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Author contact	wouter.lanneer@esat.kuleuven.be *+ 32 16 32 79 75*
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α-Fair Dynamic Spectrum Management for QRD-Based Precoding with User Encoding Ordering in Downstream G.fast Transmission

Wouter Lanneer, *Student Member, IEEE,* Paschalis Tsiaflakis, *Member, IEEE,*
Jochen Maes, *Senior Member, IEEE,* and Marc Moonen, *Fellow, IEEE*

***Abstract*—In next generation DSL networks such as G.fast, employing discrete multi-tone transmission in high frequencies up to 212 MHz, the crosstalk among lines reaches very high levels. To precompensate the crosstalk in downstream transmission, QRD-based precoding has been proposed as a near-optimal dynamic spectrum management (DSM) technique. However, the performance of QRD-based precoding is greatly affected by the user encoding ordering (UEO). Since current multi-tone UEO methods are rather heuristic in the way they approach fairness, we develop in this paper a set of novel DSM algorithms for joint power allocation and UEO that enforce a generalized α-fairness policy. Since finding the globally optimal UEO entails a combinatorial optimization problem with excessive computational complexity, an iterative algorithm is proposed that uses per-tone exhaustive searches (PTES's) and provides near-optimal approximate solutions. To further reduce the computational complexity, two suboptimal methods are suggested to replace the expensive PTES's, leading to two additional α-fair DSM algorithms that are tractable for large scenarios against little performance loss. Simulations of a G.fast cable binder show that the α-fair DSM algorithms achieve an efficient trade-off between fairness and performance in contrast to current UEO methods.**

***Index Terms*—DSL, G.fast, dynamic spectrum management (DSM), nonlinear precoding (NLP), user encoding ordering**

I. Introduction

PUSHED by increasing data rate demand and heavy competition, the International Telecommunication Union (ITU) has introduced a next generation digital subscriber lines (DSL) access technology called G.fast [2]. G.fast marks the beginning of "ultrabroadband copper access" offering gigabit (i.e. fiber-like) transmission speeds, which are achieved by employing discrete multi-tone transmission (DMT) in a broad spectrum up to 212 MHz over very short copper telephony lines (below 100 m).

Unfortunately, the broad spectrum also renders the use of dynamic spectrum management (DSM) techniques for crosstalk precompensation in downstream G.fast transmission very challenging. In particular at high frequencies the crosstalk among lines (i.e. users) in the same cable binder reaches very high levels. Foremost the traditional linear zero-forcing (ZF) precoder [3] suffers from a reduced signal-to-noise ratio (SNR), due to the large precompensation signals needed penalizing the line transmit power spectral densities (PSDs). The linear ZF precoder may be boosted by using power allocation (PA) optimization [4], and by replacing the ZF criterion with the minimum mean squared error (MMSE) criterion which tolerates some residual crosstalk at the benefit of increased SNR [5].

An alternative DSM technique for improving the SNR is nonlinear precoding (NLP) which can be combined with either the ZF or the theoretically-optimal MMSE criterion. In this paper we pragmatically focus on ZF-NLP by means of QRD-based precoding [6] which allows for easier expressions compared to MMSE-NLP, against only a very small performance loss [5]. NLP entails here the dirty paper coding-like sequential encoding of the user transmit signals in order to pre-subtract the crosstalk from previously encoded users without transmit power penalties. This may be implemented in practice with Tomlinson-Harashima precoding (THP) [7], [8]. Since users encoded first typically experience higher data rate gains than users encoded later, the multi-tone user encoding ordering (UEO) greatly affects the user data rate performance. Clearly, this user data rate variance is undesired and calls for fairness-aware multi-tone UEO methods that balance the user data rates.

However, current multi-tone UEO methods are rather heuristic in the way they approach fairness and likely suffer from suboptimality. The most well-known method is the V-BLAST scheme [9] which provides the UEO that maximizes the minimum SNR for each tone separately (i.e. per-tone max-min fairness). In G.fast simulations, V-BLAST is shown to be outperformed by an efficient heuristic method called dynamic ordering (DO) proposed in [10]. Another heuristic UEO method seeking fairness is the projected sorting method [11]. All these UEO methods assume a fixed PA obtained by a simplified scaling such as e.g. the column norm based scaling (CNBS) method [4]. On the other hand, ignoring fairness and

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W. Lanneer and M. Moonen are with the STADIUS Center for Dynamical Systems, Signal Processing and Data Analytics, Dept. of Electrical Engineering (ESAT), KU Leuven, BE 3000 Leuven, Belgium (e-mail: {wouter.lanneer, marc.moonen}@esat.kuleuven.be).

P. Tsiaflakis and J. Maes are with the Copper Access and Indoor Team of Nokia Bell Labs, BE 2018 Antwerp, Belgium (e-mail: {paschalis.tsiaflakis, jochen.maes}@nokia-bell-labs.com).

merely targeting performance (i.e. maximizing the sum-rate or mean-rate) is easier. For this case some efficient UEO methods exist that provide near-optimal solutions, like the successive ordering search (SOS) [12] and the max-sum sorted QRD method [13], [14][1].

In this paper, to offer an alternative to the heuristic fairness approach of the current UEO methods in downstream G.fast transmission, we develop a set of DSM algorithms for QRD-based precoding with joint PA and UEO that enforce a generalized α-fairness policy. This α-fairness approach is widely used in wireless communication (see e.g. [15]) and allows an optimal trade-off between performance (in the sense of user mean-rate) and fairness (in the sense of user min-rate).

Unfortunately, finding the globally optimal UEO entails a combinatorial optimization problem with an exponential complexity in the number of tones and users, which makes the considered optimization problem numerically infeasible. Therefore, an iterative block coordinate ascent (BCA) algorithm is proposed that uses a per-tone exhaustive search (PTES) and provides near-optimal approximate solutions. This algorithm will be referred to as α-fair PTES [1].

Notwithstanding having a linear complexity in the number tones, α-fair PTES continues to be prohibitively complex for scenarios with a large number of users, e.g. more than five users in a G.fast scenario. Therefore, to further reduce the computational complexity of α-fair PTES, two efficient sub-optimal methods, i.e. SOS [12] and derivative based ordering (DBO), are suggested to replace the expensive PTES. This leads to two additional α-fair DSM algorithms, referred to as α-fair SOS and α-fair DBO, with a polynomial complexity in the number of users. Simulations of a measured G.fast cable binder show that the α-fair DSM algorithms achieve an efficient performance-fairness trade-off, in contrast the current UEO methods, such as DO [10], which provide only a single operation point. In particular, α-fair SOS may serve as an upper bound for practical UEO methods in G.fast, whereas α-fair DBO has the lowest computational complexity and may be seen as an efficient extension of the DO method with joint α-fair PA and UEO optimization.

The paper is organized as follows. In Section II downstream G.fast transmission QRD-based precoding with UEO and the α-fair DSM maximization problem are introduced. In Section III α-fair PTES is presented, which solves this problem approximately with PTES's. In Section IV, the complexity of α-fair PTES is further reduced with SOS and DBO. In Section V α-fair DSM is compared with the current UEO methods in a G.fast context. Finally, Section VI concludes the paper.

Notations: We use lowercase boldface letters to denote vectors and uppercase boldface letters for matrices. We use $\mathbf{I}_A$ as the identity matrix of size A, $(.)^T$ as the transpose, $(.)^H$ as Hermitian transpose, $\mathcal{E}\{.\}$ as expectation, $[x]_a^b$ as $\max(a, \min(x, b))$, $[\mathbf{X}]_{i,j}$ as the i,j-th element of $\mathbf{X}$, $\text{diag}\{\mathbf{X}\}$ as a diagonal matrix with $\{[\mathbf{X}]_{ii}, \forall i\}$ on the main diagonal, $|\cdot|$ as the absolute value.

[1]The greedy sorted QRD method proposed in [13] for maximizing the min-rate is simply tweaked for maximizing the sum-rate by replacing the argmin by argmax [14].

II. Problem Formulation

A. Transmission Model

We consider downstream transmission in a wireline DSL cable binder consisting of N interfering users (or lines) and K tones (or frequency sub-carriers). Assuming standard synchronous DMT, transmission is modeled independently on tones $k = [1, \cdots, K]$ as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k. \tag{1}$$

$\mathbf{x}_k \triangleq [x_k^1, \ldots, x_k^N]$ is the vector of transmitted signals at the DPU on tone k, with $\mathcal{E}\{|x_k^n|^2\}$ being the transmit power on line n. $\mathbf{y}_k$ is the vector of received signals on tone k. $\mathbf{z}_k$ is the vector of uncorrelated additive noise samples on tone k, with $\sigma_k \triangleq \mathcal{E}\{|z_k^n|^2\}$ denoting the noise power. $\mathbf{H}_k \triangleq [h_k^{n,m}]$ denotes the $N \times N$ channel matrix on tone k. The diagonal elements of $\mathbf{H}_k$ contain the direct-channels whereas the off-diagonal elements contain the crosstalk channels.

Although the direct channels of $\mathbf{H}_k$ typically are dominant below 30 MHz (i.e. $|h_k^{n,n}| \gg |h_k^{n,m}|, m \neq n$), measurements have shown that this is not the case anymore for higher frequencies of G.fast where the direct channels may even be smaller than the crosstalk channels [16]. This in particular makes it important to consider improved vectoring schemes like QRD-based precoding with UEO. We assume perfect knowledge of the channel matrices.

