The Role of Mathematical Competencies in Engineering Research: Insights from Digital Signal Processing PhD Scholars

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Abstract

In the Digital Signal Processing (DSP) research community, mathematical concepts are deeply embedded in the way signals and systems are represented and used (Broesch 2008). Researchers assume familiarity with both the technological and mathematical concepts that underly signals and systems. This study seeks to explore how engineering researchers perceive and apply mathematical competencies within their research, specifically in DSP. Engineering students often need convincing to see the relevance of mathematics to their future profession (Coupland 2008). Understanding how advanced researchers apply mathematical competencies could offer valuable insights. This study aims to explore the mathematical competencies that PhD researchers in DSP apply in their work, including competencies like reasoning mathematically, using aids and tools, and communicating mathematically (Winkelman 2009). While there is existing literature on professional competencies and the role of mathematics in industry, this paper focuses specifically on the perspectives of DSP engineering researchers in academia. This is a case study used to illustrate what engineers think and perceive the mathematical competencies as and how they connect it to their research. We aim to extend these findings to other engineering disciplines, providing a broader understanding of the role of mathematics in engineering research.

Introduction

Competencies are not unheard of when we think of education and curriculum. There is a variety of competencies ranging from professional, inter-disciplinary and long-term competencies that are currently being researched with the aim of designing a good curriculum for engineering (Alpers 2020). We have always argued that the mathematics that is taught in engineering curricula is relevant, cohesive and important. Mathematical competencies are one of the fundamental groups of competencies that exist and are developed in every engineering curriculum, in most cases implicitly (Wong et al 2022). This paper is based on a case study involving 13 engineering PhD students in the DSP community of KU Leuven to indulge in an exploratory analysis which allowed explicit reflection and discussion on these mathematical competencies.

While professional competencies and industry applications of mathematics have been studied, less is known about how academic researchers apply mathematical competencies in DSP. The motivation of this research is twofold. Firstly, in the context of research on mathematical competencies, the perceptions and discussions of PhD researchers help us

consolidate the definitons of these competencies. Secondly, the conclusions can be used to see the applications of these competencies in the activities of DSP research. The questions that we can answer based on this study help us understand the role of these competencies in DSP research and can be broadly summarised below:

- How do DSP researchers perceive the importance of different mathematical competencies in their work?

- How do DSP researchers apply mathematical competencies in their daily research activities?

Methodology

In this study, we employed a workshop-based case study approach to explore how DSP researchers perceive the relevance of different mathematical competencies in their work. The workshop involved 13 PhD researchers specializing in DSP, each receiving a handout (see Appendix) with a description of one of seven mathematical competencies, with a description (Firouzian 2015), based on the framework by Niss & Højgaard (2019). Participants were asked to rate the importance of their assigned competency on a continuous scale from 0 to 5, reflecting its relevance to their research. Additionally, they provided qualitative examples of how they applied the competency in their work. Competencies were assigned randomly to ensure a broad distribution of perspectives. This approach allowed for an exploration of competencies without leading participants toward specific areas of interest. This method allowed us to gather both quantitative data on perceived importance and qualitative insights into real-world applications. The qualitative data were categorised into recurring or highly emphasised themes discussed during the workshop as well as written answers in the handouts, to find insights into the use and perception of these mathematical competencies in DSP research. The methodology not only aimed to collect data but also provided a reflective space for the researchers, allowing them to evaluate their own proficiency in these competencies and encouraging a deeper understanding of their own research practices in relation to mathematical tools and concepts.

Findings and Discussion

The findings revealed that all competencies were rated highly, with participants identifying their importance in DSP research. Researchers were able to explicitly connect these competencies to their work. They provided detailed examples of how these competencies are applied in signal processing algorithms, optimization and acoustic modelling. The observed data also highlights how mathematical competencies are both visible and implicit in their daily research practices. Table 1 shows a concise summary of descriptions of each competency by each researcher and their scoring on a scale from 0 to 5 indicating how they rate themselves in terms of mastering the competency as well as how important they think it is in their field of expertise. The last column indicates the authors' list of extracted themes/ideas that can be concluded from the workshop after

analysing the observations. The table is followed by a discussion of each competency where the reflections of the authors as well as the participants have been portrayed.

