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Optimally regularized recursive least squares for acoustic echo cancellation[1]

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OPTIMALLY REGULARIZED RECURSIVE LEAST SQUARES FOR ACOUSTIC ECHO CANCELLATION

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ABSTRACT

In adaptive filtering algorithms for acoustic echo cancellation it is appropriate to regularize the correlation matrix update and hence to improve the algorithm's numerical stability. Traditional regularization of recursive least squares (RLS) and underdetermined RLS algorithms (such as the affine projection algorithm) is done by adding a scaled identity matrix to the adaptive filter input correlation matrix, which comes down to Levenberg-Marquardt regularization (LMR). It will be shown that from a Bayesian linear minimum mean square error point of view, the optimal regularization method for least-squares based adaptive filtering algorithms is in fact a Tikhonov regularization (TR) using a regularization matrix that incorporates prior knowledge on the unknown room impulse response. We will propose both Levenberg-Marquardt regularized and Tikhonov regularized recursive least squares algorithms and explore their convergence behaviour with different regularization matrices. Furthermore some remedies will be suggested for reducing the regularized RLS complexity to $O(n_F^2)$.

1. INTRODUCTION

In a typical acoustic echo cancellation (AEC) setup as depicted in **Figure 1**, a loudspeaker signal $u(t)$ coming from the far-end side is broadcast in the near-end room. Due to reflections at the room boundaries, an undesired echo $x(t)$ of the loudspeaker signal is picked up by the microphone in addition to the near-end signal $v(t)$, i.e. $y(t) = x(t)+v(t)$. If the room transfer function is modelled as a finite impulse response (FIR) filter with $n_F + 1$ filter taps,

$$\mathbf{f} \triangleq \begin{bmatrix} f_0 & f_1 & \dots & f_{n_F} \end{bmatrix}^T, \tag{1}$$

then the echo signal can be considered as a filtered version of the loudspeaker signal,

$$x(t) = \mathbf{u}^T(t)\mathbf{f}, \tag{2}$$

with

$$\mathbf{u}^T(t) \triangleq \begin{bmatrix} u(t) & u(t-1) & \dots & u(t-n_F) \end{bmatrix}, \tag{3}$$

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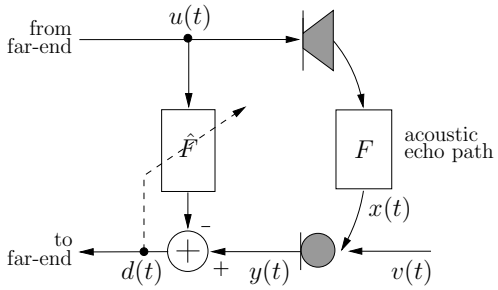


Figure 1: A typical acoustic echo cancellation (AEC) setup.

and it may be predicted using an estimate $\hat{\mathbf{f}}(t)$ of the room impulse response (RIR):

$$\hat{y}(t) = \mathbf{u}^T(t)\hat{\mathbf{f}}(t). \tag{4}$$

The main challenge in acoustic echo cancellation is to obtain a reliable RIR estimate $\hat{\mathbf{f}}(t)$, even during double-talk when a disturbing near-end signal is present, and to track the RIR as the acoustic environment changes. It is well-known that the recursive least squares (RLS) algorithm [1] offers a highly desirable tracking performance and behaves in a relatively robust way in a continuous double-talk situation [2]. Moreover numerically stable fast versions of the RLS algorithm have been derived, which reduce the computational complexity to $O(n_F)$ [3].
The RLS algorithm using an exponential window with forgetting factor λ is given by:

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mathbf{R}^{-1}(t)\mathbf{u}(t)\varepsilon(t), \tag{5}$$

$$\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \mathbf{u}(t)\mathbf{u}^T(t), \tag{6}$$

$$\varepsilon(t) = y(t) - \mathbf{u}^T(t)\hat{\mathbf{f}}(t-1). \tag{7}$$

If the loudspeaker signal $u(t)$ is not persistently exciting, a situation which is not unusual in room acoustics applications due to the coloration of audio signals and the large RIR order, then the loudspeaker signal correlation matrix $\mathbf{R}(t)$ in (6) may be ill-conditioned. This will typically lead to a slower convergence or even instability of the adaptive algorithm, certainly in double-talk situations. The conditioning of $\mathbf{R}(t)$ cannot be improved by soft-constrained initialization, i.e. $\mathbf{R}(0) = \alpha\mathbf{I}$ with α some small constant [1], because the exponentially windowed algorithm forgets the initialization along with the older data. A popular remedy however is to add an appropriately scaled version of $\alpha\mathbf{I}$ to $\mathbf{R}(t)$ in each recursion step, which leads to the regularized correlation matrix update [4]:

$$\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \mathbf{u}(t)\mathbf{u}^T(t) + (1-\lambda)\alpha_{\text{LMR}}\mathbf{I}. \tag{8}$$