In G.fast per-line spectral mask constraints are included, in order to not generate too much interference into other technologies, and combined with per-line aggregated transmit power (ATP) constraints [17]. These power constraints are given as[2]

$$\mathcal{E}\{|x_k^n|^2\} \leq P_k^{\text{mask}}, \qquad \forall k, n, \tag{2}$$

$$\sum_k \mathcal{E}\{|x_k^n|^2\} \leq P^{\text{total}}, \qquad \forall n. \tag{3}$$

B. QRD-Based Precoding with UEO

A well-known implementation of ZF-NLP with sequential crosstalk pre-subtraction is THP [6]. It is based on the QRD of the conjugate transposed channel matrix, and involves feedforward and feedback filtering of the user data vector $\mathbf{u}_k \triangleq [u_k^1, \ldots, u_k^N]$ (see Fig. 1), where the user data signals $\{u_k^n\}$ are assumed to be uncorrelated and consist of unit power and zero mean QAM symbols, i.e. $\mathcal{E}\{\mathbf{u}_k \mathbf{u}_k^H\} = \mathbf{I}_N$.

THP can adopt any UEO into the sequential crosstalk pre-subtraction process. It entails a user permutation which is enforced by left-multiplication with the $N \times N$ permutation matrix $\mathbf{E}_k$ defined as

$$[\mathbf{E}_k]_{n,m} \triangleq \begin{cases} 1 & \text{if } m = \pi_k^n \\ 0 & \text{else,} \end{cases} \tag{4}$$

where $\boldsymbol{\pi}_k \triangleq [\pi_k^1, \ldots, \pi_k^N]$ is the UEO vector with π_k^i designating the i-th encoded user on tone k. This means that $\mathbf{E}_k[u_k^1, \ldots, u_k^N]^T = [u_k^{\pi_k^1}, \ldots, u_k^{\pi_k^N}]^T$. Furthermore, left-multiplication of a matrix with $\mathbf{E}_k^T$ leads to the inverse row

[2]Note that $P^{\text{ATP}} \triangleq \overline{P}^{\text{ATP}}/\Delta_f$ with $\overline{P}^{\text{ATP}}$ and Δ_f denoting the ATP and tone spacing, respectively.

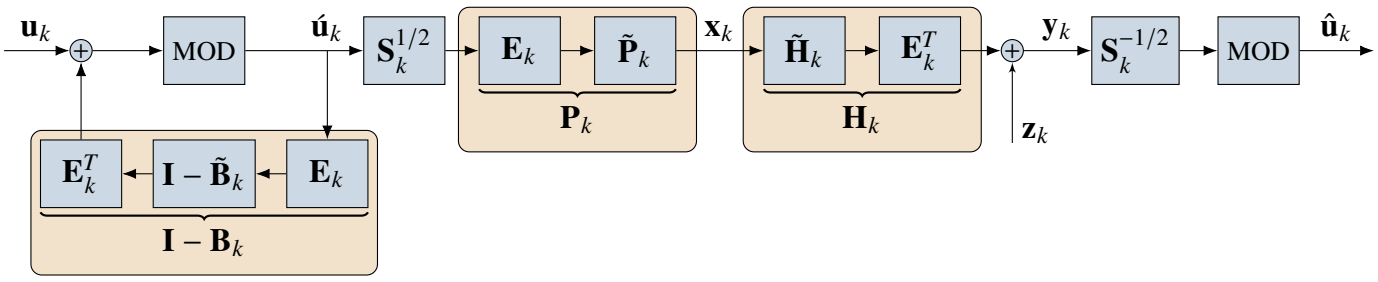

Fig. 1. Block diagram of the THP implementation with UEO for tone k.

permutation, as $\mathbf{E}_k^T\mathbf{E}_k = \mathbf{I}_N$. Similarly, right-multiplication of a matrix with $\mathbf{E}_k^T$ and $\mathbf{E}_k$ will result into the permutation and inverse permutation of the columns, respectively.

To include the UEO into the common THP representation, as explained in [6], we extend the QRD of the conjugate transposed channel matrix with the permutation matrix $\mathbf{E}_k$, i.e.,

$$(\mathbf{E}_k\mathbf{H}_k)^H \overset{\text{QRD}}{=} \mathbf{Q}_k\mathbf{R}_k, \tag{5}$$

where $\mathbf{Q}_k$ is a unitary matrix and $\mathbf{R}_k^H \triangleq [r_k^{n,m}]$ is a lower triangular matrix. Now,

$$\mathbf{H}_k = \mathbf{E}_k^T \underbrace{\mathbf{R}_k^H\mathbf{Q}_k^H}_{\tilde{\mathbf{H}}_k}, \tag{6}$$

where $\tilde{\mathbf{H}}_k$ may be seen as the re-ordered channel matrix according to $\boldsymbol{\pi}_k$. Then $\tilde{\mathbf{P}}_k \triangleq \mathbf{Q}_k\text{diag}(\mathbf{R}_k^H)^{-1}$ leads to the final $N \times N$ feedforward precoder matrix

$$\mathbf{P}_k = \mathbf{Q}_k\text{diag}(\mathbf{R}_k^H)^{-1}\mathbf{E}_k. \tag{7}$$

Hence the transmit vector is

$$\mathbf{x}_k = \mathbf{P}_k\mathbf{S}_k^{1/2}\acute{\mathbf{u}}_k, \tag{8}$$

where $\acute{\mathbf{u}}_k \triangleq [\acute{u}_k^1, \ldots, \acute{u}_k^N]$ is the data vector after the feedback filtering (see Fig. 1). The diagonal gain scaling matrix $\mathbf{S}_k \triangleq \text{diag}\{\mathbf{s}_k\}$ with $\mathbf{s}_k \triangleq [s_k^1, \ldots, s_K^N]$ models the per-user transmit powers on tone k. This leads to the following transmit PSD on line n and tone k

$$\mathcal{E}\{|x_k^n|^2\} = \sum_m \left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^m. \tag{9}$$

The sequential crosstalk pre-subtraction according to the UEO is realized by the $N \times N$ feedback filtering matrix $\mathbf{I}-\mathbf{B}_k$. The pre-subtracted signals are prevented from increase in transmit power by applying a nonlinear modulo operation to both dimensions of the QAM constellation (see [6] for exact formulas). Thus $\acute{\mathbf{u}}_k$ is computed sequentially from the first to the last encoded user data signal (i.e. from user π_k^1 to π_k^N) as

$$\acute{u}_k^{\pi_k^n} = \text{mod}\left[u_k^{\pi_k^n} - \sum_{m=1}^{n-1}[\tilde{\mathbf{B}}_k]_{n,m}\acute{u}_k^{\pi_k^m}\right], \qquad \forall n. \tag{10}$$

with $\tilde{\mathbf{B}}_k$ a lower triangular matrix defined as

$$[\tilde{\mathbf{B}}_k]_{n,m} = \begin{cases} r_k^{n,m}\sqrt{s_k^{\pi_k^m}} \Big/ r_k^{m,m}\sqrt{s_k^{\pi_k^n}} & \text{if } m \le n \\ 0 & \text{else,} \end{cases} \tag{11}$$

while the final $\mathbf{B}_k = \mathbf{E}_k^T\tilde{\mathbf{B}}_k\mathbf{E}_k$ is no longer lower triangular.

At the receiver a second modulo operation is required to make the first modulo operation transparent [6], i.e.,

$$\hat{u}_k^{\pi_k^n} = \text{mod}\left[\frac{y_k^{\pi_k^n}}{\sqrt{s_k^{\pi_k^n}}}\right] \simeq u_k^{\pi_k^n} + \frac{z_k^{\pi_k^n}}{\sqrt{s_k^{\pi_k^n}}}. \tag{12}$$

In practice the modulo operation at the transmitter results in a precoding loss, since it causes the signals $\{\acute{u}_k^n\}$ to be approximately uniformly distributed, and hence to have more energy than the QAM signals $\{u_k^n\}$. Another incurred loss is that the modulo operation at the receiver increases the number of nearest neighbors at the boundary of the QAM constellations, eventually leading to an increased number of bit errors. These losses are most pronounced for low SNR [18]. Since they have only a small impact on the total data rates, these losses are not taken into account in the simulations in Section V.

From (12) it is seen that the number of achievable bits that can be loaded on tone k for user n is given as

$$b_k^n = \log_2\left(1 + \Gamma^{-1}\sigma_k^{-1}s_k^n\right), \tag{13}$$

where $\Gamma > 0$ dB denotes the so-called capacity gap. It models the performance gap between a practical DSL system which transmits QAM constellation points and the theoretical capacity which assumes ideal Gaussian signaling. It is a function of the desired BER, coding gain, and noise margin [19]. Further, the total data rate of user n is $R_n = f_s\sum_k b_k^n$, where f_s is the DMT symbol rate.

