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Abstract	Full-duplex (FDX) transmission is a promising technique emerging in DSL networks that theoretically may double the spectral efficiency by simultaneously transmitting in the downstream (DS) and upstream (US) on the same frequency band. Unfortunately, this may lead to severe near-end crosstalk (NEXT) interference in addition to the usual far-end crosstalk (FEXT) among the lines within a cable binder. To limit the NEXT impact by balancing the user transmit powers, tailored vectoring-based dynamic spectrum management (DSM) techniques are vital. In this paper, we develop a DSM algorithm for the specific case of perfect NEXT cancellation at the access node. This assumption in combination with US-DS duality theory allows to reformulate the DS-US structure of the non-convex weighted sum-rate maximization problem into an easier US-US structure, which can be solved with low-complexity iterative fixed-point power updates. Simulations of G.fast multi-user FDX transmission demonstrate significant improvements over time division duplex transmission.
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Vectoring-Based Dynamic Spectrum Management for G.fast Multi-User Full-Duplex Transmission

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***Abstract*—Full-duplex (FDX) transmission is a promising technique emerging in DSL networks that theoretically may double the spectral efficiency by simultaneously transmitting in the downstream (DS) and upstream (US) on the same frequency band. Unfortunately, this may lead to severe near-end crosstalk (NEXT) interference in addition to the usual far-end crosstalk (FEXT) among the lines within a cable binder. To limit the NEXT impact by balancing the user transmit powers, tailored vectoring-based dynamic spectrum management (DSM) techniques are vital. In this paper, we develop a DSM algorithm for the specific case of perfect NEXT cancellation at the access node. This assumption in combination with US-DS duality theory allows to reformulate the DS-US structure of the non-convex weighted sum-rate maximization problem into an easier US-US structure, which can be solved with low-complexity iterative fixed-point power updates. Simulations of G.fast multi-user FDX transmission demonstrate significant improvements over time division duplex transmission.**

I. Introduction

Full-duplex (FDX) transmission is a promising technique emerging in DSL networks that theoretically may double the spectral efficiency by simultaneously transmitting in the downstream (DS) and upstream (US) on the same frequency band. In early stages of lab testing, single-user FDX seems to exceed expectations. For instance, with XG-FAST (a potential future-generation of the latest G.fast DSL standard [1]) an aggregate data rate of 8.8 Gb/s has been achieved on a single 30m copper line during trials at Nokia Bell Labs in Antwerp, while the combination of FDX and multi-line bonding promises aggregate rates exceeding 20 Gb/s [2].

Unfortunately, FDX transmission in a multi-user DSL scenario leads to severe near-end crosstalk[1] (NEXT) interference in some cases, in addition to the usual far-end crosstalk (FEXT) inteference, which is also present in standard time division duplex (TDD) transmission. Both NEXT and FEXT result from the electro-magnetic coupling between lines in the same cable binder (see Fig. 1). NEXT interference originates from modems transmitting at the same side of the network as the receiving victim modem, while FEXT interference originates from modems transmitting at the other side of the network. Intra-modem NEXT is also referred to as echo. NEXT may be completely mitigated at the access node (AN) based on the available inter-user signal coordination. Analog hybrids at the transceivers isolate the transmit signals from the receive signals, such that the required dynamic range of the analog-to-digital converters is not substantially increased. The residual NEXT is then further removed digitally using vectoring techniques. Obviously, due to the lack of signal coordination between the customer premise equipments (CPEs), inter-user NEXT at the CPEs cannot be vectored and therefore may strongly degrade performance.

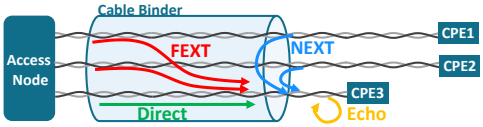

Fig. 1. In a G.fast multi-user FDX scenario users experience both near-end and far-end crosstalk at the customer side.

To limit the residual NEXT impact for DSL multi-user FDX transmission, tailored vectoring-based dynamic spectrum management (DSM) techniques are vital. Finding the optimal DS and US vectoring schemes in order to maximize the data rates is however a non-convex problem. Two algorithms are proposed in [3] that alternate between DS and US optimization using conventional US-DS duality theory. In each iteration the US parameters are optimized in the primal domain with a DS-US structure for fixed DS parameters; and vice versa, for fixed US parameters, the DS parameters are optimized in the dual domain with a reversed US-DS structure. The DS/US parameters are optimized using computationally expensive optimal spectrum balancing

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[1]NEXT is referred to as "self-coupling" or "self-interference" in wireless full-duplex communication systems.

