots \\ B_{J1}(\omega) & \dots & B_{JM}(\omega) \end{bmatrix}$$

- least squares (LS) frequency domain criterion:

$$\min_{\boldsymbol{\theta}} \sum_{\omega} \mathbf{E}^H(\omega,\boldsymbol{\theta})\mathbf{E}(\omega,\boldsymbol{\theta}) \quad \text{with} \quad \mathbf{E}(\omega,\boldsymbol{\theta}) = \boldsymbol{\mathcal{B}}(\omega)\mathbf{S}(\omega) - A(\omega)\mathbf{Y}(\omega)$$

- equivalent QP formulation:

$$\begin{aligned} \min_{\boldsymbol{\theta}} \quad & \boldsymbol{\theta}^T \left(\sum_{\omega} \boldsymbol{\Phi}(\omega) \right) \boldsymbol{\theta} \\ \text{s. t.} \quad & a_0 = 1 \end{aligned}$$

$$\boldsymbol{\Phi}(\omega) = \left[\begin{array}{c|c} \left(\mathbf{S}^*(\omega)\mathbf{S}^T(\omega)\right) \otimes \left(\mathbf{I}_J \otimes \left(\mathbf{z}_Q(\omega)\mathbf{z}_Q^H(\omega)\right)\right) & \operatorname{vec}\left(\mathbf{Y}(\omega)\mathbf{S}^H(\omega)\right) \otimes \left(\mathbf{z}_Q(\omega)\mathbf{z}_P^H(\omega)\right) \\ \hline \operatorname{vec}\left(\mathbf{Y}(\omega)\mathbf{S}^H(\omega)\right)^H \otimes \left(\mathbf{z}_P(\omega)\mathbf{z}_Q^H(\omega)\right) & \mathbf{Y}^H(\omega)\mathbf{Y}(\omega)\mathbf{z}_P(\omega)\mathbf{z}_P^H(\omega) \end{array} \right]$$

Identification of Black-Box Wave Propagation Models (3)

Hybrid identification

- incorporation of structural information requires to:
  1. discretize wave equation in space and time
  2. rewrite wave equation in terms of pole-zero model parameter vector
- **Main result:**

$$\nabla^2 u(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} u(\mathbf{r},t) = s(\mathbf{r},t) \Rightarrow \boldsymbol{\Xi}(\omega)\underline{\boldsymbol{\theta}} = \mathbf{0}$$

  - spatiotemporal → spatiospectral domain (Helmholtz equation)
  - substitute black-box model + exploit common denominator
  - define FEM grid (think of increasing number of observers $J \rightarrow K$)
  - define extended parameter vector $\underline{\boldsymbol{\theta}}$ $[(K(Q+1)+P+1)\times 1]$
  - apply FEM to obtain set of linear equations (“Galerkin equations”)
  - rewrite Galerkin equations in terms of extended parameter vector
- $\boldsymbol{\Xi}(\omega)$ is sparse & highly structured matrix, depending on FEM grid, FEM basis functions, point source positions, and source spectra

Identification of Black-Box Wave Propagation Models (4)

Hybrid identification

- proposed hybrid identification method:

$$\min_{\underline{\boldsymbol{\theta}}} \quad \underline{\boldsymbol{\theta}}^T \mathbf{C}^T \left(\sum_{\omega} \boldsymbol{\Phi}(\omega) \right) \mathbf{C} \underline{\boldsymbol{\theta}}$$

$$\text{s. t.} \quad \begin{cases} \boldsymbol{\Xi}(\omega_1)\underline{\boldsymbol{\theta}} = \mathbf{0} \\ \vdots \\ \boldsymbol{\Xi}(\omega_L)\underline{\boldsymbol{\theta}} = \mathbf{0} \\ a_0 = 1 \end{cases}$$

with selection matrix **C** defined such that $\mathbf{C}\underline{\boldsymbol{\theta}} = \boldsymbol{\theta}$

- large-scale equality-constrained QP:
  - data-based objective in terms of original parameter vector
  - equality constraints in terms of extended parameter vector are used to impose structural information at *L* selected frequencies

Identification of Black-Box Wave Propagation Models (5)

What if point source positions are unknown?