C. α-Fair Dynamic Spectrum Management

Instead of the traditional weighted sum-rate, we consider an objective based on the generalized α-fairness utility functions of the user data rates $\{R_n\}$ [20]. This leads to the following α-fair DSM maximization problem in the UEO $\{\boldsymbol{\pi}_k\}$ and PA $\{\mathbf{s}_k\}$ variables

$$\underset{\{\boldsymbol{\pi}_k\},\{\mathbf{s}_k\in\mathcal{D}_k\}}{\text{maximize}} \; f_o(\{\boldsymbol{\pi}_k,\mathbf{s}_k\}) = \begin{cases} \sum_n w_n\dfrac{R_n^{1-\alpha}}{1-\alpha} & \alpha \ne 1, \alpha \ge 0 \\ \sum_n w_n\log(R_n) & \alpha = 1 \end{cases}$$
$$\text{s.t.} \qquad \sum_m \left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^m \le P_k^{\text{mask}}, \qquad \forall k,n. \tag{14}$$

In the objective function $f_o(\{\boldsymbol{\pi}_k,\mathbf{s}_k\})$, w_n is the weight of user n, and $\alpha \in [0,\infty)$ controls the trade-off between performance (user sum-rate or mean-rate) on the one hand,

Algorithm 1: α-fair PTES

Initialize $\forall k$:
  $\boldsymbol{\pi}_k = (1, \cdots, N)$
  $\mathbf{s}_k = \mathbf{0}_N$
repeat
  for $k = 1 \cdots K$ **do**
    // Exhaustive search on tone k
    foreach $\boldsymbol{\pi}_k$ **do**
      $\mathbf{s}_k \leftarrow \mathrm{PA}(k, \boldsymbol{\pi}_k, \alpha)$
      if f_o is increased **then**
        Store $\{\mathbf{s}_k, \boldsymbol{\pi}_k\}$
      end
    end
  end
until the function value f_o in (14) is converged

and fairness (user min-rate), on the other hand. In particular, $\alpha \to \infty$ corresponds to max-min fairness (which can be considered the fairest allocation), $\alpha = 1$ corresponds to the so-called proportional fairness policy, and $\alpha = 0$ corresponds to weighted-sum maximization (which is the most performant allocation). In addition, the maximum bit loading constraint (i.e. a bit cap $b^{\max}$) is translated here into a PSD cap in (14) where $\mathcal{D}_k \triangleq \left\{ s_k^n | 0 \le s_k^n \le \Gamma \sigma_k \left(2^{b^{\max}} - 1\right), \forall n \right\}$.

Further, notice that the ATP constraints (3) are omitted in (14), because these are always observed to be inactive in our simulations in Section V. However, in Section III-C, it is shown how the proposed algorithms can be extended to handle these ATP constraints.

III. α-Fair PTES

Problem (14) is nonconvex since finding the optimal per-tone UEOs $\{\boldsymbol{\pi}_k\}$ entails a combinatorial optimization problem whose complexity increases exponentially with the number of tones K and the number of users N. For a fixed set of $\{\boldsymbol{\pi}_k\}$, however, the remaining optimal PA problem is convex and can be solved efficiently. The combinatorial problem thus can be considered as the maximization over various concave functions (one for each set of $\{\boldsymbol{\pi}_k\}$) leading to a non-smooth objective function. Unfortunately, for any practical G.fast scenario, the naive approach of exhaustive search (ES) over the $(N!)^K$ possible orderings to find the globally optimal solution of (14) is intractable. Therefore we develop in this section an iterative algorithm that avoids the exponential complexity in K and computes a near-optimal approximate solution of (14).

A. Tackling the Exponential Complexity in K

A feasible approach to approximately solve (14) is the use of an iterative BCA of the objective function [21], also known as the Gauss-Seidel method. A resource block r_k corresponds here to the encoding orders and transmit powers on a particular tone k, i.e., $r_k \triangleq [\boldsymbol{\pi}_k, \mathbf{s}_k]$. In a given iteration t, the BCA method generates the next iterate $\mathbf{r}^{(t+1)} \triangleq [r_1^{(t+1)}, \ldots, r_K^{(t+1)}]$ using the current iterate $\mathbf{r}^{(t)} \triangleq [r_1^{(t)}, \ldots, r_K^{(t)}]$, by successively solving the following problem for all tones $k = [1, \ldots, K]$ [3]:

$$r_k^{(t+1)} = \underset{\xi \in \mathcal{R}_k}{\arg\max}\ f_o\left(r_1^{(t+1)}, \ldots, r_{k-1}^{(t+1)}, \xi, r_{k+1}^{(t)}, \ldots, r_K^{(t)}\right) \quad (15)$$

with $\mathcal{R}_k$ denoting the constraint set of resource block k. Nevertheless, optimizing over encoding order $\boldsymbol{\pi}_k$ for tone k still requires an ES over all possible $\boldsymbol{\pi}_k$, where for each $\boldsymbol{\pi}_k$ the optimal PA has to be calculated. Solving subproblem (15) for tone k thus corresponds to an outer maximization in $\boldsymbol{\pi}_k$ and an inner maximization in $\mathbf{s}_k$ for fixed $\boldsymbol{\pi}_k$. The outer maximization is formulated as

$$\underset{\boldsymbol{\pi}_k}{\text{maximize}} \quad h_k(\boldsymbol{\pi}_k) \quad (16)$$

whilst the inner maximization corresponds to

$$h_k(\boldsymbol{\pi}_k) = \underset{\mathbf{s}_k \in \mathcal{D}_k}{\text{maximize}}\ f_o\left(r_1^{(t+1)}, \ldots, r_{k-1}^{(t+1)}, \boldsymbol{\pi}_k, \mathbf{s}_k, r_{k+1}^{(t)}, \ldots, r_K^{(t)}\right)$$
$$\text{s.t.} \quad \sum_m \left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^m \le P_k^{\text{mask}}, \qquad \forall n. \quad (17)$$

A complete algorithmic description is then given as Alg. 1. The algorithm will be referred to as α-fair PTES, since, in each iteration t, the K subproblems (15) are solved using ES's over $N!$ possible orderings in (16). For each searched $\boldsymbol{\pi}_k$, the optimal PA subroutine (Alg. 2) is called to solve problem (17), which is specified in Section III-B. Moreover, each iterate obtained after solving subproblem (15) for tone k is feasible, such that Alg. 1 complies with the definition of a real-time DSM algorithm [22]. Further, the following convergence result for α-fair PTES is outlined below.

Theorem 1: The sequence of objective values $\left\{f_o\left(\mathbf{r}^{(t+1)}\right)\right\}$ generated by α-fair PTES is monotonically nondecreasing every iteration and is guaranteed to converge. Moreover, every limit point of $\{\mathbf{r}^{(t+1)}\}$ is globally optimal for (14) with respect to the PA $\{\mathbf{s}_k\}$ if the UEO $\{\boldsymbol{\pi}_k\}$ is assumed to be fixed.

Proof: First, observe from

$$\frac{\partial^2}{\partial (s_k^n)^2} f_o(\mathbf{r}) = -\left(1 + \frac{\alpha}{R_n^\alpha}\right) \frac{1}{R_n^\alpha} \frac{w_n f_s / \log(2)}{(\Gamma \sigma_k + s_k^n)^2}, \quad \forall k, n, \quad (18)$$

which is always strictly negative due to R_n^α and s_k^n being both positive and bounded from above, that (14) is strictly convex in $\mathbf{s}_k$ [15], [21, Prop. B.4]. This thus means that the PA problem in (17) is convex and has a unique solution. Further, define $\mathbf{z}_k^{(t)} \triangleq [r_1^{(t+1)}, \ldots, r_k^{(t+1)}, r_{k+1}^{(t)}, \ldots, r_K^{(t)}]$. Then, successively solving (15) for all tones k with (16) and (17) leads to the optimal solution, which yields

$$f_o\left(\mathbf{r}^{(t+1)}\right) \ge f_o\left(\mathbf{z}_{K-1}^{(t)}\right) \ge f_o\left(\mathbf{z}_{K-2}^{(t)}\right) \ge \cdots \ge f_o\left(\mathbf{z}_1^{(t)}\right) \ge f_o\left(\mathbf{r}^{(t)}\right)$$
$$\text{for } l = 1, 2, \ldots \quad (19)$$

Combining (19) with $f_o(\mathbf{r})$ being bounded from above establishes that the objective value sequence $f_o\left(\mathbf{r}^{(t)}\right)$ converges to a finite positive real number [21, Prop. A.3]. Now, consider a limit point $\mathbf{r}^*$ of $\{\mathbf{r}^{(t)}\}$ (there exists at least one due to the finite number of possible UEOs and the compact set of feasible PAs). Then, unfortunately, it is hard to make any claims about the optimality of $\mathbf{r}^*$ with respect to the UEOs $\{\boldsymbol{\pi}_k\}$, due to the discrete nature of $\{\boldsymbol{\pi}_k\}$. However, when assuming $\{\boldsymbol{\pi}_k\}$ to be

[3] In fact any permutation of $[1, \ldots, K]$ may be used as cyclic update order.

fixed, the limit point is the global optimum of (14) in $\{\mathbf{s}_k\}$. This can be shown by using the fact that f_o is a differentiable and strictly convex function in $\mathbf{s}_k$ for any $\alpha \in [0,\infty)$ over a convex and compact set for each tone k, such that the BCA method converges to the globally optimal PA for any fixed UEO [21, Ch. 2.7]. ∎

We remark that only for the case $\alpha = 0$, when the original objective f_o simplifies to the weighted sum-rate function, one iteration over all tones k is sufficient to achieve the global optimum with respect to both $\{\boldsymbol{\pi}_k\}$ and $\{\mathbf{s}_k\}$.

B. The Power Allocation Subroutine

The complete PA subroutine, listed as Alg. 2, solves the strictly convex PA problem (17) for a given UEO $\boldsymbol{\pi}_k$ on tone k in a straightforward manner. Since general-purpose convex optimizers would increase the computational complexity too much, we propose the conditional gradient method, which solves a sequence of linear approximations and converges to the unique solution of (17) [23].