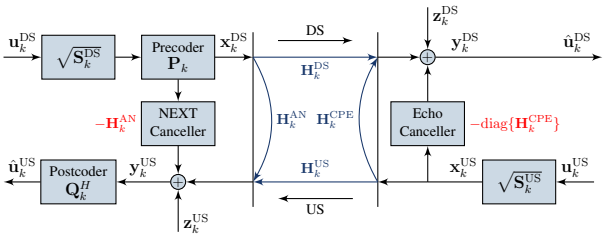

Fig. 2. Transmitter and receiver structure for FDX at tone k.

(OSB)-like [4] per-tone discrete exhaustive power or bit searches.

In this paper, we propose a novel low-complexity vectoring-based DSM algorithm for the specific case of perfect NEXT cancellation at the access node. This assumption in combination with US-DS duality theory allows to reformulate the DS-US structure of the optimization problem into an easier US-US structure. As a result, the novel algorithm can use distributed spectrum balancing (DSB)-like iterative fixed-point power updates, which already have proven to be very effective and powerful in half-duplex DSL scenarios [5]–[7]. The resulting algorithm is referred to as FDX-DSB. It encompasses the multiple access channel (MAC)-DSB [6] and the broadcast channel (BC)-DSB [7] algorithm as special cases for half-duplex US and DS transmission, respectively.

II. System Model

A. FDX Transmission Model

We consider FDX transmission in a G.fast DSL cable binder consisting of N lines each connecting one user. For the specific case of perfect NEXT cancellation at the AN (see Fig. 2) and standard synchronous discrete-multi-tone modulation, the transmission is modeled independently on each tone or frequency sub-carrier $k = [1, \ldots, K]$ as

$$\begin{bmatrix} \mathbf{y}_k^{\mathrm{DS}} \\ \mathbf{y}_k^{\mathrm{US}} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k^{\mathrm{DS}} & \mathbf{H}_k^{\mathrm{CPE}} \\ \mathbf{0} & \mathbf{H}_k^{\mathrm{US}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^{\mathrm{DS}} \\ \mathbf{x}_k^{\mathrm{US}} \end{bmatrix} + \begin{bmatrix} \mathbf{z}_k^{\mathrm{DS}} \\ \mathbf{z}_k^{\mathrm{US}} \end{bmatrix}. \quad (1)$$

$\mathbf{x}_k^{\mathrm{DS}} \triangleq [x_{k,1}^{\mathrm{DS}}, \ldots, x_{k,N}^{\mathrm{DS}}]$ is the DS transmit vector at tone k, where $x_{k,n}^{\mathrm{DS}}$ is the DS signal transmitted on line n and tone k. $\mathbf{y}_k^{\mathrm{DS}}$ and $\mathbf{z}_k^{\mathrm{DS}}$ have similar structures. $\mathbf{y}_k^{\mathrm{DS}}$ is the DS receive vector on tone k. $\mathbf{z}_k^{\mathrm{DS}}$ is the vector of uncorrelated additive noise signals at the CPE-side on tone k, without loss of generality $\mathcal{E}\{|z_{k,n}^{\mathrm{DS}}|^2\} = 1, \forall n$. $\mathbf{H}_k^{\mathrm{DS}}$ denotes the $N \times N$ DS FEXT matrix on tone k. The diagonal elements of $\mathbf{H}_k^{\mathrm{DS}}$ contain the direct channels whilst the off-diagonal elements contain the FEXT channels. $\mathbf{H}_k^{\mathrm{CPE}}$ denotes $N \times N$ NEXT matrix at tone k at the CPE. The diagonal elements of $\mathbf{H}_k^{\mathrm{CPE}}$ contain the echo channels whilst the off-diagonal elements contain the NEXT channels. The US quantities $\mathbf{x}_k^{\mathrm{US}}, \mathbf{y}_k^{\mathrm{US}}, \mathbf{z}_k^{\mathrm{US}}, \mathbf{H}_k^{\mathrm{US}}$ are similarly defined. The $N \times N$ NEXT matrix at the AN (denoted by $\mathbf{H}_k^{\mathrm{AN}}$ in Fig. 2) is set to the zero matrix. Note that the diagonal elements of $\mathbf{H}_k^{\mathrm{CPE}}$ are set to zero when considering also perfect echo cancellation at the CPE-side. Perfect knowledge of all channel matrices is assumed.