Competency	Observation Notes	Self	Importance	Main Extracted Idea
Thinking Mathematically	Identifying reusable parts of derivations for generalized algorithms (e.g., acoustic echo cancellation)	3	4	**Generalizing** mathematical structures from specific derivations. **Recognising** relevant parts of a derivation.
Communicating Mathematically	Understanding literature, discussing ideas conceptually, refining derivations, aligning notation, communicating research	3.5, 3.5	4, 5	**Conceptual communication** of mathematics across expert and non-expert audiences.
Using Aids and Tools	Using built-in MATLAB functions, developing new tools, balancing external and self-made tools, optimizing software	4	5	**Strategic use** and critical understanding of mathematical tools.
Symbols and Formalisms	Translating between notations across fields; clear expression needed, notational abuse	2.5, 4	5, 2	**Mastery** and **translation** of symbolic mathematical language, **variability** of symbols across different domains
Mathematical Modelling	Evaluating assumptions; model selection based on context (e.g., GW detectors, sound localization)	3.5	4	Critical evaluation and adaptation of mathematical models, focusing on the **assumptions** made
Representing Mathematically	Choosing appropriate representations (time vs frequency domain); transformation benefits solution properties	3.8, 3	5, 4	Strategic **selection** and **transformation** of representations
Reasoning Mathematically	Building/testing tools; proving convergence; combining models from data	4, 3.8	5, 4	Building, **validating**, and refining mathematical reasoning

Table 1: Overview of the seven mathematical competencies under consideration Niss & Højgaard (2019), with summarized observation notes from handouts given out during the workshop, ratings on a 5-scale indicating self-proficiency in and importance of each

competency, and main extracted ideas from the observed as well as written text. Each competency was analyzed by one or two PhD researchers.

Thinking Mathematically

"How does a species like Homo sapiens learn to think mathematically?" (Tall 2013) It is a question that educators and students have thought about for decades and there is abundance of literature that illustrates the answer to this question in mathematics, philosophy and the sciences. We also sense the vastness of this question and in terms of definitions this is the one that is the most difficult one to put in a closed bracket. It is not surprising that this is one of the toughest competencies to describe as it sits as an umbrella term for all the mathematics that we observe and do.

Mathematical thinking includes the ability to understand and judge what kind of questions can be answered using mathematics and hence where a mathematical approach might be helpful (Alpers et al 2013). In terms of recognising their alignment with the competency, the PhD researcher made an analogy to their work with acoustic echo and noise cancellation. The researcher describes the competency with respect to their work as *identifying* certain parts of a *mathematical derivation* that could lead up to an *initial algorithm*. This in itself is a strong statement as it allows us to see that there exists a potential algorithm that needs to be understood or created and is recognised as well as the mathematical approach that can be taken in order to do so. They further talk about *reusing* or *generalising* what is found to apply to other spaces. Thinking mathematically was evidenced in the reflection about generalizing derivations for example, identifying reusable parts of an integrated algorithm for combined acoustic echo cancellation and noise reduction to extend it for additional interference sources. Rated 3 in self-assessment and 4 in importance, this highlights the skill of abstracting core mathematical structures from specific cases.

Communicating Mathematically

Mathematics is a language and there are formal rules to use it unambigously. In general, engineering students make a concious effort to use appropriate terminologies only if the environment of learning has facilitated both thinking and communication skills (Rahman et al 2012). Most often a researcher needs to present their work in the community that understands all its definitions (conferences and publications) as well as communicate their work to a group of people who are not experts in that field (outreach programs and seminars). This idea led to some substancial reflections that established a description of this competency, based on the response by two researchers. They broke down the competency in 3 major chunks: Understanding literature and own research, collaborating with peers, and aligning definitions and notation. One researcher stated that the competency is used to connect different sources and then *linking* it to their own research in order to establish *connections*.

One key quote was, "Discussing ideas with others requires understanding the maths on a conceptual level," indicating that presenting work, even preliminary findings, demands clarity of thought and expression. This researcher rated their communication competency at 3.5 but rated its importance at a strong 5. This is not the case only with researchers in DSP but for all of mathematics related research. It allows to present one's work in seminars and conferences and hence it is essential to know one's own work well. However researchers are simultaneously answering research questions as well as defining and finding new results, especially in engineering. This can put the researcher in a tight spot as they are still trying to convey to an audience about something that is not yet found or defined conceptually well.