The key idea is here to linearize the objective function of (17) in the data rates $\{R_n\}$, by iteratively using the Taylor expansion of $f_o(\{R_n\})$ around $\{\bar{R}_n\}$

$$\begin{aligned}\tilde{f}_o(\{R_n\}) &\triangleq f_o(\{\bar{R}_n\}) + \sum_n w_n \frac{\partial f_o}{\partial R_n}\bigg|_{\bar{R}_n} \left(R_n - \bar{R}_n\right) \\ &= \sum_n \frac{w_n}{\bar{R}_n^\alpha} R_n + \text{const.}, \end{aligned} \tag{20}$$

where $\{\bar{R}_n\}$ is the set of data rates obtained by the current iterate $\mathbf{s}_k^{(i)}$ together with $\boldsymbol{\pi}_k$ and the other resource blocks $\{r_{k'}|\forall k' \neq k\}$ which are fixed, and const. makes the approximation tight. $\tilde{f}_o$ may be interpreted as a weighted sum-rate function with weights $w_n/\bar{R}_n^\alpha$. Then, the current iterate $\mathbf{s}_k^{(i)}$ is updated to the next feasible transmit power vector by

$$\mathbf{s}_k^{(i+1)} = (1-\gamma)\mathbf{s}_k^{(i)} + \gamma \hat{\mathbf{s}}_k, \tag{21}$$

where $\gamma \in [0,1]$ is the step-size that needs to be properly chosen to guarantee convergence [23], and $\hat{\mathbf{s}}_k$ is the solution of the following approximation of (17) based on (20):

$$\begin{aligned}\hat{\mathbf{s}}_k = \underset{\mathbf{s}_k \in \mathcal{D}_k}{\text{argmax}} \quad & \sum_n \frac{w_n f_s b_k^n}{\bar{R}_n^\alpha} + \text{const.} \\ \text{s.t.} \quad & \sum_m \left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^m \leq P_k^{\text{mask}}, \qquad \forall n. \end{aligned} \tag{22}$$

The main advantage of this approach is that the Lagrange dual function of (22) is easily evaluated with a closed-form solution. In addition, setting the number of inner iterations to 1 or 2 is already sufficient, since for a typical G.fast DSL scenario the bit rate for one tone is small compared to the total data rate (i.e. $\bar{R}_n \gg f_s b_k^n, \forall k$). Then, because strong duality holds, (22) is equivalent to the dual problem

$$\underset{\boldsymbol{\lambda}_k \geq \mathbf{0}}{\text{minimize}}\ g_k(\boldsymbol{\lambda}_k) \tag{23}$$

with

$$\begin{aligned} &g_k(\boldsymbol{\lambda}_k) \\ &= \underset{\mathbf{s}_k \in \mathcal{D}_k}{\text{maximize}} \sum_n \left\{ \frac{w_n f_s b_k^n}{\bar{R}_n^\alpha} - \lambda_k^n \sum_m \left(\left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^m - P_k^{\text{mask}} \right) \right\} \end{aligned} \tag{24}$$

where $\boldsymbol{\lambda}_k \triangleq [\lambda_k^1, \cdots, \lambda_k^N]$ is the vector of non-negative dual Lagrange variables for the spectral mask constraints at tone k. The Lagrange dual function $g_k(\boldsymbol{\lambda}_k)$ is evaluated using the following water-filling type expression that satisfies the KKT conditions

$$s_k^n = \left[\frac{w_n f_s/\log(2)}{\bar{R}_n^\alpha \left(\sum_m \lambda_k^m \left|[\mathbf{P}_k]_{m,n}\right|^2 \right)} - \Gamma \sigma_k \right]_0^{s_k^{\max}}. \tag{25}$$

To compute the optimal dual Lagrange variables of (23), we propose the standard ellipsoid method [24] attack on constrained convex minimization problems. The ellipsoid method is a so-called cutting-plane method and can be considered as a generalization of the one-dimensional bisection method to higher dimensions. It generates a sequence of ellipsoids with decreasing volume that are guaranteed to contain the optimal set of Lagrange multipliers. The main idea is that by evaluating the subgradient at a certain ellipsoid center, the half ellipsoid not containing the optimal set of Lagrange multipliers can be eliminated. In comparison with subgradient updating, where convergence speed depends heavily on the choice of the step size, the ellipsoid method appears to converge faster and is more stable across a wide variety of scenarios [25]. An ellipsoid with center $\mathbf{z}_k$ and size and shape defined by the positive-semidefinite matrix $\mathbf{A}_k$ is defined as

$$\mathcal{E}_k(\mathbf{A}_k, \mathbf{z}_k) \triangleq \left\{ \mathbf{x} | (\mathbf{z}_k - \mathbf{x})^T \mathbf{A}_k^{-1} (\mathbf{z}_k - \mathbf{x}) \leq 1 \right\}. \tag{26}$$

The square roots of the eigenvalues of $\mathbf{A}_k$ are the lengths of the semi-axes of $\mathcal{E}_k(\mathbf{A}_k, \mathbf{z}_k)$.

Given an initial ellipsoid that contains the optimal $\boldsymbol{\lambda}_k^*$ for tone k, the ellipsoid is iteratively cut in two based on the subgradient $\mathbf{d}_k$. The subgradient $\mathbf{d}_k \triangleq [d_k^1, \cdots, d_k^N]$ evaluated at the ellipsoid center $\mathbf{z}_k$ is calculated using following expression for d_k^n (see [25])

$$d_k^n = \left(P_k^{\text{mask}} - \sum_m \left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^{*,m} \right), \qquad \forall n, \tag{27}$$

where the transmit powers $\{s_k^{*,m}, \forall m\}$ are evaluated in (25) at $\boldsymbol{\lambda}_k = \mathbf{z}_k$. The half ellipsoid described by

$$\mathcal{E}_k(\mathbf{A}_k, \mathbf{z}_k) \cap \{\mathbf{z}_k | \mathbf{d}_K^T (\mathbf{z}_k - \mathbf{x}) \geq 0\} \tag{28}$$

is then guaranteed to contain the optimal set of dual Lagrange variables. Then, a new minimum volume ellipsoid $\mathcal{E}_k^+(\mathbf{A}_k^+, \mathbf{z}_k^+)$ that contains this half ellipsoid is formed as follows

$$\tilde{\mathbf{d}}_k = \frac{\mathbf{d}_k}{\sqrt{\mathbf{d}_k^T \mathbf{A}_k \mathbf{d}_k}} \tag{29a}$$

$$\mathbf{z}_k^+ = \mathbf{z}_k - \frac{1}{N+1} \mathbf{A}_k \tilde{\mathbf{d}}_k \tag{29b}$$

$$\mathbf{A}_k^+ = \frac{N^2}{N^2-1} \left(\mathbf{A}_k - \frac{2}{N+1} \mathbf{A}_k \tilde{\mathbf{d}}_k \tilde{\mathbf{d}}_k^T \mathbf{A}_k \right), \tag{29c}$$

where $\tilde{\mathbf{d}}_k$ is the normalized subgradient, see [24] for more details. This is a so-called *objective iteration*.

When $\mathbf{z}_k$ is infeasible, $\mathcal{E}_k^+(\mathbf{A}_k^+, \mathbf{z}_k^+)$ is computed similar to the objective iteration, but using the violated positivity constraint function's subgradient instead of (27) (i.e. $\mathbf{d}_k = -\mathbf{e}_i$ when $z_k^i < 0$ for some i). This is a so-called *constraint iteration* where all discarded points are infeasible. Note that in an objective iteration, all discarded points have an objective value greater than or equal to the current, feasible point. Thus in both cases, no minimizers are discarded, so that each next generated ellipsoid still contains all the minimizers that are in the initial ellipsoid.

To design an initial ellipsoid that is guaranteed to contain the optimal set of Lagrange multipliers $\boldsymbol{\lambda}_k^*$, we state following proposition (similar to the proposition in [25] for PA in the case of a multi-carrier interference channel subject to only ATP constraints).

Proposition 1: For the approximate N-user α-fair PA problem (22) of tone k with a fixed UEO $\boldsymbol{\pi}_k$, the optimal set of Lagrange dual variables $\boldsymbol{\lambda}_k^*$ must satisfy

$$\lambda_k^{n,*} \leq \lambda_k^{n,\max}, \forall n, \tag{30}$$

with

$$\lambda_k^{n,\max} \triangleq \max_m \left\{ \frac{w_m f_s/\log(2)}{|[\mathbf{P}_k]_{n,m}|^2 (\bar{R}_m)^\alpha} \left(\frac{1}{\Gamma \sigma_k} \right) \right\}, \tag{31}$$

where $w_n/(\bar{R}_n)^\alpha$ is the virtual weight of user n, and $\mathbf{P}_k$ is the precoder matrix given $\boldsymbol{\pi}_k$.

Proof: See appendix A. ∎

Using the above result and a geometric argument, the initial ellipsoid $\mathcal{E}_k^0(\mathbf{A}_k^0, \mathbf{z}_k^0)$ containing the optimal $\{\lambda_k^n\}$ for tone k may be chosen as

$$\mathbf{A}_k^0 = a\mathbf{I}_N \text{ with } a = \sum_m (\lambda_k^{m,\max}/2)^2 \tag{32a}$$

$$\mathbf{z}_k^0 = \left[\lambda_k^{1,\max}/2, \cdots, \lambda_k^{N,\max}/2 \right]^T. \tag{32b}$$

C. Active Per-Line ATP Constraints

α-fair PTES can be extended to handle active per-line ATP constraints [given in (3)] as well. The basic idea is to use dual decomposition to decouple the ATP constraints across the tones, by augmenting the objective function $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ with a weighted sum of the per-line ATP constraints, leading to the following Lagrangian:

$$f_g(\{\boldsymbol{\pi}_k, \mathbf{s}_k\}, \boldsymbol{\theta}) = f_0(\{\boldsymbol{\pi}_k, \mathbf{s}_k\}) - \sum_n \theta^n \left(\sum_k \sum_m \left| [\mathbf{P}_k]_{n,m} \right|^2 s_k^m - P^{\text{ATP}} \right) \tag{33}$$

where $\boldsymbol{\theta} \triangleq \{\theta^1, \cdots, \theta^N\}$ are the non-negative dual Lagrange variables. For each fixed $\boldsymbol{\theta}$, the Lagrangian (33) can be maximized by Alg. 1, where the original objective $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ needs to be replaced by $f_g(\{\boldsymbol{\pi}_k, \mathbf{s}_k\}, \boldsymbol{\theta})$. The only difference for the PA subroutine (Alg. 2) is that in (25) the term λ_k^m should be replaced by $\lambda_k^m + \theta^m$, and that (31) should be modified accordingly (which is detailed in Appendix A).