In G.fast per-line power spectral density (PSD) mask constraints are included, in order to not generate too much interference into other technologies, which are combined with per-line aggregated transmit power (ATP) constraints [8]. These per-line power constraints are given as follows in DS/US direction

$$\mathcal{E}\{|x_{k,n}^{\mathrm{DS/US}}|^2\} \leq P_k^{\mathrm{mask}}, \qquad \forall k, n \quad (2)$$

$$\sum_k \mathcal{E}\{|x_{k,n}^{\mathrm{DS/US}}|^2\} \leq P^{\mathrm{ATP}}, \qquad \forall n \quad (3)$$

with $P^{\mathrm{ATP}} = \overline{P}^{\mathrm{ATP}}/\Delta_f$ and Δ_f denoting the tone spacing.

The number of achievable bits that can be loaded on tone k of user n in DS/US direction is modeled by

$$b_{k,n}^{\mathrm{DS/US}} = \log_2\left(1 + \frac{\mathrm{SNR}_{k,n}^{\mathrm{DS/US}}}{\Gamma}\right), \quad (4)$$

where Γ denotes the capacity gap which takes practical QAM implementations into account, and is a function of the desired BER, coding gain, and noise margin. $\mathrm{SNR}_{k,n}^{\mathrm{DS/US}}$ denotes the signal-to-interference-and-noise ratio for user n and tone k in the DS/US direction (and will be defined later). Typically, a maximum bitloading $b^{\max}$ is also imposed. The total DS/US data rate of user n is $R_n^{\mathrm{DS/US}} = f_s \sum_k b_{k,n}^{\mathrm{DS/US}}$, where f_s is the DMT symbol rate.

At the AN, vectoring or signal coordination of the lines within the cable binder is possible. DS vectoring corresponds to precoding of the DS data signals, whilst US vectoring corresponds to postcoding of the US receive signals.

B. Nonlinear DS Vectoring

In the DS direction, we consider nonlinear precoding (NLP), in which the DS user data signals are sequentially encoded in order to pre-subtract the FEXT from previously encoded users without transmit power penalties. This corresponds to optimal dirty paper coding and may be implemented in practice with e.g. Tomlinson-Harashima precoding. Implementing NLP also requires a subtraction or encoding order of the lines which is assumed to be given by the user index without loss of generality.

The DS transmit signals are formed by gain scaling the DS data vector $\mathbf{u}_k^{\mathrm{DS}} \triangleq [u_{k,1}^{\mathrm{DS}}, \ldots, u_{k,N}^{\mathrm{DS}}]$ and multiplication with the $N \times N$ precoding matrix $\mathbf{P}_k = [\mathbf{p}_{k,1}, \ldots, \mathbf{p}_{k,N}]$ (see left-hand side of Fig. 2)

$$\mathbf{x}_k^{\mathrm{DS}} = \mathbf{P}_k \sqrt{\mathbf{S}_k^{\mathrm{DS}}} \mathbf{u}_k^{\mathrm{DS}}, \quad (5)$$

where the DS data vector satisfies $\mathcal{E}\{\mathbf{u}_k^{\mathrm{DS}}\mathbf{u}_k^{\mathrm{DS,H}}\} = \mathbf{I}_N$ and $\mathbf{S}_k^{\mathrm{DS}} \triangleq \mathrm{diag}\{\mathbf{s}_k^{\mathrm{DS}}\}$ is a diagonal matrix with $\mathbf{s}_k^{\mathrm{DS}} \triangleq [s_{k,1}^{\mathrm{DS}}, \ldots, s_{k,N}^{\mathrm{DS}}]$ defining the DS transmit powers at tone k. The DS transmit power on line n and tone k is then coupled across all users and is computed as

$$\mathcal{E}\{|x_{k,n}^{\mathrm{DS}}|^2\} = \left[\mathbf{P}_k\mathbf{S}_k^{\mathrm{DS}}\mathbf{P}_k^H\right]_{n,n}. \tag{6}$$