In [5] this type of regularization is called *Levenberg-Marquardt regularization (LMR)*, because of the similarity with the non-linear optimization techniques proposed by Levenberg [6] and Marquardt [7].
A second and very similar type of regularization was proposed by Tikhonov et al. [8] and can be understood by considering the off-line least squares RIR estimate:

$$\hat{\mathbf{f}}(t) = (\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{y} \tag{9}$$

with

$$\mathbf{U}^T \triangleq \begin{bmatrix} u(t) & u(t-1) & \dots & u(1) \\ u(t-1) & u(t-2) & \dots & u(0) \\ \vdots & \vdots & \ddots & \vdots \\ u(t-n_F) & u(t-n_F-1) & \dots & u(1-n_F) \end{bmatrix} \tag{10}$$

$$\mathbf{y}^T \triangleq \begin{bmatrix} y(t) & y(t-1) & \dots & y(1) \end{bmatrix}. \tag{11}$$

In case $\mathbf{U}^T\mathbf{U}$ is ill-conditioned a so-called ill-posed problem occurs, which may be transformed into a reasonably well-posed problem (i.e. with a unique solution) by applying *Tikhonov regularization (TR)*:

$$\hat{\mathbf{f}}(t) = (\mathbf{U}^T\mathbf{U} + \alpha_{\text{TR}}\mathbf{I})^{-1}\mathbf{U}^T\mathbf{y}. \tag{12}$$

This estimate can be written in a recursive least squares form as follows:

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mathbf{R}^{-1}(t)\Big(\mathbf{u}(t)\varepsilon(t) + (1-\lambda)\alpha_{\text{TR}}\hat{\mathbf{f}}(t-1)\Big) \tag{13}$$

$$\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \mathbf{u}(t)\mathbf{u}^T(t) + (1-\lambda)\alpha_{\text{TR}}\mathbf{I}, \tag{14}$$

with $\varepsilon(t)$ as defined in (7). If we rewrite the update in (13) as

$$\hat{\mathbf{f}}(t) = \Big(\mathbf{I} - (1-\lambda)\alpha_{\text{TR}}\mathbf{R}^{-1}(t)\Big)\hat{\mathbf{f}}(t-1) + \mathbf{R}^{-1}(t)\mathbf{u}(t)\varepsilon(t), \tag{15}$$

it may be clear that in [9] this algorithm was called the leaky RLS algorithm because of its equivalence with the more well-known leaky LMS (LLMS) update [10]:

$$\hat{\mathbf{f}}(t) = (1-\mu\alpha_{\text{LLMS}})\hat{\mathbf{f}}(t-1) + \mu\mathbf{u}(t)\varepsilon(t). \tag{16}$$

Finally, we note that without the exponential window (i.e. $\lambda = 1$) the Tikhonov regularized RLS (TR-RLS) algorithm as defined by (13)-(14)-(7) is identical to the Levenberg-Marquardt regularized RLS (LMR-RLS) algorithm as given by (5)-(8)-(7).
The performance of the LMR-RLS and TR-RLS algorithms depends heavily on the choice of the respective regularization parameters α_{LMR} and α_{TR}. Many choices for these parameters have been suggested in the literature, ranging from "any small number" to a parameter incorporating prior knowledge on the near-end signal or on the unknown RIR or on both. Most often the regularization parameter is an estimate of the long-term near-end signal power, as suggested in [11]. In **Section 2** we will look at the choice of α from a Bayesian linear minimum mean square error (LMMSE) point of view and conclude that the optimal regularization parameter indeed depends on both the near-end signal characteristics and on the statistical assumptions about the unknown RIR. Moreover, we will suggest a more general approach to regularization by allowing for a regularization matrix that is not a scaled identity matrix. In this way it becomes possible to include more prior knowledge on the unknown RIR in the adaptive algorithm, which will appear to be beneficial to its convergence behaviour.
Then in **Section 3** we will explain how prior knowledge can be gathered from the acoustic setup to construct an appropriate regularization matrix, as proposed earlier in [12]. We will illustrate the potential benefit of the proposed regularization techniques with the simulation results given in **Section 4**.