Algorithm 2: Power Allocation (PA) subroutine

Procedure PA$(k, \boldsymbol{\pi}_k, \alpha)$
- $\mathbf{P}_k = \mathbf{Q}_k \text{diag}(\mathbf{R}_k^H)^{-1}$ using (5)
- Initialize $\mathbf{s}_k^{(0)} \in \mathcal{D}_k$
- **for** $i = 0 \cdots I$ **do**
  - Calculate $\bar{R}_n, \forall n$
  - Initialize ellipsoid with (32)
  - **repeat**
    - **if** $z_k^i < 0$ for any i **then** /* Infeasible */
      - $\mathbf{d}_k = -\mathbf{e}_i$
    - **else** /* Feasible */
      - Calculate $\hat{\mathbf{s}}_k$ with (25)
      - Calculate $\mathbf{d}_k$ with (27)
    - **end**
    - Cut ellipsoid with (29)
  - **until** $\mathbf{z}_k \geq \mathbf{0}$ and $\sqrt{\mathbf{d}_k^T \mathbf{A}_k \mathbf{d}_k} < \epsilon$
  - Update $\mathbf{s}_k^{(i+1)} = (1-\gamma)\mathbf{s}_k^{(i)} + \gamma \hat{\mathbf{s}}_k$, for $\gamma = \frac{2}{2+i}$
- **end**
- **return** $\mathbf{s}_k = \mathbf{s}_k^{(I+1)}$

The remaining difficulty is now searching the optimal set of Lagrange multipliers $\boldsymbol{\theta}$, such that the ATP constraint for each line n is either tight or inactive, i.e., such that $\theta^n \left(\sum_k \sum_m \left| [\mathbf{P}_k]_{n,m} \right|^2 s_k^m - P^{\text{ATP}} \right) = 0, \forall n$. A standard approach for this is subgradient updating [25], by means of

$$\theta^n = \left[\theta^n + \epsilon \left(\sum_k \sum_m \left| [\mathbf{P}_k]_{n,m} \right|^2 s_k^m - P^{\text{ATP}} \right) \right]_0, \qquad \forall n, \tag{34}$$

where ϵ is a pre-defined step-size, which should be chosen sufficiently small in order to guarantee convergence [25]. For this dual decomposition step, we may claim optimality by referring to the "zero duality gap"-result for multi-tone systems when the number of tones is large [25], [26].

IV. Efficient per-tone UEO methods

The α-fair PTES algorithm presented in Section III is an iterative method for approximately solving problem (14) that eliminates the exponential complexity in the number of tones K. Nevertheless, the complexity for the ES to obtain the optimal UEOs in (16) is still exponential in the number of users N. For scenarios with e.g. more than five users, this algorithm continues to be prohibitively complex. To reduce the computational complexity of α-fair PTES, we propose two specific suboptimal methods with polynomial complexity in N to replace the ES in (16).

A. Successive Ordering Search

The successive ordering search (SOS) method as proposed in [12] is a way to approximate the PTES with polynomial complexity in N. Interestingly, SOS is shown in simulations to provide solutions very close to those of the PTES when maximizing the sum-rate. SOS initializes problem (16) with a given UEO $\boldsymbol{\pi}_k$ and its corresponding objective value. Then it

Algorithm 3: α-fair SOS

Replace the ES in Algorithm 1 with:
Set ordering $\boldsymbol{\pi}_k$ best so far and update $\mathbf{s}_k \leftarrow \mathtt{PA}(k, \boldsymbol{\pi}_k, \alpha)$
repeat
  for $n = 1 \cdots N$ **do**
    Find p such that $\pi_k^p = n$
    for $d = 1$ ***to*** $N, d \neq p$ **do**
      Set $\acute{\boldsymbol{\pi}}_k$ to (35)
      $\mathbf{s}_k \leftarrow \mathtt{PA}(k, \acute{\boldsymbol{\pi}}_k, \alpha)$
      if f_o is increased **then**
        Set $\boldsymbol{\pi}_k = \acute{\boldsymbol{\pi}}_k$
        Store $\{\mathbf{s}_k, \boldsymbol{\pi}_k\}$
      end
    end
  end
until $\boldsymbol{\pi}_k$ converges

successively changes the position of one user, while keeping the relative positions of the other users the same, continuously storing the best positions so far. More specific, for each user n which has, say, position p in the given initial UEO $\boldsymbol{\pi}_k$, the $N-1$ other positions are tested. The tested $\{\acute{\boldsymbol{\pi}}_k\}$ are obtained by moving user $n = \pi^p$ from position p to d and squeezing the rest of $\boldsymbol{\pi}_k$ such that

$$\acute{\boldsymbol{\pi}}_k = \left(\pi_k^1, \cdots, \pi_k^{d-1}, \pi_k^p, \pi_k^{d+1}, \cdots, \pi_k^{p-1}, \pi_k^{p+1}, \cdots, \pi_k^N\right), \quad (35)$$

for every $d \neq p$. For each tested $\acute{\boldsymbol{\pi}}_k$, the PA subroutine is called and the objective function is evaluated. If there is an objective value increase, the UEO $\boldsymbol{\pi}_k$ is replaced by $\acute{\boldsymbol{\pi}}_k$. This process of finding the best position for user n (relative to the other users) is successively performed for all users. Convergence is guaranteed by the fact that the objective value $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ is strictly increasing after each iteration. Since $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ is upper bounded, SOS must converge to a certain (possibly) suboptimal UEO $\boldsymbol{\pi}_k$. Typically only a few iterations are needed.

SOS may be efficiently implemented when constructing the QRDs of a new UEO using the QRD of the previously tested UEO. When switching two adjacent columns in a matrix, only two columns of the original Q matrix and two rows of the original R matrix have to be recomputed to obtain the new QRD. The QRD's computational complexity is then $O(N^2)$ instead of the standard $O(N^3)$. When organizing the successive position updates such that always two adjacent users are switched in (35), this property can be exploited and the complexity of one iteration is $O(N^4)$ [i.e. $N(N-1)$ QRD updates of $O(N^2)$].

A refinement of the α-fair PTES algorithm where the ES for solving subproblem (16) is replaced by SOS, is listed as Alg. 3. The algorithm is referred to as α-fair SOS. Note that α-fair SOS corresponds to an inexact BCA method, since (16) is only being solved approximately, in contrast to α-fair PTES. However, as long as the SOS method is initialized with the UEO of the previous iterate $\boldsymbol{\pi}_k^{(t)}$ for each tone k, a monotonically nondecreasing objective value f_o in (19) is guaranteed. Hence, the convergence result for α-fair PTES in Theorem 1 is also valid for α-fair SOS. A difference, however, is that for $\alpha = 0$ the obtained solution is not necessarily globally optimal anymore with respect to $\{\boldsymbol{\pi}_k\}$.

B. Derivative Based Ordering

To obtain simulation times approaching those of the fast heuristic UEO methods, we propose another greedy UEO method with even lower complexity than SOS to replace the ES in (16). It is referred to as derivative based ordering (DBO) and leads to the α-fair DBO algorithm and listed as Alg. 4.

The key observation is that improving the position in the UEO for a given user and tone, while keeping the relative positions of the other users the same, will always increase the SNR for that given user and tone. To see this, consider the QRD of the permuted and conjugated channel matrix $(\mathbf{E}_k\mathbf{H}_k)^H$ as a Gram-Schmidt procedure. Here each row vector $\overline{\mathbf{h}}_k^{\pi_k^p}$ of $\mathbf{H}_k$ is projected into the orthogonal complement of the $p-1$ previously projected vectors, with the length of the orthogonal projected vector given by $|r_k^{\pi_k^p, \pi_k^p}|$. Moreover, each additional vector in the orthogonal complement may only decrease $|r_k^{\pi_k^p, \pi_k^p}|$. Therefore improving the position p of user n on tone k (i.e. $n = \pi_k^p$) to d with $d < p$, whilst keeping the relative positions of the other users the same, will result into $|r_k^{\pi_k^d, \pi_k^d}| > |r_k^{\pi_k^p, \pi_k^p}|$. Since $\|[\mathbf{P}_k]_{\text{col } n}\| = 1/|r_k^{\pi_k^d, \pi_k^d}|$, user n will hence experience an improved SNR for the same physical line transmit power.

Based on above observation, each iteration the gradient of the objective function $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ in the transmit powers $\mathbf{s}_k$ for tone k is first calculated, i.e.,

$$D_k^n \triangleq \frac{w_n f_s/\log(2)}{(R_n)^\alpha}\left(\frac{1}{\Gamma\sigma_k + s_k^n}\right), \forall n, \quad (36)$$

after which the UEO $\boldsymbol{\pi}_k$ is updated based on the computed gradients such that $D_k^{\pi_k^1} \geq \cdots \geq D_k^{\pi_k^N}$. Consequently, the user with the largest derivative (i.e. the user whose increase in SNR will increase the objective $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ most) is encoded first, getting its possible smallest precoder column norm assigned. Conversely, the user with the smallest derivative (i.e. the user whose decrease in SNR will affect $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ least) is encoded last, getting its largest possible precoder column norm assigned. Then, for each $\boldsymbol{\pi}_k$, the PA subroutine is called and the objective function is evaluated.