The SNR for user n and tone k in the DS with NLP is [9]

$$\mathrm{SNR}_{k,n}^{\mathrm{DS}} = \frac{s_{k,n}^{\mathrm{DS}}|\overline{\mathbf{h}}_{k,n}^{\mathrm{DS}}\mathbf{p}_{k,n}|^2}{\hat{\sigma}_{k,n} + \sum_{m>n} s_{k,m}^{\mathrm{DS}}|\overline{\mathbf{h}}_{k,n}^{\mathrm{DS}}\mathbf{p}_{k,m}|^2}, \tag{7}$$

where $\hat{\sigma}_{k,n} = 1 + \sum_m \left|\left[\mathbf{H}_k^{\mathrm{CPE}}\right]_{n,m}\right|^2 \mathcal{E}\{|x_{k,m}^{\mathrm{US}}|^2\}$ equals the DS noise plus NEXT interference from the US into the DS, and $\overline{\mathbf{h}}_{k,n}^{\mathrm{DS}}$ denotes the nth row of $\mathbf{H}_k^{\mathrm{DS}}$.

C. Nonlinear US Vectoring

In the US direction, on the other hand, we consider nonlinear postcoding in which the US user data signals are sequentially decoded in order to successively remove the FEXT from previously decoded users. Here the decoding order is assumed to be the reversed user index, i.e., user N is decoded first, user 1 is decoded last.

As of consequence, the US transmit vector is simply the gain scaled US data vector $\mathbf{u}_k^{\mathrm{US}}$. This leads to $\mathcal{E}\{\mathbf{x}_k^{\mathrm{US}}\mathbf{x}_k^{\mathrm{US},H}\} \triangleq \mathbf{S}_k^{\mathrm{US}} = \mathrm{diag}\{\mathbf{s}_k^{\mathrm{US}}\}$, with $\mathbf{s}_k^{\mathrm{US}} \triangleq [s_{k,1}^{\mathrm{US}}, \ldots, s_{k,N}^{\mathrm{US}}]$ modeling the US transmit powers (see right-hand side of Fig. 2).

The optimal nonlinear postcoder can easily be shown to be the well-known minimum-mean-squared-error generalized decision feedback equalizer (MMSE-GDFE). The MMSE-GDFE leads to the following epxression for the SNR for user n and tone k in the US [6]

$$\mathrm{SNR}_{k,n}^{\mathrm{US}} = s_{k,n}^{\mathrm{US}}\mathbf{h}_{k,n}^{\mathrm{US},H}\left(\mathbf{I}_N + \sum_{m<n} s_{k,m}^{\mathrm{US}}\mathbf{h}_{k,m}^{\mathrm{US}}\mathbf{h}_{k,m}^{\mathrm{US},H}\right)^{-1}\mathbf{h}_{k,n}^{\mathrm{US}} \tag{8}$$

where $\mathbf{h}_{k,n}^{\mathrm{US}} \triangleq \left[\mathbf{H}_k^{\mathrm{US}}\right]_{\mathrm{col}\ n}$. The corresponding MMSE postcoder matrix $\mathbf{Q}_k^H = [\mathbf{q}_{k,1}, \ldots, \mathbf{q}_{k,N}]^H$ is defined as

$$\mathbf{q}_{k,n} = \left(\mathbf{I}_N + \sum_{m\le n} s_{k,m}^{\mathrm{US}}\mathbf{h}_{k,m}^{\mathrm{US}}\mathbf{h}_{k,m}^{\mathrm{US},H}\right)^{-1}\mathbf{h}_{k,n}^{\mathrm{US}}s_{k,n}^{\mathrm{US}}. \tag{9}$$

Optimization of the US parameters is easy due to its convenient analytical structure. From (8), we see that the US data rates are solely dependent of the scalar transmit powers $\mathbf{s}_k^{\mathrm{US}}$. Once these are calculated, the postcoder matrices may be obtained by using the closed-form expression in (9).

III. FDX-DSB

In this section we develop a low-complexity algorithm that computes a local optimum of the weighted sum-rate maximization problem for the G.fast DSL multi-user FDX transmission, which takes the following form:

