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| **Author contact** | wouter.lanneer@esat.kuleuven.be *+ 32 (0)16 32 79 75* |
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# Linear and nonlinear precoding based dynamic spectrum management for downstream vectored G.fast transmission

Wouter Lanneer and Marc Moonen
KU Leuven, Dept. of Electrical Engineering (ESAT),
STADIUS Center for Dynamical Systems,
Signal Processing and Data Analytics
{wouter.lanneer, marc.moonen}@esat.kuleuven.be

Paschalis Tsiaflakis and Jochen Maes
Bell Labs, Alcatel-Lucent, Belgium
{paschalis.tsiaflakis, jochen.maes}@alcatel-lucent.com

***Abstract*—In the G.fast frequency range (up to 212 MHz), the diagonal dominance structure of the channel matrix is no longer valid at the higher frequencies. As a result, the linear Zero Forcing (ZF) precoder in combination with dynamic spectrum management (DSM) is no longer near-optimal for downstream vectored G.fast transmission. To boost performance, we develop a novel low-complexity algorithm for both linear and non-linear precoding based DSM that maximizes the weighted line sum-rate under realistic per-line total power and per-tone spectral mask constraints. The algorithm relies on a Lagrange multiplier based transformation of the downstream dual decomposition approach formulation into its dual upstream formulation, together with a low-complexity iterative fixed-point formula to solve the resulting upstream problems. Simulations with measured G.fast channel data up to both 106 and 212 MHz are provided revealing a significantly increased performance of this algorithm over linear ZF precoding.**

## I. Introduction

In Digital Subscriber Lines (DSL)-systems, the main source of performance degradation is the crosstalk interference between different copper lines in the same cable bundle. This problem has been addressed with the introduction of vectoring (also known as DSM-level 3) for VDSL2, which removes the crosstalk interference [1]. Since in the VDSL2 frequency range (below 30 MHz) the channel matrix typically has a diagonal dominance structure, the linear Zero Forcing (ZF)-precoder in combination with dynamic spectrum management (DSM) is near-optimal for downstream vectored transmission [2]. However G.fast, a new standard approved by the International Telecommunication Union (ITU), already exploits a much broader spectrum up to 106 MHz, while a 212 MHz profile is under definition [3]. At these high frequencies, the diagonal dominance structure of the channel matrix is no longer valid [4]. As a result, the ZF-precoder is not near-optimal anymore and, in addition, suffers from increased per-tone and per-line total transmit powers due the large precompensation signals needed for crosstalk cancellation [5].

In the context of these developments, we focus on downstream linear (LP) and nonlinear (NLP) precoding based DSM in order to maximize the weighted line sum-rate under realistic per-line total power and per-tone spectral mask constraints. These downstream rate maximization problems are typically non-convex and therefore difficult to solve.

In a wireless and single carrier context, downstream rate maximization problems under per-line power constraints have been shown to be equivalent with their more easily solvable dual upstream problems by incorporating an unknown noise covariance matrix [6]. This leads to a MinMax optimization of the weighted sum-rate over transmit covariance matrices and the unknown noise covariance matrix. A simpler, but equivalent, approach where only the transmit covariance matrices have to be optimized, is the Broadcast Channel Optimal Spectrum Balancing (BC-OSB) [7]. BC-OSB has been proposed for optimal power allocation in a DSL system employing Dirty Paper Coding (DPC) under per-line total power constraints. It consists of a dual decomposition approach where the Lagrange dual function is transformed based on the Lagrange multipliers before exploiting upstream-downstream (US-DS)-duality. The resulting dual upstream Lagrangian is then maximized by per-tone discrete exhaustive searches, which however have large computational complexity. This in particular makes the BC-OSB infeasible for LP, due to its much larger search space of transmit covariance matrices needed compared to NLP.

In this paper, we focus on extending the BC-OSB approach to also include per-tone spectral mask constraints on top of the per-line total power constraints and considering LP as well as NLP. The latter is enabled by proposing a low complexity method to maximize the upstream Lagrangian by means of an iterative fixed-point formula. The resulting algorithm is referred to as Broadcast Channel Distributed Spectrum Balancing (BC-DSB).

This paper is organized as follows. Section II introduces the downstream system model and corresponding line sum-rate maximization problem. Section III presents BC-DSB. Section IV provides a performance comparison of BC-DSB in a G.fast context. Finally, section V concludes this paper.