DBO with (36) may be seen as an improved version of the heuristic UEO method presented in [10] and referred to as DO. DO sets the UEO for tone k as the ascending order of the aggregated user bit loadings from tone 1 up to $k-1$. This corresponds to the weights $1/R_n$ for $\alpha = 1$ in (36) which compensate for the total date rate obtained in the previous iteration. Additionally, (36) takes the gradient of $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$ into account, such that the resources on tone k are efficiently allocated to users with low total data rates and low SNR values (such that an increase in SNR has a large effect).

The iterative updating process of DBO is however not guaranteed to converge. Due to its heuristic nature, it is possible for DBO to cycle through a few sets of $\boldsymbol{\pi}_k$'s. This

Algorithm 4: α-fair DBO

Initialize $\forall k$:
  $\boldsymbol{\pi}_k$ with maxsum SQRD algorithm [13]
  $\mathbf{s}_k \leftarrow \mathtt{PA}(k, \boldsymbol{\pi}_k, 0)$
Replace the ES in Algorithm 1 with:
Set ordering $\boldsymbol{\pi}_k$ best so far and update $\mathbf{s}_k \leftarrow \mathtt{PA}(k, \boldsymbol{\pi}_k, \alpha)$
repeat
  $D_k^n = \dfrac{w_n f_s/\log(2)}{(R_n)^\alpha}\left(\dfrac{1}{\Gamma\sigma_k + s_k^n}\right), \forall n$
  Sort $\hat{\boldsymbol{\pi}}_k$ such that $D_k^{\hat{\pi}_k^1} \geq \cdots \geq D_k^{\hat{\pi}_k^N}$
  $\mathbf{s}_k \leftarrow \mathtt{PA}(k, \hat{\boldsymbol{\pi}}_k, \alpha)$
  if f_o is increased **then**
    Set $\boldsymbol{\pi}_k = \hat{\boldsymbol{\pi}}_k$
    Store $\{\mathbf{s}_k, \boldsymbol{\pi}_k\}$
  end
until $\hat{\boldsymbol{\pi}}_k$ converges

issue may simply be resolved with proper implementation. For instance, the number of iterations may be restricted with $\boldsymbol{\pi}_k$ being selected from the iteration with the highest objective function. Alternatively, the iteration may be stopped when a cycle is detected. When always initializing $\boldsymbol{\pi}_k$ with the UEO of the previous iterate, solving (16) for each tone k with DBO is bound to non-decrease the objective function $f_o(\{\boldsymbol{\pi}_k, \mathbf{s}_k\})$. As a result, α-fair DBO is an inexact BCA method for solving (14) with monotonically nondecreasing objective values in (19). Therefore, the convergence result stated in Theorem 1 for α-fair PTES is also valid for α-fair DBO.

Initializing the α-fair DBO with the fast sorted QRD algorithm for maximizing the sum rate [13] has shown to work well in our simulations. It ensures a high initial mean-rate performance before trading performance for more fairness.

C. Complexity Comparison

The computational complexity of the proposed α-fair DSM algorithms (as summarized in Table I) are mainly determined by the number of QRDs and PA subroutine calls. A QRD (with complexity $O(N^3)$) is necessary for the precoder matrix computation according to each tested UEO $\boldsymbol{\pi}_k$ at tone k. In addition, given the precoder matrix the PA subroutine then solves (17) for tone k (with linear complexity in N). Let I_1 be the number of outer iterations to approximately solve (14). Further, let I_2 and I_3 be the number of inner iterations needed by SOS and DBO to approximately solve (16), respectively.

Table II shows the required number of outer iterations I_1 for both α-fair SOS and DBO to converge in the 10-user G.fast scenario of Section V-B. The stopping criterion is when the relative objective function f_o increase between two consecutive iterations drops below 10^{-4}. Observe that the required number of outer iterations increases for increasing α. The average number of inner iterations is between 1-3 for α-fair SOS and 3-6 for α-fair DBO with stopping after a cycle is detected.

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON

Algorithm	Precoder Matrix Computation	PA Subroutine Calls
α-fair ES	$O((N!)^K K N^3)$	$I_1 K (N!)^K$
α-fair PTES	$O(I_1 K (N!) N^3)$	$I_1 K (N!)$
α-fair SOS	$O(I_1 I_2 K N^4)$	$I_1 I_2 K N^2$
α-fair DBO	$O(I_1 I_3 K N^3)$	$I_1 I_3 K$
Heuristic UEO Methods	$O(K N^3)$	K

TABLE II
NUMBER OF OUTER ITERATIONS I_1 REQUIRED FOR α-FAIR SOS AND DBO FOR A 10-USER G.FAST SCENARIO (SEE SECTION V-B)

α	0	1	2	5	10	15	20	25
α-fair SOS	1	2	2	5	9	11	16	18
α-fair DBO	1	2	2	3	4	6	9	12
α	30	40	50	75	100	125	150	
α-fair SOS	21	25	31	41	51	54	62	
α-fair DBO	11	16	23	30	34	46	49	

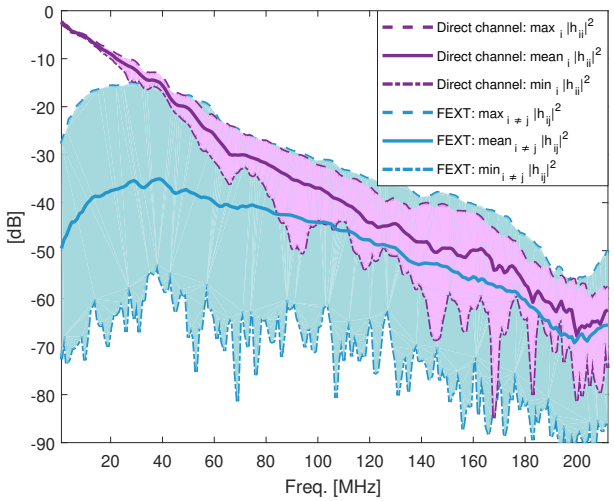

Fig. 2. Direct and crosstalk channel magnitude characteristics of measured 10-line 80m cable binder. Values are smoothed by averaging over 1 MHz bin.

The computational complexity comparison shows that the complexity of α-fair PTES is significantly reduced by replacing the PTES with efficient suboptimal methods, against acceptable performance losses as will be shown in the next section. α-fair DBO in particular comes close to the fast and heuristic UEO methods like e.g. DO.

V. SIMULATIONS

In this section the performance of the proposed α-fair DSM algorithms is evaluated for the downstream G.fast 212 MHz profile. Unless stated otherwise, the considered scenarios adopt G.9700 spectral masks [17] ranging from −65 dBm/Hz to −79

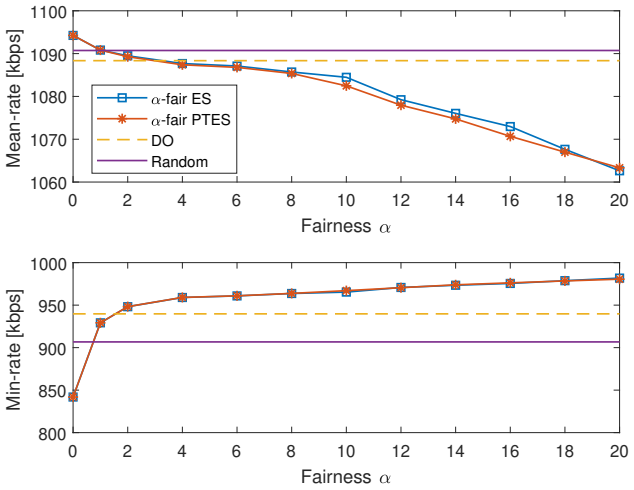

Fig. 3. Fairness-performance trade-off for a $N = 4$ and $K = 4$ scenario.

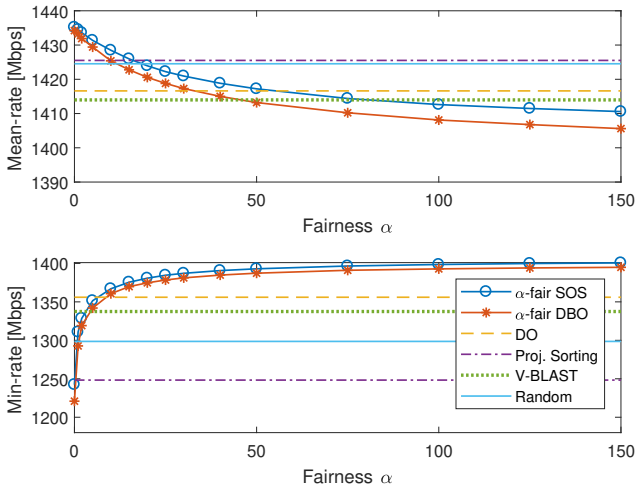

Fig. 4. Fairness-performance trade-off for various UEO methods evaluated in a $N = 10$ G.fast cable binder.

dBm/Hz, 8 dBm per-line ATP constraints[4], a 14 bit cap and a background noise PSD of −140 dBm/Hz. The tone spacing is 51.75 kHz and the symbol rate is 48 kHz. The capacity gap Γ is set to 9.45 dB. Furthermore, the channel matrices have been obtained by measurements of a cable binder consisting of 10 lines of 80 m. It is noted that the observed crosstalk levels in this particular cable binder are rather high compared to other G.fast measurements (see Fig. 2). For all simulations, equal user weights are considered, i.e., $\{w_n = 1, \forall n\}$.