$$\begin{aligned}
\underset{\{\mathbf{P}_k\},\{\mathbf{s}_k^{\mathrm{DS/US}}\succeq\mathbf{0}\}}{\text{maximize}} \quad & \sum_n w_n^{\mathrm{DS}}R_n^{\mathrm{DS}} + \sum_n w_n^{\mathrm{US}}R_n^{\mathrm{US}} & \\
\text{s.t.} \quad & \sum_k \left[\mathbf{P}_k\mathbf{S}_k^{\mathrm{DS}}\mathbf{P}_k^H\right]_{n,n} \le P^{\mathrm{ATP}} & \forall n \\
& \left[\mathbf{P}_k\mathbf{S}_k^{\mathrm{DS}}\mathbf{P}_k^H\right]_{n,n} \le P_k^{\mathrm{mask}} & \forall k,n \\
& \sum_k \left[\mathbf{S}_k^{\mathrm{US}}\right]_{n,n} \le P^{\mathrm{ATP}} & \forall n \\
& \left[\mathbf{S}_k^{\mathrm{US}}\right]_{n,n} \le P_k^{\mathrm{mask}} & \forall k,n
\end{aligned} \tag{10}$$

where $w_n^{\mathrm{DS/US}}$ is the weight of user n in the DS/US direction.

To solve (10), we adopt the typical dual decomposition approach of DSM algorithms by applying Lagrangian relaxation with e.g. standard subgradient based updating of the Lagrange multipliers, and by relying on the "zero-duality gap"-result[2] for multi-carrier systems [10]. The Lagrangian is then given by

$$\begin{aligned}
\mathcal{L} = & \underbrace{\sum_n w_n^{\mathrm{DS}}R_n^{\mathrm{DS}} - \sum_k \mathrm{Tr}\{\mathbf{\Theta}_k^{\mathrm{DS}}\mathbf{P}_k\mathbf{S}_k^{\mathrm{DS}}\mathbf{P}_k^H\}}_{\mathcal{L}^{\mathrm{DS}}} \\
& + \underbrace{\sum_n w_n^{\mathrm{US}}R_n^{\mathrm{US}} - \sum_k \mathrm{Tr}\{\mathbf{\Theta}^{\mathrm{US}}\mathbf{S}_k^{\mathrm{US}}\}}_{\mathcal{L}^{\mathrm{US}}} \\
& + \underbrace{\sum_n \left[\left(\theta_n^{\mathrm{DS}} + \theta_n^{\mathrm{US}}\right)P^{\mathrm{ATP}} + \sum_k \lambda_{k,n}P_k^{\mathrm{mask}}\right]}_{\tilde{P}}.
\end{aligned} \tag{11}$$

$\mathbf{\Theta}_k^{\mathrm{DS}} = \mathrm{diag}\{\theta_n^{\mathrm{DS}} + \lambda_{k,n}\}$ is a diagonal matrix containing the Lagrange multipliers that correspond to the DS per-line ATP and PSD mask constraints at tone k, respectively. Whereas $\mathbf{\Theta}^{\mathrm{US}} = \mathrm{diag}\{\theta_n^{\mathrm{US}}\}$ contains the Lagrange multipliers corresponding to the US per-line ATP constraints. The Lagrange dual function is then defined as the unconstrained maximization of the Lagrangian for a given set of Lagrange multipliers

$$\begin{aligned}
& g(\{\mathbf{\Theta}_k^{\mathrm{DS}}\}, \mathbf{\Theta}^{\mathrm{US}}) = \\
& \max_{\{\mathbf{P}_k\},\{\mathbf{s}_k^{\mathrm{DS/US}}\in\mathcal{D}_k\}} \mathcal{L}\left(\{\mathbf{\Theta}_k^{\mathrm{DS}}\}, \mathbf{\Theta}^{\mathrm{US}}, \{\mathbf{P}_k\}, \{\mathbf{s}_k^{\mathrm{DS/US}}\}\right)
\end{aligned} \tag{12}$$

[2]Although the "zero-duality gap"-result of [10] is generally only valid for the per-line ATP constraints, assuming a zero-duality gap results in an algorithm that seems to work well in practice.

where $\mathcal{D}_k \triangleq \{s_{k,n}^{\mathrm{DS/US}} | s_{k,n}^{\mathrm{DS}} \geq 0 \text{ and } 0 \leq s_{k,n}^{\mathrm{US}} \leq P_k^{\mathrm{mask}}, \forall n\}$. The master dual optimization problem is

$$\underset{\{\mathbf{\Theta}_k^{\mathrm{DS}} \succeq \mathbf{0}\}, \mathbf{\Theta}^{\mathrm{US}} \succeq \mathbf{0}}{\text{minimize}} \; g(\{\mathbf{\Theta}_k^{\mathrm{DS}}\}, \mathbf{\Theta}^{\mathrm{US}}). \tag{13}$$

Maximizing the Lagrangian in (12) is difficult since it has a DS-US structure that is non-convex and, on top of that, is coupled due the NEXT from the US into the DS. The difficulty arises from the fact that a DS problem typically is solved in its dual US domain. We tackle this difficulty by reformulating the Lagrangian into an equivalent dual Lagrangian which has a full US-US structure, i.e., a dual US and a primal US term.