- $\boldsymbol{\Xi}(\omega)$ depends on point source positioning matrix $\boldsymbol{\Psi}$
- $\boldsymbol{\Psi}$ contains barycentric point source coordinates relative to FEM grid $\Rightarrow$ sparse, nonnegative, columns sum up to 1
- if initial estimate $\hat{A}(\omega)$ of pole-zero model denominator's frequency response function is available, the Galerkin equations can be rewritten in terms of $\underline{\mathbf{b}}_m$ and $\mathrm{vec}(\boldsymbol{\Psi})$
- $\mathrm{vec}(\boldsymbol{\Psi})$ can then be estimated by solving a sparse approximation problem with additional structural constraints

$$\min_{\boldsymbol{\zeta}} \; \|\boldsymbol{\Gamma}_L \mathbf{F}\boldsymbol{\zeta} + \boldsymbol{\Gamma}_R \hat{\mathbf{a}}\|_2 + \|\boldsymbol{\Delta}(\hat{\mathbf{a}}, \omega)\boldsymbol{\zeta}\|_2 + \lambda\|\boldsymbol{\psi}\|_1$$

$$\text{s. t.} \; \begin{cases} (\mathbf{I}_M \otimes \mathbf{1}_{1\times K})\,\boldsymbol{\psi} &= \mathbf{1}_{M\times 1} \\ \boldsymbol{\psi} &\geq \mathbf{0} \end{cases}$$