Using Aids and Tools

This is a competency that was discussed quite easily. All researchers agreed that MATLAB optimization routines were the ones that were mentioned explicitly. They reiterated the fact that there are existing (previously designed) solvers that are used in optimization problems. “However many times we are making these tools/aids ourselves from scratch. We work a lot on developing tools using equations from maths. But you cannot do everything, so rely on the tools other people have developed,” explained an engineers way of thinking perfectly!

Mathematics is increasingly mediated by computational tools, from built-in functions in software like MATLAB to specialized solvers for optimization problems. The competency of using aids and tools emphasizes not just the ability to use these tools but also understanding their underlying principles and limitations. As one researcher stated, while these tools can simplify calculations, a deep understanding of how they work is necessary to use them effectively.

In the reflection, a strong emphasis was placed on using tools like built-in MATLAB functions and *optimization solvers,* while also developing custom tools when necessary. The researcher scored themselves highly (Self: 4; Importance: 5), acknowledging that reliance on external aids must be balanced with understanding the underlying mathematics. As noted, "to tighten a screw you better need a screwdriver," illustrating the pragmatism behind tool use. Self-ratings suggest that researchers feel confident in using these aids, especially when *applying existing tools* to their research. However, research in mathematical tool usage indicates that an over-reliance on software without understanding its underlying mechanics can lead to a superficial understanding of the problem at hand.

Symbols and Formalisms

The use of symbols and formalism is an essential part of mathematical communication. However, as one researcher noted, different fields and papers often use different notations, which can be a source of confusion. Understanding how to translate and work with various symbols is a vital skill for researchers and can further be further broken down to the idea of Symbol Sense (Arcavi, A. 2005). Symbol sense broadly describes a

student's tendencies when using symbols and notation.This competency is frequently used when writing scientific articles, where precision in notation is necessary to convey complex ideas clearly.

The researcher's reflection revealed the constant negotiation involved in interpreting differing notational systems across fields and translating complex ideas into mathematical expressions for publication. While self-rated at 2.5 but recognizing its importance at 5, the reflection noted the challenge of "*notation abuse*" being common but needing attention, particularly when translating rather abstract concepts like Riemannian manifolds into practical applications. Understanding formal symbols requires unpacking the *logical structures* behind them, a necessary skill when bridging theory and application (Selden & Selden 1995). Self-ratings suggest students find this competency somewhat challenging, likely due to the complexity and variability of symbols across different domains. Mastery of this competency is necessary not only for academic writing but also for understanding the works of others.

Mathematical Modelling

Even though this competency is most mentioned in engineering mathematics, a systematic literature review indicated that there is a necessity for further theoretical work on conceptualizing mathematical modelling competencies as well as its implementation at various stages in education (Cevikbas et al. 2022). Mathematical modelling involves translating real-world problems into mathematical structures, often underpinned by assumptions that must be critically evaluated. As one researcher pointed out, decisions about which model to use and how to represent signals and systems depend heavily on the problem at hand. This competency is particularly relevant in fields like sound source localization and signal processing, where models are used to interpret data and make predictions. Mathematical modelling appeared prominently in the reflections, where the researcher described *evaluating assumptions* in physical models (e.g., likelihood assumptions in gravitational wave detection, model choices in supernova waveforms). This reflective critical stance was rated 3.5 in self-assessment and 4 for research.

Mathematical modelling constitutes a major challenge for students of different skill levels (Blum & Niss, 1991). Researchers self-assessed themselves moderately in this area, indicating that while they recognize the importance of modelling, they may not always feel confident in applying models to new situations. Effective modelling requires moving fluidly between real-world phenomena and abstract representations, explicitly considering and adjusting the assumptions embedded in models, precisely the skill the researcher reported using when debating the relevance and generality of models.

Mathematical Representation

Mathematical representation is essential in conveying complex concepts, particularly in fields like signal processing where problems are often expressed in the *time or frequency domain.* To solve increasingly complex problems, the biological brain needs to organize concepts in ways that enable them to be manipulated easily and often these entities,

known as Procepts, can represent a concept as well as a process, simultaneously. An elementary procept is the amalgam of three components: a process that produces a mathematical object, and a symbol that is used to represent either process or object (Tall 2013). This aligns very well with the researchers' reflections, as different representations of the same problem can lead to insights about the problem's various properties that may not be visible in one representation. The researcher demonstrated a nuanced understanding of multiple representations, noting the choice between time and frequency domain formulations depending on problem properties (e.g. common-zeros problems). They reflected on the shift from variational models in the time domain to iterative solutions in the frequency domain, a practice that earned a 3 in self-assessment and a 4 in importance. Researchers rated themselves and expressed that they understand the value of using multiple representations to clarify and solve problems. Mastery of this competency enables the researcher to communicate their ideas more effectively and tackle a wider range of problems.