Included in the comparison are three aforementioned state-of-the-art UEO algoritms that tackle the PA and UEO problem separately. V-BLAST [9] computes the UEO that maximizes the minimum $|r_k^{\pi_k^n,\pi_k^n}|$ (which corresponds to maximizing the minimum SNR in case of a flat PA) for each tone k separately. Given $\{\boldsymbol{\pi_k}\}$, the optimal PA subroutine (with α set to zero) is called for each tone. DO [10] sequentially runs through the tones, for each tone k, choosing $\boldsymbol{\pi}_k$ as the order of the total bit rate aggregated over tones 1 to $k-1$, and then calling the PA subroutine for tone k. Projected sorting [11] is another variant of the sorted QRD algorithm [13]. The best of four randomly generated UEOs $\{\boldsymbol{\pi}_k\}$ is also included for comparison (labeled as 'Random').

A. Four-User and Four-Tone Scenario

The first simulation considers a specific scenario consisting of $N = 4$ users and $K = 4$ tones, which are selected from the 10-user cable binder. These small numbers are necessary in order to be able to assess the optimality of α-fair PTES in comparison with the globally optimal but computationally complex α-fair ES. For this scenario we assume a flat spectral mask of −76 dBm/Hz with neither ATP constraints nor a bit cap.

Fig. 3 shows the performance (mean-rate) versus fairness (min-rate) trade-off obtained by α-fair ES, α-fair PTES, DO and random ordering. As can be seen from the figure, the curve for α-fair ES and for α-fair PTES are very close to each other with ES having only a slightly increased mean-rate. It is remarked that for this small scenario there are a number of very suboptimal local optima of (14) compared to the global optimum. To obtain a solution very close to the global optimum, the update order of successively solving the per-tone subproblems in (15) is of great importance such that it is advantageous to perform multiple runs with different update orderings. Interestingly however, for the full 10-user cable binder with $K = 4096$ tones it appears that there are many local optima that result in almost identical user data rates. This then reduces the importance of the update ordering for larger scenarios.

B. 10-User G.fast Scenario

In this section the complete 10-user cable binder is simulated to evaluate the performance of the α-fair DSM algorithms. For $N = 10$ the α-fair PTES algorithm is not computationally practical and therefore omitted here. Fig. 4 shows the performance (mean-rate) versus fairness (min-rate) trade-off curve in function of α obtained by the various algorithms. Additionally, the user data rate distribution of $\{0, 1, 10, 150\}$-fair SOS and DO is given in Fig. 5, where the distributions are shown as cumulative density functions (CDFs).

Foremost, the α-fair DSM methods achieve a trade-off between performance and fairness that provides additional gains over the solutions obtained by the current UEO methods. For instance, for large α (i.e. max-min fairness case), the min-rate with DO is 1356 Mbps, whilst α-fair SOS increases this to over 1400 Mbps. Moreover, for the same min-rate, α-fair SOS and DBO achieve a higher mean-rate than the current UEO methods. Further, $\alpha = 1$ (i.e. proportional fairness) offers the largest fairness gain without almost no mean-rate loss. Another interesting operating point is $\alpha = 10$ for which α-fair SOS achieves for every user a higher data rate than DO (Fig. 5).

[4]In our simulations with the G.9700 power specification typically all ATP constraints are inactive due to the PSD mask and bit cap constraints.

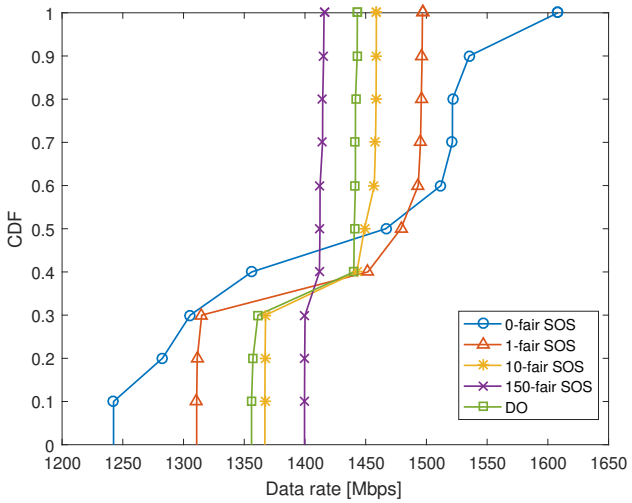

Fig. 5. Data rate distribution as a CDF obtained by $\alpha = \{0, 1, 10, 150\}$-fair SOS and DO for a $N = 10$ G.fast cable binder.

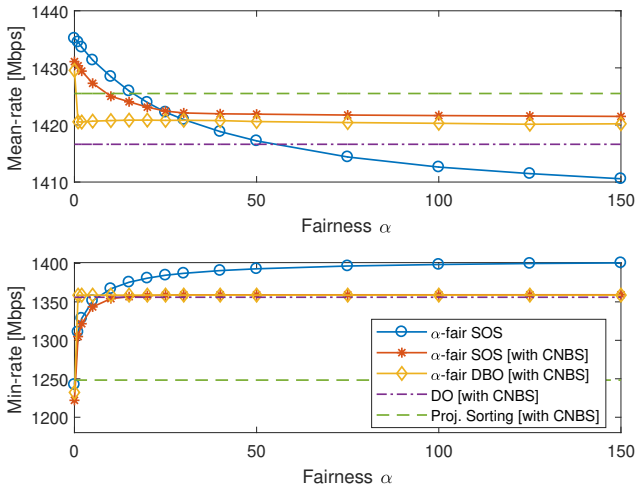

Fig. 6. Fairness-performance trade-off in a $N = 10$ G.fast cable binder. α-fair DSM with jointly optimization of the UEO and PA is more powerful than merely combining UEO with a simplified scaling such as the CNBS [4].

We remark also that DO achieves a higher min-rate and mean-rate than V-BLAST, confirming the results in [10].

The economies of scale are also active, meaning that when increasing α, initially for a small performance loss a significant increase in fairness is acquired, after which increasingly more performance is sacrificed for only little extra fairness. This steep initial fairness gain mainly stems from the UEOs being assigned in favor of the weakest users. Only for far larger α's, the PA also starts to really kick in as strong users back off their transmit power in order to produce smaller precompensation signals in favor of weak users. As a result, $\alpha = 150$-fair SOS obtains almost perfect equal user data rates (Fig. 5).

Moreover, the α-fair DSM approach of jointly tackling the UEO and PA is quite powerful in G.fast transmission. To illustrate this, a comparison is made with algorithms where the optimal PA subroutine (Alg. 2) is replaced by a simplified scaling such as the CNBS method [4], which is incompatible with the α-fairness policy. This means the UEO algorithms are now combined with CNBS. In the simulated 10-user cable binder, three users achieve clearly lower data rates due to asymmetric channel characteristics (see Fig. 5). Therefore, there is a large gap between the min-rate and mean-rate curves of the UEO algorithms combined with the CNBS (see Fig. 6), despite all best UEO positions are assigned to these three weaker users. The data rate gap is substantially decreased when jointly optimizing the UEO and PA according the α-fairness policy, because then stronger users additionally back off their transmit power in order to produce smaller precompensation signals, leading to smaller transmit power penalties for the three weaker users.

Fig. 7 shows the mean-rates and min-rates of α-fair SOS versus the number of outer iterations. The obtained rates with DO (which only require a single iteration) are included as well for reference. We stress that only the α-fair objective f_o is bound to monotonically non-decrease after each iteration, and not necessarily the mean-rates or the min-rates, which are shown here. Convergence is typically fast for moderate α (see also Table II). Moreover, since every iterate is feasible, the proposed α-fair DSM algorithms can be considered as real-time DSM algorithms [22].

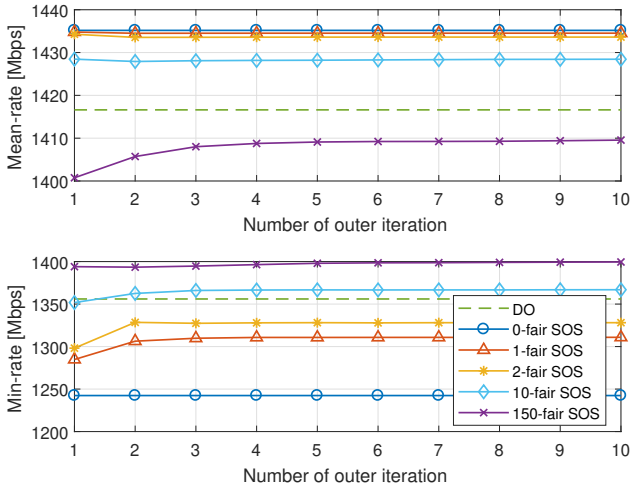

Fig. 7. The convergence in terms of mean-rate and min-rate of α-SOS for various α-values. The data-rates obtained by DO are given as reference.

Finally, for a small number of users, α-fair SOS and DBO are observed to operate close to the computationally complex α-fair PTES. This is illustrated in Fig. 8 where various UEO methods are evaluated when varying the number of users N in the 10-line cable binder and considering all $\binom{10}{N}$ combinations[5]. Additionally, Fig. 2 in [1] shows the mean-rates and min-rates of all proposed α-fair DSM algorithms for various α-values in a 5-user G.fast scenario. In particular, α-fair SOS achieves a trade-off almost identical to α-fair PTES. α-fair SOS is thus the best (tractable) algorithm available for larger scenarios and is expected to operate close to the optimal

[5]Note that not all data points of α-fair PTES are plotted due to prohibitive computational complexity.