data term

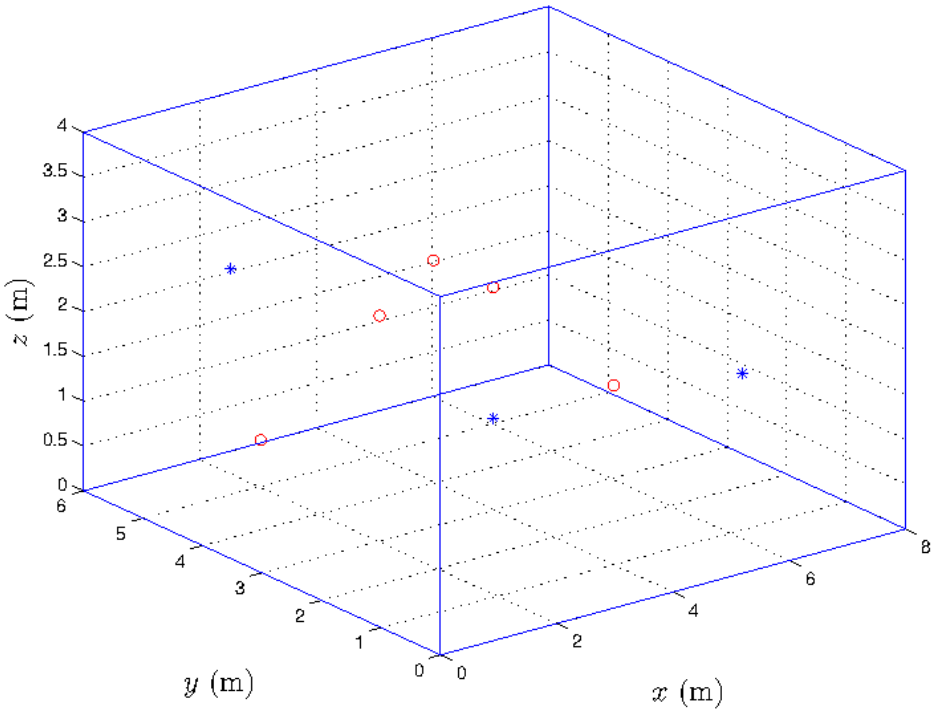
Galerkin equations

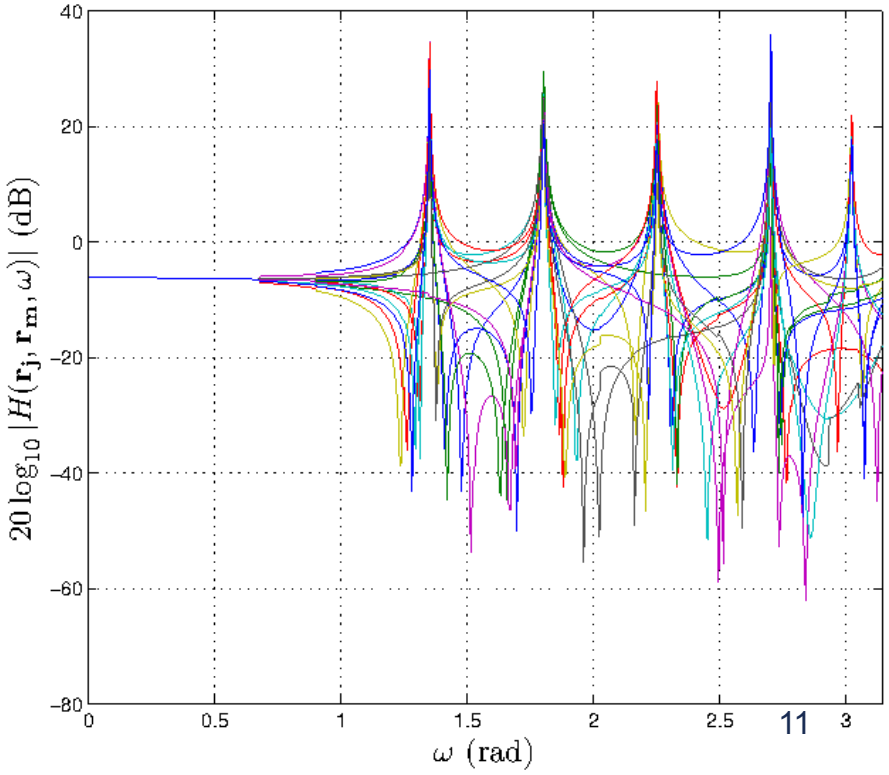
sparsity and structural constraints

Simulation results (1)

Simulation scenario

- application: indoor acoustic wave propagation
- scenario: $8 \times 6 \times 4$ m room, $M = 3$ sources, $J = 5$ sensors

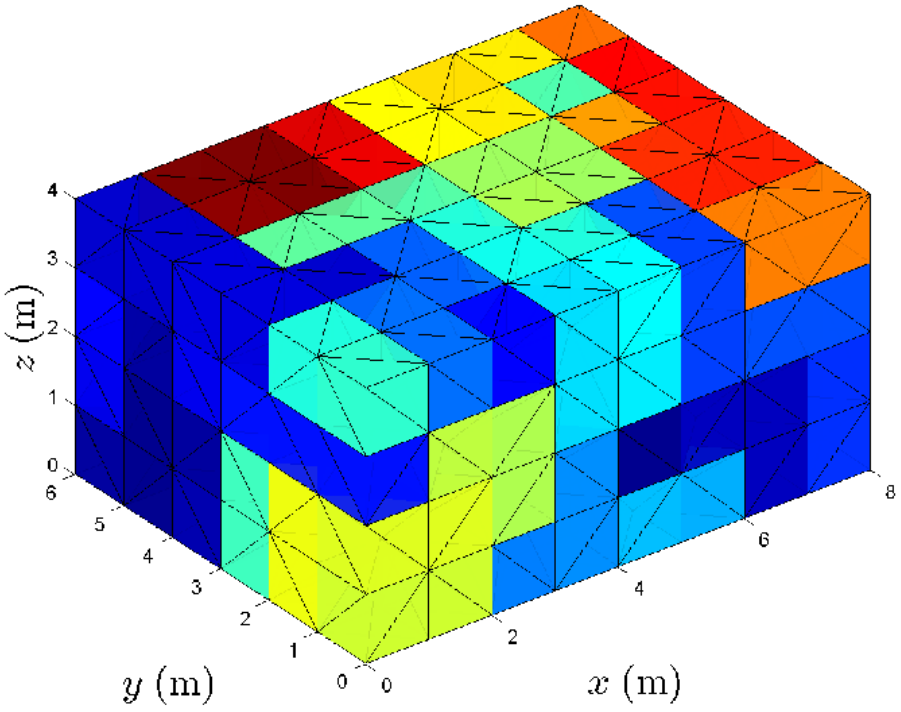




Simulation results (2)

Simulation parameters

- data set: $N = 1000$ samples, $\mathrm{SNR} = 0$ dB, $f_s = 100$ Hz
- model order: $Q = P = 12$
- FEM mesh: 315 grid points, 1152 tetrahedral elements