Reasoning Mathematically

Reasoning mathematically is a fundamental competency that underpins the entire process of mathematical inquiry. It involves logical thinking, deductive reasoning, and the ability to prove or disprove hypotheses based on sound mathematical principles. Researchers engage in formal mathematical reasoning (e.g., algebraic manipulation, proofs) and informal reasoning (e.g., intuition, estimation) when solving real engineering problems. As one researcher highlighted, mathematical reasoning is particularly critical in algorithmic development, where *iterative processes* must be justified and optimized.

Strong mathematical reasoning skills are developed over time, with Researchers progressing from more intuitive approaches to more formal, rigorous methods. This competency is essential not just for solving existing problems but also for creating new mathematical tools and algorithms. Mathematical reasoning was deeply embedded in the researcher's daily practice, particularly in building and proving iterative algorithms, comparing models, and *connecting theoretical results to empirical data*. Rated highly (Self: 4; Importance: 5), the reflection described "developing new tools based on intuition first, then testing optimality," and "pairing two models based on real data visualization."

Conclusions

This reflective study highlights how essential mathematical competencies are woven into the daily practices of researchers in Digital Signal Processing. Researchers establish that they do not simply apply mathematics but instead they actively shape, adapt, and refine it within their evolving research contexts. The observations show that while competencies are recognized as highly important, they also present significant challenges. Furthermore, the balance between using established tools and creating new ones reflects a deeper understanding of when to trust existing methods and when to innovate. Translating mathematical models across domains (e.g., time to frequency), handling formalisms with care, and critically evaluating model assumptions all emerge as core competencies needed for high-quality DSP research. The reflections also underscore the real-world tensions:

how to communicate incomplete or evolving ideas clearly, how to align notation across fields, and how to reason rigorously even when facing intuitive or uncertain grounds.

Ultimately, these findings suggest that fostering these mathematical competencies should be seen not as peripheral but central to training for engineering research. The results suggest that mathematical competencies are integral to DSP engineering research. Strengthening them can significantly enhance the ability of researchers to innovate, collaborate across disciplines, and contribute impactful results to the broader signal processing community. Additionally, by examining how these competencies are used in advanced research, we can better align teaching methods to meet the needs of future engineers. Further research could extend these findings to other engineering disciplines, deepening our understanding of the role of mathematics in engineering research and education.