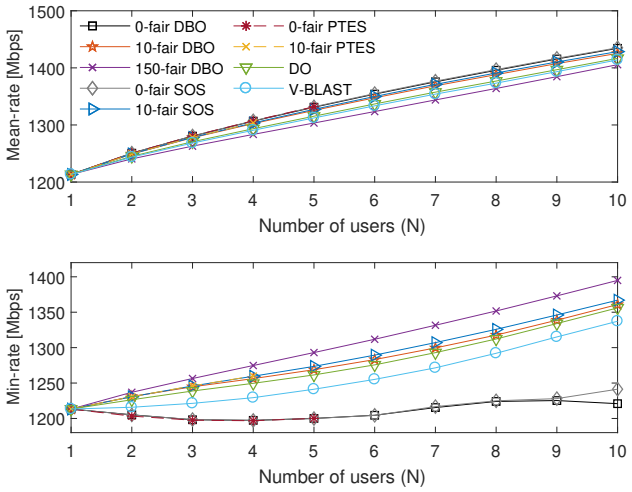

Fig. 8. The mean-rate and min-rate versus the number of users for various UEO methods in a 10-line G.fast cable binder.

trade-off in Fig 4. It may serve as an upper bound for practical UEO methods. On the other hand, α-fair DBO offers a much lower simulation time at an acceptable performance loss. The lower algorithm complexity translates mainly into a lower performance (mean-rate) curve, whilst the fairness (min-rate) curve is less sensitive.

VI. Conclusion

Efficient α-fair DSM algorithms for QRD-based precoding with joint PA and UEO in G.fast downstream transmission have been presented in this paper. Since computing the optimal UEO scales exponentially with the number of tones K and users N, computing a globally optimal solution for any realistic G.fast scenario is intractable. Motivated by this, α-fair PTES has been proposed that solves the optimization problem with coordinated ascent using PTES's in an iterative fashion, which leads to a linear complexity in K and is tractable up to 5 users. Additionally, two efficient suboptimal methods (α-fair SOS and DBO) are proposed that approximately solve the per-tone subproblems and further reduce the complexity, which allows to handle larger scenarios as well. Simulations with measured G.fast channel data have indicated that the proposed α-fair DSM algorithms are capable of achieving an efficient trade-off between performance and fairness, in contrast to the current UEO methods like DO [10] which provide only a single operation point. In particular, α-fair SOS may serve as an upper bound for practical UEO methods in G.fast, whereas α-fair-DBO is a low-complexity DSM algorithm providing additional gains over the DO method.

Appendix A
Proof of Proposition 1

The optimal $\boldsymbol{\lambda}_k^*$ has to satisfy the KKT conditions of problem (22), where we assume that f_o is being replaced by f_g in (33) in order to handle ATP constraints as well. For clarity, we first define the Lagrangian of this extended problem (22) as follows:

$$\mathcal{L} = \sum_n \left\{ \frac{w_n f_s b_k^n}{\bar{R}_n^\alpha} - \left(\theta^n + \lambda_k^n\right) \sum_m \left|[\mathbf{P}_k]_{n,m}\right|^2 s_k^m \right\} + \sum_n \lambda_k^n P_k^{\text{mask}} + \sum_n \theta^n P^{\text{ATP}} + \text{const.} \tag{37}$$

The KKT conditions are obtained by differentiating the Lagrangian of (17) with respect to s_k^m and extracting $\lambda_k^{n,*}$ to one side of the equation:

$$\lambda_k^{n,*} = \frac{w_m f_s/\log(2)}{|[\mathbf{P}_k]_{n,m}|^2 \left(\bar{R}_m\right)^\alpha} \left(\frac{1}{\Gamma\sigma_k + s_k^m} \right) - \frac{1}{|[\mathbf{P}_k]_{n,m}|^2} \times \left(\sum_{p\neq n} \lambda_k^{p,*} |[\mathbf{P}_k]_{p,m}|^2 + \sum_p \theta^p |[\mathbf{P}_k]_{p,m}|^2 \right), \forall m. \tag{38}$$

Each $\lambda_k^{n,*}$ thus has to satisfy a set of N conditions due to the precoder matrix that couples the line transmit powers between the users. The set of conditions (38) may be relaxed to the following upper bound

$$\lambda_k^{n,*} \leq \max_m \left\{ \frac{w_m f_s/\log(2)}{|[\mathbf{P}_k]_{n,m}|^2 \left(\bar{R}_m\right)^\alpha} \left(\frac{1}{\Gamma\sigma_k + s_k^m} \right) - \frac{1}{|[\mathbf{P}_k]_{n,m}|^2} \sum_p \theta^p |[\mathbf{P}_k]_{p,m}|^2 \right\} \tag{39}$$

$$\leq \max_m \left\{ \left[\frac{w_m f_s/\log(2)}{|[\mathbf{P}_k]_{n,m}|^2 \left(\bar{R}_m\right)^\alpha} \left(\frac{1}{\Gamma\sigma_k + s_k^m} \right) - \frac{1}{|[\mathbf{P}_k]_{n,m}|^2} \sum_p \theta^p |[\mathbf{P}_k]_{p,m}|^2 \right]_0 \right\}, \tag{40}$$

$$\leq \max_m \left\{ \left[\frac{w_m f_s/\log(2)}{|[\mathbf{P}_k]_{n,m}|^2 \left(\bar{R}_m\right)^\alpha} \left(\frac{1}{\Gamma\sigma_k} \right) - \frac{1}{|[\mathbf{P}_k]_{n,m}|^2} \sum_p \theta^p |[\mathbf{P}_k]_{p,m}|^2 \right]_0 \right\}, \tag{41}$$

where (39) results from the terms corresponding to $\{\lambda_k^{p,*} \forall p \neq n\}$ in (38) being all negative, and (40) results from the optimal $\lambda_k^{n,*}$ being non-negative, while (41) results from the transmit powers s_k^m being non-negative. Note that in case $\boldsymbol{\theta} = \mathbf{0}_N$ (i.e. inactive ATP constraints) condition (31) is recovered.

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Wouter Lanneer (S'15) received the M.Sc. degree in electrical engineering from the KU Leuven, Leuven, Belgium, in 2014, where he is currently pursuing the Ph.D. degree under the supervision of Prof. M. Moonen at the Electrical Engineering Department of KU Leuven. He was a Visiting Researcher at the University of Toronto in 2018. He is involved in joint projects with KU Leuven and Nokia Bell Labs, Antwerp, Belgium. His research focuses on signal processing and optimization for digital communication systems with an emphasis on ultra-broadband DSL access networks.



Paschalis Tsiaflakis (S'06-M'09) received his M.Sc. and Ph.D. degree in electrical engineering from the KU Leuven, Leuven, Belgium, in 2004 and 2009, respectively. He has co-supervised five Ph.D. students and has conducted research with Princeton University in 2007, UCLA in 2010, the Université Catholique de Louvain in 2011, and Tsinghua University in 2013. Since 2013, he has been a Research and Innovation Engineer with Nokia Bell Labs, where he drives innovation into next generation communication systems. He contributes to standardization bodies and has filed over 20 patents. He was a recipient of the Nokia Innovation Award in 2017, the Nokia Spotlight Recognition Award in 2016, the Belgian Young ICT Personality Award in 2010, two IEEE ICC best paper awards in 2013, and the Best Multimedia Master Thesis Prize Award in 2001.



Jochen Maes (SM'11) received the M.Sc. degree in physics and the Ph.D. degree in science from KU Leuven, Leuven, Belgium. He joined Bell Labs in 2006, where he continuously shifts the limits of broadband. He heads the Fixed Networks Group of Nokia Bell Labs focused on innovations in optical and copper access networks. He contributes to several standardization bodies. His work received the Broadband Infovision Awards in 2010 and 2014, and the Bell Labs President's Award in 2011. He is Bell Labs Fellow since 2016.



Marc Moonen (M'94-SM'06-F'07) is a Full Professor at the Electrical Engineering Department of KU Leuven, Leuven, Belgium, where he is heading a research team working in the area of numerical algorithms and signal processing for digital communications, DSL and audio signal processing.

He received the 1994 KU Leuven Research Council Award, the 1997 Alcatel Bell (Belgium) Award (with Piet Vandaele), the 2004 Alcatel Bell (Belgium) Award (with Raphael Cendrillon), and was a 1997 Laureate of the Belgium Royal Academy of Science. He received journal best paper awards from the IEEE Transactions on Signal Processing (with Geert Leus and with Daniele Giacobello) and from Elsevier Signal Processing (with Simon Doclo).

He was the chairman of the IEEE Benelux Signal Processing Chapter from 1998 to 2002, a member of the IEEE Signal Processing Society Technical Committee on Signal Processing for Communications, and President of EURASIP (European Association for Signal Processing), from 2007 to 2008 and from 2011 to 2012.

He has served as an Editor-in-Chief for the EURASIP Journal on Applied Signal Processing from 2003 to 2005, an Area Editor for Feature Articles in IEEE Signal Processing Magazine from 2012 to 2014, and has been a member of the editorial board of Signal Processing, IEEE Transactions on Circuits and Systems II, IEEE Signal Processing Magazine, Integration-the VLSI Journal, EURASIP Journal on Wireless Communications and Networking and EURASIP Journal on Advances in Signal Processing.