Key is to view in the DS the NEXT interference from the US explicitly as background noise, i.e.,

$$\mathbf{y}_k^{\mathrm{DS}} = \mathbf{H}_k^{\mathrm{DS}} \mathbf{x}_k^{\mathrm{DS}} + \underbrace{\mathbf{H}_k^{\mathrm{CPE}} \mathbf{x}_k^{\mathrm{US}} + \mathbf{z}_k^{\mathrm{DS}}}_{\mathbf{z}_k'^{\mathrm{DS}}}, \tag{14}$$

such that a virtual pre-whitening may be applied to the DS receive signals

$$\begin{aligned} \tilde{\mathbf{y}}_k^{\mathrm{DS}} &= \mathbf{\Sigma}_k^{-1/2} \mathbf{y}_k^{\mathrm{DS}} \\ &= \underbrace{\mathbf{\Sigma}_k^{-1/2} \mathbf{H}_k^{\mathrm{DS}}}_{\widetilde{\mathbf{H}}_k^{\mathrm{DS}}} \mathbf{x}_k^{\mathrm{DS}} + \tilde{\mathbf{z}}_k^{\mathrm{DS}}, \end{aligned} \tag{15}$$

with $\mathbf{\Sigma}_k = \mathrm{diag}\{\mathbf{I}_N + \mathbf{H}_k^{\mathrm{CPE}} \mathbf{S}_k^{\mathrm{US}} \mathbf{H}_k^{\mathrm{CPE},H}\}$. Now $\tilde{\mathbf{z}}_k^{\mathrm{DS}}$ satisfies $\mathcal{E}\{\tilde{\mathbf{z}}_k^{\mathrm{DS}} \tilde{\mathbf{z}}_k^{\mathrm{DS},H}\} = \mathbf{I}_N$. The pre-whitening $\mathbf{\Sigma}_k^{-1/2}$ thus effectively moves the NEXT at the CPE-side into the DS FEXT channel matrix, such that (15) corresponds to a mere broadcast channel with $\widetilde{\mathbf{H}}_k^{\mathrm{DS}}$ as equivalent channel matrix.

As a direct consequence, the US-DS duality theory from [7] may be applied to the DS direction. This duality theory states that the same set of rates in the dual US domain as in the primal DS domain can be achieved under the same virtual sum-power constraint for a fixed set of DS Lagrange multipliers $\{\mathbf{\Theta}_k^{\mathrm{DS}}\}$. This dual US domain corresponds to a so-called MAC and is obtained by switching the role of precoder and postcoder, by Hermitian transposing the channel matrix, and by scaling the dual US noise powers with the DS Lagrange multipliers, i.e.,

$$\hat{\mathbf{y}}_k^{\mathrm{US}} = \mathbf{P}_k^H \left(\widetilde{\mathbf{H}}_k^{\mathrm{DS},H} \hat{\mathbf{x}}_k^{\mathrm{US}} + \underbrace{(\mathbf{\Theta}_k^{\mathrm{DS}})^{1/2} \hat{\mathbf{z}}_k^{\mathrm{US}}}_{\hat{\mathbf{z}}_k'^{\mathrm{US}}} \right), \tag{16}$$

where $\mathbf{P}_k^H$ is the $N \times N$ dual US postcoder matrix at tone k. Observe that $\mathcal{E}\{\hat{\mathbf{z}}_k'^{\mathrm{US}} \hat{\mathbf{z}}_k'^{\mathrm{US},H}\} = \mathbf{\Theta}_k^{\mathrm{DS}}$. Furthermore, we define the diagonal dual US transmit covariance matrix as $\hat{\mathbf{S}}_k^{\mathrm{US}} \triangleq \mathbf{\Sigma}_k^{-1} \mathcal{E}\{\hat{\mathbf{x}}_k^{\mathrm{US}} \hat{\mathbf{x}}_k^{\mathrm{US},H}\}$, such that the diagonal pre-whitening filter $\mathbf{\Sigma}_k^{-1/2}$ ends up scaling the virtual sum-power instead of the FEXT channel matrix $\widetilde{\mathbf{H}}_k^{\mathrm{DS},H}$. This virtual sum-power then corresponds to $\sum_k \mathrm{Tr}\{\mathbf{\Theta}_k^{\mathrm{DS}} \mathbf{P}_k \mathbf{S}_k^{\mathrm{DS}} \mathbf{P}_k^H\} = \sum_k \mathbf{\Sigma}_k \hat{\mathbf{S}}_k^{\mathrm{US}}$. Finally, considering the MMSE-GDFE-based postcoder, the SNR of user n and tone k in the dual US is expressed as follows