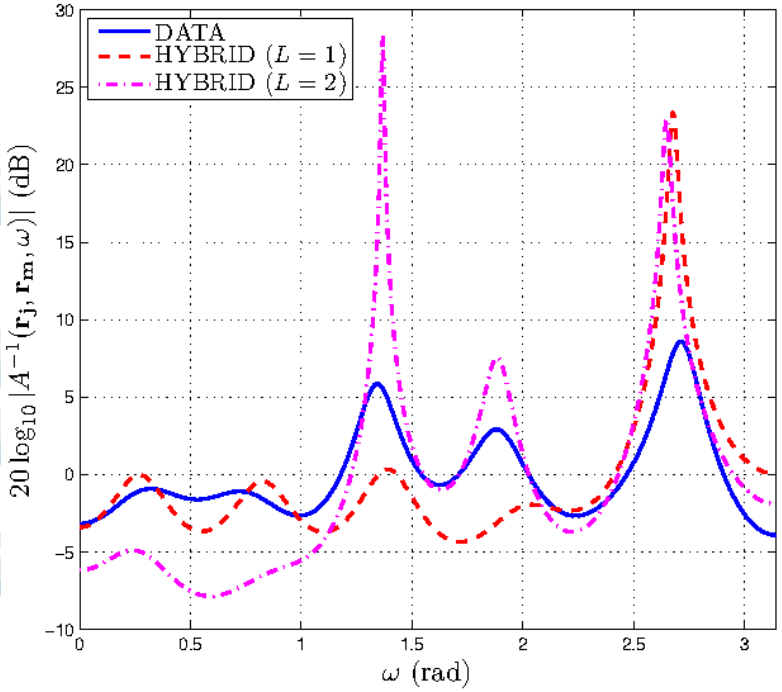


Simulation results (3)

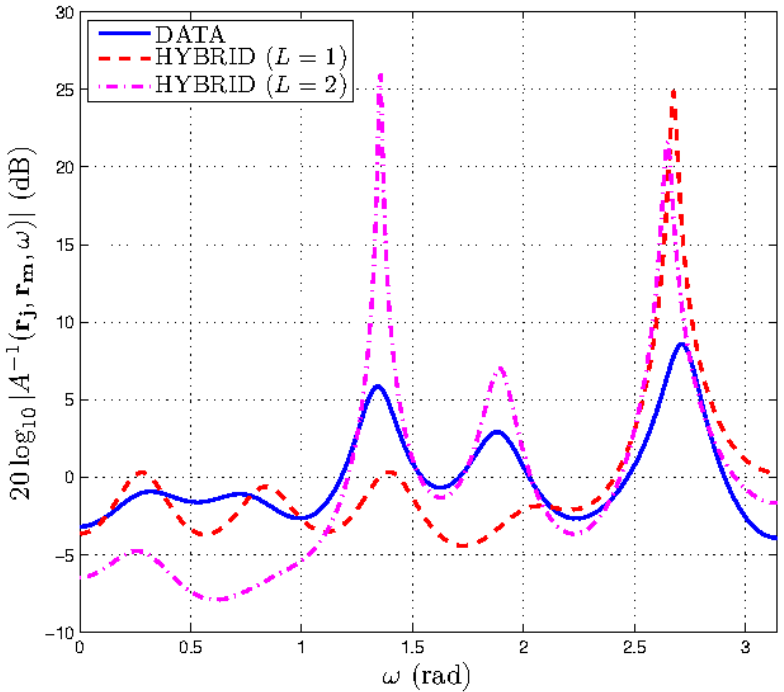
Simulation results

- comparison of inverse denominator frequency magnitude response $|\hat{A}(\omega)|$ obtained using different methods
- $L = 1 : \omega_1 = 2.7018 \text{ rad}$ (4th resonance frequency)
- $L = 2 : \omega_1 = 2.7018 \text{ rad}, \omega_2 = 1.3509 \text{ rad}$ (4th,1st resonance freq.)

exact source positioning matrix



estimated source positioning matrix



Conclusion

- Novel approach to identification of MIMO-LTI wave propagation models having common-denominator pole-zero parametrization
- Traditional data-based identification can be improved by incorporating physical wave propagation model
- FEM discretization yields high-dimensional, sparse, and structured linear set of equations that can be imposed at frequencies where high model accuracy is desired
- Proposed hybrid identification approach consists in sequentially solving two large-scale convex optimization problems:
  - sparse approximation problem for estimating point source positioning matrix appear in FEM Galerkin equations
  - equality-constrained QP for estimating common-denominator pole-zero model parameters