2. OPTIMALLY REGULARIZED RLS

In a Bayesian LMMSE framework a linear RIR estimate is sought for that minimizes the MMSE criterion,

$$\min_{\hat{\mathbf{f}}(t)} E\Big\{\text{tr}\big\{\big(\hat{\mathbf{f}}(t)-\mathbf{f}\big)\big(\hat{\mathbf{f}}(t)-\mathbf{f}\big)^T\big\}\Big\}, \tag{17}$$

under the assumption that the unknown RIR $\mathbf{f}$ and the near-end signal vector $\mathbf{v} \triangleq [v(t) \ \dots \ v(1)]^T$ are drawn from uncorrelated zero-mean random processes which may be characterized by their covariance matrices:

$$\text{cov}(\mathbf{f}) = E\big\{(\mathbf{f}-E\{\mathbf{f}\})(\mathbf{f}-E\{\mathbf{f}\})^T\big\} \triangleq \mathbf{R}_{\mathbf{f}}, \tag{18}$$

$$\text{cov}(\mathbf{v}) = E\big\{(\mathbf{v}-E\{\mathbf{v}\})(\mathbf{v}-E\{\mathbf{v}\})^T\big\} \triangleq \mathbf{R}_{\mathbf{v}}. \tag{19}$$

The Bayesian LMMSE estimate $\hat{\mathbf{f}}(t)$ is then given by the mean of the posterior probability density function of $\mathbf{f}$ after the data have been recorded [13]:

$$\hat{\mathbf{f}}(t) = p(\mathbf{f}|\mathbf{y}) = (\mathbf{U}^T{\mathbf{R}_{\mathbf{v}}}^{-1}\mathbf{U} + {\mathbf{R}_{\mathbf{f}}}^{-1})^{-1}\mathbf{U}^T{\mathbf{R}_{\mathbf{v}}}^{-1}\mathbf{y}. \tag{20}$$

In the sequel we will assume that $\mathbf{v}$ is drawn from a white random process with possibly time-dependent variance, such that $\mathbf{R}_{\mathbf{v}}$ is a diagonal matrix, i.e.

$$\mathbf{R}_{\mathbf{v}} = \begin{bmatrix} \sigma_v^2(t) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_v^2(1) \end{bmatrix}. \tag{21}$$

In case $\mathbf{v}$ is not white, e.g. if $v(t)$ is a speech signal, the prediction error approach in [2] may be used to estimate $\mathbf{R}_{\mathbf{v}}$ concurrently with the RIR.
The Bayesian LMMSE estimate in (20) with (21) can be written in a recursive least squares form with exponential windowing, which leads to an optimally regularized TR-RLS algorithm:

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mathbf{R}^{-1}(t)\Big(\frac{1}{\sigma_v^2(t)}\mathbf{u}(t)\varepsilon(t) - (1-\lambda){\mathbf{R}_{\mathbf{f}}}^{-1}\hat{\mathbf{f}}(t-1)\Big), \tag{22}$$

$$\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \frac{1}{\sigma_v^2(t)}\mathbf{u}(t)\mathbf{u}^T(t) + (1-\lambda){\mathbf{R}_{\mathbf{f}}}^{-1}, \tag{23}$$

with $\varepsilon(t)$ as defined in (7). It can be seen that the above update equations are equivalent to the classical TR-RLS equations if $\sigma_v^2(t){\mathbf{R}_{\mathbf{f}}}^{-1} = \alpha_{\text{TR}}\mathbf{I}$, i.e. if the unknown RIR $\mathbf{f}$ is assumed to be drawn from a stationary white random process ($\mathbf{R}_{\mathbf{f}} = \sigma_f^2\mathbf{I}$) and if the near-end signal variance is assumed to be constant ($\sigma_v^2(t) = \sigma_v^2, \ \forall t$) and if the regularization parameter is chosen as

$$\alpha_{\text{TR}} = \frac{\sigma_v^2}{\sigma_f^2}. \tag{24}$$

Table 1: Optimally regularized RLS algorithms

TR-RLS - Tikhonov Regularized Recursive Least Squares

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \mathbf{R}^{-1}(t)\Big(\frac{1}{\sigma_v^2(t)}\mathbf{u}(t)\varepsilon(t) - (1-\lambda){\mathbf{R_f}}^{-1}\hat{\mathbf{f}}(t-1)\Big), \quad (25)$$

$$\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \frac{1}{\sigma_v^2(t)}\mathbf{u}(t)\mathbf{u}^T(t) + (1-\lambda){\mathbf{R_f}}^{-1}, \quad (26)$$