$$\hat{\mathrm{SNR}}_{k,n}^{\mathrm{US}} = \hat{s}_{k,n}^{\mathrm{US}} \overline{\mathbf{h}}_{k,n}^{\mathrm{DS}} \left(\mathbf{\Theta}_k^{\mathrm{DS}} + \sum_{m<n} \hat{s}_{k,m}^{\mathrm{US}} \overline{\mathbf{h}}_{k,m}^{\mathrm{DS},H} \overline{\mathbf{h}}_{k,m}^{\mathrm{DS}} \right)^{-1} \overline{\mathbf{h}}_{k,n}^{\mathrm{DS},H} \tag{17}$$

where the decoding order is the reverse of the encoding order (i.e. user 1 is decoded last, user N is decoded first) as US-DS duality dictates.

As a result, we are able to define the following equivalent Lagrangian of (11) that needs to be maximized in the dual Lagrange function (12) each iteration of the subgradient search, for a given set of Lagrange multipliers $\{\mathbf{\Theta}_k^{\mathrm{DS}}, \mathbf{\Theta}^{\mathrm{US}}\}$

$$\mathcal{L}' = \underbrace{\sum_n w_n^{\mathrm{DS}} \hat{R}_n^{\mathrm{US}} - \sum_k \mathrm{Tr}\{\mathbf{\Sigma}_k \hat{\mathbf{S}}_k^{\mathrm{US}}\}}_{\hat{\mathcal{L}}_k^{\mathrm{US}}} + \mathcal{L}^{\mathrm{US}} + \tilde{P}. \tag{18}$$

Although (18) is still non-convex, it has an easier full US-US structure that solely depends on scalar user transmit powers $\{\hat{\mathbf{s}}_k^{\mathrm{US}}, \mathbf{s}_k^{\mathrm{US}}\}$. For instance, these transmit powers can be calculated using DSB-like iterative fixed-point power updates that solve $\partial \mathcal{L}' / \partial \{\hat{s}_{k,n}^{\mathrm{US}}, s_{k,n}^{\mathrm{US}}\} = 0, \forall k, n$, which are based on sequential convex approximation, and already have proven to be very effective and powerful in half-duplex DSL scenarios [5]–[7]. Moreover, once these powers are calculated, the corresponding DS transmit powers and precoding matrices can be obtained using SNR duality transformations [7]. The resulting algorithm (listed in Alg. 1) is referred to as FDX-DSB and will be elaborated in a forthcoming report, further details are omitted here for brevity.

We emphasize that FDX-DSB avoids the alternating optimization in [3] between a primal domain with a DS-US structure and a dual domain with a reversed US-DS structure, which requires on top of that the use of computationally expensive OSB-like [4] per-tone discrete exhaustive power or bit searches. On the other hand, without perfect NEXT cancellation at the AN, this approach of FDX-DSB is not feasible due to the additional NEXT coupling from the DS into the US.

When $w_n^{\mathrm{DS}} = 0, \forall n$ (no active DS transmission), FDX-DSB simplifies to MAC-DSB [6] for half-duplex US scenarios. On the other hand, when $w_n^{\mathrm{US}} = 0, \forall n$ (no active US transmission), FDX-DSB simplifies to BC-DSB [7] for half-duplex DS scenarios.