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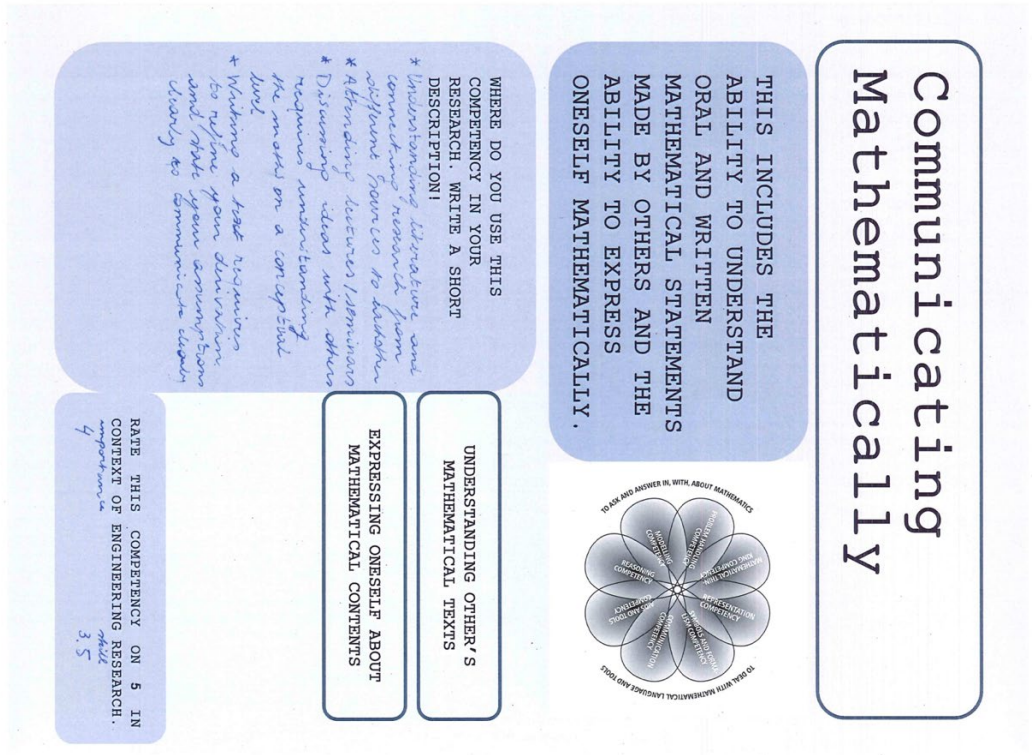
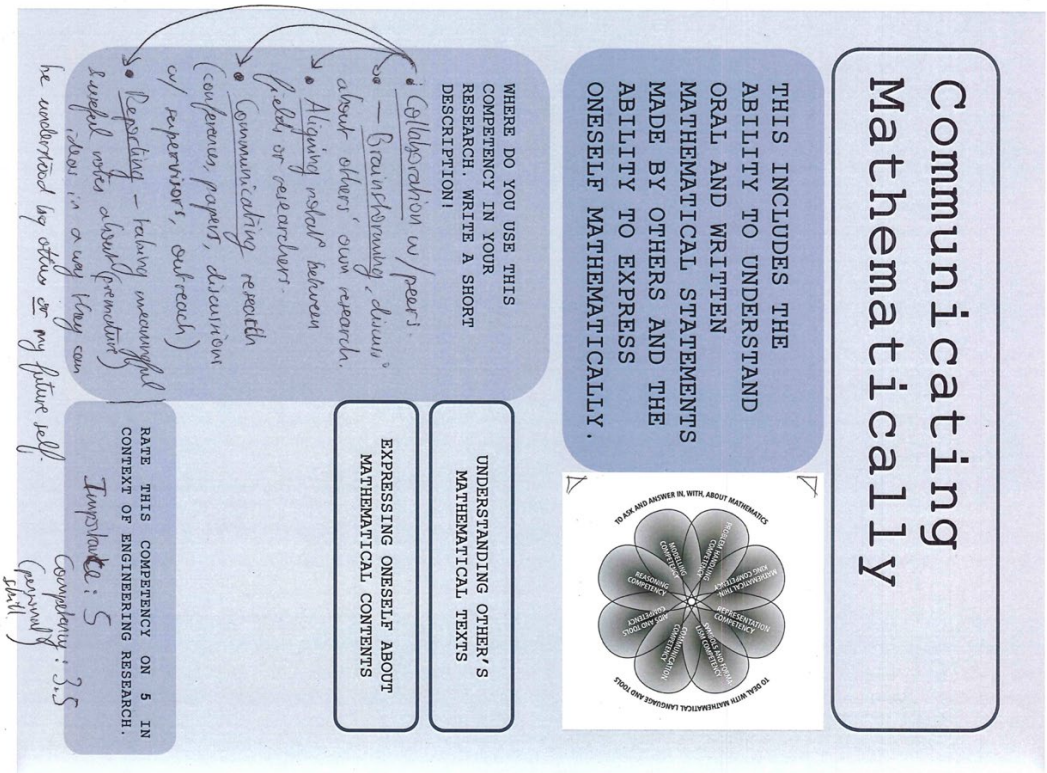
Appendix A


Communicating Mathematically
THIS INCLUDES THE ABILITY TO UNDERSTAND ORAL AND WRITTEN MATHEMATICAL STATEMENTS MADE BY OTHERS AND THE ABILITY TO EXPRESS ONESELF MATHEMATICALLY.
TO ASK AND ANSWER IN, WITH, ABOUT MATHEMATICS
TO DEAL WITH MATHEMATICAL LANGUAGE AND TOOLS
UNDERSTANDING OTHER'S MATHEMATICAL TEXTS
EXPRESSING ONESELF ABOUT MATHEMATICAL CONTENTS
WHERE DO YOU USE THIS COMPETENCY IN YOUR RESEARCH. WRITE A SHORT DESCRIPTION!
RATE THIS COMPETENCY ON 5 IN CONTEXT OF ENGINEERING RESEARCH.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