$$\varepsilon(t) = y(t) - \mathbf{u}^T(t)\hat{\mathbf{f}}(t-1). \quad (27)$$

LMR-RLS - Levenberg-Marquardt Regularized RLS

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{f}}(t-1) + \frac{1}{\sigma_v^2(t)}\mathbf{R}^{-1}(t)\mathbf{u}(t)\varepsilon(t), \quad (28)$$

$$\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \frac{1}{\sigma_v^2(t)}\mathbf{u}(t)\mathbf{u}^T(t) + (1-\lambda){\mathbf{R_f}}^{-1}, \quad (29)$$

$$\varepsilon(t) = y(t) - \mathbf{u}^T(t)\hat{\mathbf{f}}(t-1). \quad (30)$$

Obviously, by combining the RLS update in (5) and the RLS a priori error in (7) with the above correlation matrix update in (23), an optimally regularized LMR-RLS algorithm can also be obtained. The optimally regularized LMR-RLS and TR-RLS algorithms are summarized in **Table 1**.

From a computational point of view it is not appealing to add a full rank matrix to the correlation matrix update in each recursion step, as in (8), (14) and (23), because then the matrix inversion lemma cannot be used to reduce the RLS complexity from $O(n_F^3)$ to $O(n_F^2)$. Some remedies for this complexity problem have been proposed in the literature. A first solution is to perform the RLS algorithm with a rectangular data window instead of an exponential window [14]. In this case the algorithm will not forget the initial regularization matrix, and the correlation matrix update reduces to a rank-two matrix update which can be calculated in $O(n_F^2)$ by applying the matrix inversion lemma twice. However, using a rectangular window is not ideal from a numerical point of view. A second solution is to approximate the regularization matrix ${\mathbf{R_f}}^{-1}$ or $\alpha\mathbf{I}$ by a sum of rank-one matrices and to add only one of these rank-one matrices in each recursion step. In this case also the correlation matrix update reduces to a rank-two matrix update which can be calculated in $O(n_F^2)$ by applying the matrix inversion lemma twice [4], [5]. A final solution is to update the eigendecomposition of the regularized correlation matrix in (8), (14) and (23), which can be done in $O(n_F^2)$ if the regularization matrix is diagonal [9], [15], using the method described in [16].

3. PRIOR KNOWLEDGE ON ROOM ACOUSTICS

A room impulse response has a very typical form, which may be characterized by three parameters, as illustrated in **Figure 2**:

- the *initial delay* d, which corresponds to the time needed by the loudspeaker sound wave to reach the microphone through a direct path (i.e. without reflections),
- the *direct path attenuation* A, which determines the peak response in the RIR, and

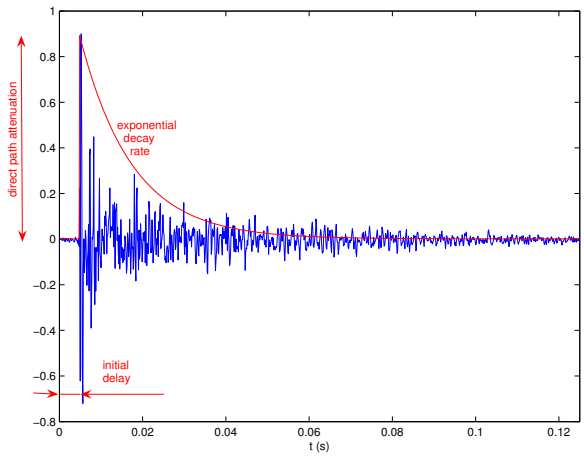


Figure 2: Room impulse response characterized by three parameters.

- the *exponential decay time constant* τ, which models the reverberant tail of the RIR.

These three parameters may be estimated from the acoustic setup (distance between loudspeaker and microphone, acoustic absorption of the walls, room volume, etc.), e.g. using Sabine's reverberation formulas [17]. Hence they can be considered as prior knowledge. If these three parameters are taken into account, a diagonal regularization matrix may be constructed as

$${}^3{\mathbf{R_f}}^{-1} \triangleq A \cdot \mathrm{diag}\Big(\underbrace{\beta, \ldots, \beta}_{d}, 1, \underbrace{e^{-2\frac{1}{\tau}}, \ldots, e^{-2\frac{n_F-d}{\tau}}}_{n_F-d}\Big),$$

where β is a small number. As an example, the RIR in **Figure 2** may be approximated by choosing $d = 38$, $A = 0.9$, $\tau = 100$ and $\beta = 10^{-6}$.