IV. Simulations

In this section, we simulate the proposed FDX-DSB algorithm for two fiber to the building (FTTB) G.fast scenarios consisting of $N = 10$ users (depicted in

Algorithm 1: FDX-DSB

repeat

Set/update $\{\mathbf{\Theta}_k^{\mathrm{DS}}\}, \mathbf{\Theta}^{\mathrm{DS}}$

repeat

$\forall k, n$: obtain $\hat{s}_{k,n}^{\mathrm{US}}$ by solving $\frac{\partial \mathcal{L}'}{\partial \hat{s}_{k,n}^{\mathrm{US}}} = 0$

$\forall k, n$: obtain $s_{k,n}^{\mathrm{US}}$ by solving $\frac{\partial \mathcal{L}'}{\partial s_{k,n}^{\mathrm{US}}} = 0$

until Convergence

Obtain $\mathbf{P}_k^H$ as dual MMSE-GDFE [7], $\forall k$

Obtain $\mathbf{S}_k^{\mathrm{DS}}$ with US-DS SNR duality transformations [7], $\forall k$

until (2) and (3) are satisfied

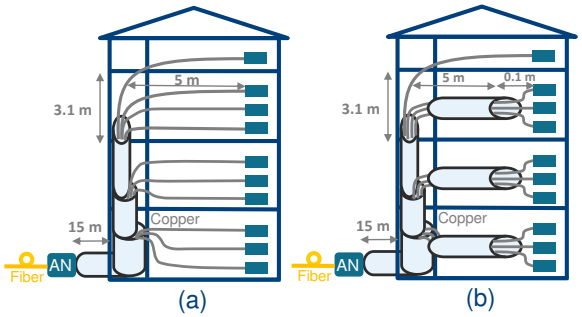

Fig. 3. 10-User fiber to the building G.fast scenarios

Fig. 3). Considering the recommended G.fast parameters [1], we set the ATP to 4 dBm while the PSD masks are obtained from [8] ranging from -65 dBm/Hz to -79 dBm/Hz. The tone spacing is 51.75 kHz and the symbol rate is 48 kHz.The capacity gap Γ is set to 10 dB and the noise PSD to -140 dBm/Hz. Bearing the short line lengths in mind, we use $b^{\max} = 15$. Both the FEXT and NEXT channel matrices are modeled based on lab measurements. The number of tones $K = 4096$ corresponding to a bandwidth up to 212 MHz. In addition to perfect echo cancellation, perfect NEXT cancellation at the AN is assumed.

The simulation reveals significant performance gains of FDX transmission over standard TDD transmission (see Fig. 4). For instance, for an average 1.5 Gbps US rate, FDX transmission achieves an average DS rate of 2.14 Gbps and 2.46 Gbps in scenario (a) and (b) respectively with $N = 10$, compared to an average DS rate of 1.35 Gbps for TDD. The slightly larger gain in scenario (a) is because of the larger separation length between the CPEs in each floor [10m in scenario (a) versus 0.2m in scenario (b)]. This means that the NEXT signals travel a larger distance over the copper lines in scenario (a) such that they are indeed attenuated more. Moreover, scenario (a) with only one active user per floor (i.e. $N = 4$) results in a lower overall NEXT level, yielding even larger FDX gains (see Fig. 4). These results demonstrate that the level of NEXT interference at the CPE is crucial in G.fast multi-user FDX transmission.

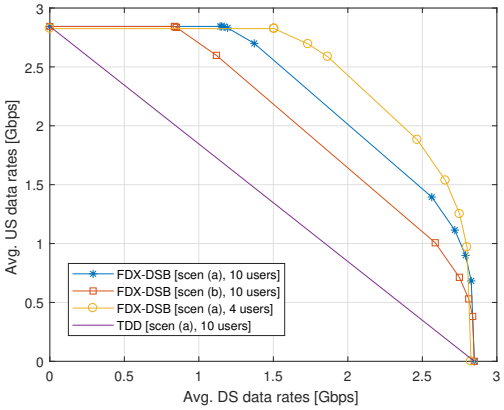

Fig. 4. The rate regions of the G.fast FDX FTTB scenarios in Fig. 3 reveal significant performance gains over standard TDD transmission.

V. Conclusion

We have proposed a novel algorithm for vectoring-based DSM in a DSL multi-user FDX scenario with perfect NEXT cancellation at the AN. The algorithm is referred to as FDX-DSB and will be elaborated in a forthcoming report. It relies on reformulating the DS-US structure of the non-convex weighted sum-rate maximization into an easier US-US structure, by exploiting US-DS duality. This avoids the alternating optimization approach of [3] between a primal domain with a DS-US structure and a dual domain with a reversed US-DS structure. The reformulated problem can then be solved with low-complexity iterative DSB-like fixed-point power updates. FDX simulations of a 10-user G.fast FTTB scenario show significant improvements over standard TDD transmission.

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