4. SIMULATION RESULTS

We will compare the convergence behaviour of the standard, non-regularized RLS algorithm and of the LMR-RLS and TR-RLS algorithms with two choices for the regularization matrix: the classical $\alpha\mathbf{I}$ and the proposed 3-parameter model ${}^3{\mathbf{R_f}}^{-1}$ of the RIR inverse covariance matrix ${\mathbf{R_f}}^{-1}$. The unknown RIR is the one depicted in **Figure 2** and has $n_F + 1 = 1000$ filter taps. The loudspeaker signal is a male speech signal sampled at $f_s = 8kHz$ and the near-end signal is a stationary white noise signal with variance $\sigma_v^2 = 10^{-3}$, such that the echo-to-near-end ratio (ENR) equals $18dB$. In the regularized algorithms, weighting with the true near-end signal variance is performed as in (22)-(23). As a consequence, the optimal choice of the regularization parameter in the LMR-RLS-$\alpha\mathbf{I}$ and TR-RLS-$\alpha\mathbf{I}$ algorithms is $\alpha = 1$. In the LMR-RLS-${}^3{\mathbf{R_f}}^{-1}$ and TR-RLS-${}^3{\mathbf{R_f}}^{-1}$ algorithms the 3-parameter model with the parameter values suggested in **Section 3** is used. The exponential forgetting factor is set to $\lambda = 0.9997$ for all algorithms. The performance measure used for comparison of the convergence behaviour is the normalized distance of the RIR estimate to the unknown RIR:

$$\text{normalized distance } (dB) = 20\log_{10}\frac{\|\hat{\mathbf{f}}(t) - \mathbf{f}\|}{\|\mathbf{f}\|}. \quad (31)$$

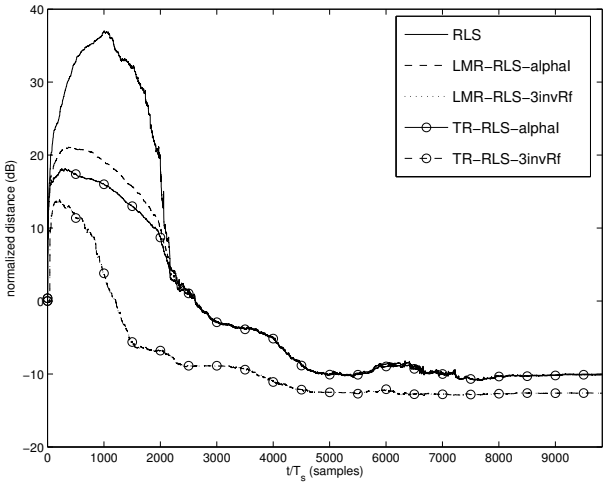


Figure 3: *Convergence curves of the RLS algorithms with different regularization methods*

The convergence curves shown in **Figure 3** reveal that the optimally regularized algorithms using the proposed 3-parameter model exhibit the fastest convergence and the lowest steady-state error. We note that the curves of the LMR-RLS-$^3\mathbf{R_f}^{-1}$ and TR-RLS-$^3\mathbf{R_f}^{-1}$ algorithms overlap entirely, such that only the TR-RLS-$^3\mathbf{R_f}^{-1}$ curve is visible. The classically regularized LMR-RLS-$\alpha\mathbf{I}$ and TR-RLS-$\alpha\mathbf{I}$ algorithms perform worse, but initially still somewhat better than the non-regularized RLS algorithm. There is a slight advantage in using the TR-RLS-$\alpha\mathbf{I}$ over the LMR-RLS-$\alpha\mathbf{I}$ algorithm.

5. CONCLUSION

In an attempt to classify the most commonly used regularization techniques for acoustic echo cancellation, we have distinguished between Levenberg-Marquardt (LMR) and Tikhonov regularization (TR). Whereas these two methods are typically applied with a scaled identity regularization matrix, we have shown that the use of a non-identity regularization matrix may improve the adaptive filter's convergence behaviour even more. From a Bayesian linear minimum mean square error point of view, the optimal regularization matrix is equal to the inverse covariance matrix of the unknown RIR and weighting should be performed with the inverse near-end signal covariance matrix. We have proposed a 3-parameter model to collect prior knowlegde on the RIR and hence to construct a diagonal matrix for optimal regularization. The benefit of using such a regularization matrix instead of the classical scaled identity matrix was illustrated by computer simulations.

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