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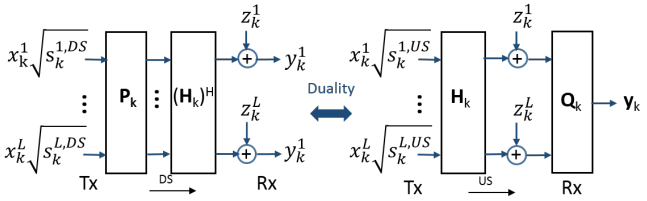


Fig. 1. Downstream (left-hand side) and its dual upstream (right-hand side) system model for transmission on tone $k$. Switching between both domains is done by interchanging the role of precoding and equalizer vectors and hermitian transposing the channel matrix.

## II. System Model and Problem Statement

We consider a downstream transmission system consisting of $L$ interfering users each possessing one G.fast line (left-hand side of Fig. 1). Transmission occurs by means of Discrete-Multi-Tone (DMT) modulation with $K$ sub-carriers or tones spaced by $\Delta_f$ Hz, and with only transmitter coordination. This scenario is referred to as the Broadcast Channel (BC). Both linear and non-linear precoding strategies are considered. Assuming perfect DMT synchronization, the downstream transmission on tone $k$ can be modeled as

$$\mathbf{y}_k^{DS} = \mathbf{H}_k^H \mathbf{P}_k (\mathbf{S}_k^{DS})^{1/2} \mathbf{x}_k^{DS} + \mathbf{z}_k. \tag{1}$$

$\mathbf{x}_k^{DS}$ is the $L$-vector of downstream data symbols of all lines on tone $k$ with identity covariance matrix. $\mathbf{y}_k^{DS}$ is the $L$-vector of received symbols on tone $k$. $\mathbf{z}_k$ is the $L$-vector of additive noise samples on tone $k$. We assume w.l.o.g. $E\{\mathbf{z}_k \mathbf{z}_k^H\} = I_L$. The matrix $\mathbf{P}_k \triangleq [\mathbf{p}_k^1, \cdots, \mathbf{p}_k^L]$ denotes the precoding vectors on tone $k$, with $\mathbf{p}_k^l$ being an L-vector with unity $l_2$-norm and representing the linear processing corresponding to the data symbol of line $l$. $(\mathbf{S}_k^{DS})^{1/2} \triangleq \text{diag}\{\sqrt{s_k^{1,DS}}, \cdots, \sqrt{s_k^{L,DS}}\}$ is a diagonal scaling matrix on tone $k$ where $s_k^{l,DS}$ represents the symbol transmit powers of line $l$. Finally, $\mathbf{H}_k \triangleq [\mathbf{h}_k^1, \cdots, \mathbf{h}_k^L]$ represents the $L \times L$ channel matrix on tone $k$, where $h_k^{l,j} \triangleq [\mathbf{H}_k]_{l,j}$ is the frequency response on tone $k$ from transmitter $j$ to receiver $l$. The diagonal elements are the direct channels, and the off-diagonal elements are the crosstalk channels. We remark that although $\mathbf{H}_k$ typically has a diagonal dominance structure (i.e. $|h_k^{l,j}| \ll |h_k^{l,l}|$) below 30 MHz, recent measurements show that this structure is not valid anymore for higher frequencies of G.fast where the direct channels may even be smaller than the crosstalk channels [4].

### A. Nonlinear Precoding

DPC [8], a nonlinear interference subtraction technique, achieves sum-capacity for the downstream channel [9]. It can be seen as the dual of the upstream capacity-achieving Minimum Mean Squared Error Generalized Decision Feedback Equalizer (MMSE-GDFE) [10]. To implement DPC, a subtraction or encoding order of the lines is required. Without loss of generality, we assume that the encoding order is given by the line index. This means that line 1, which is encoded first, sees all other lines as interference; while line $L$, which is encoded last, has the interference subtracted from all other lines. For this NLP based transmission, the weighted line sum-rate is

$$C^{DS} = \sum_k \sum_l w_l \log_2 \left(1 + \frac{s_k^{l,DS} |(\mathbf{h}_k^l)^H \mathbf{p}_k^l|^2}{\Gamma(1 + \sum_{j>l} s_k^{j,DS} |(\mathbf{h}_k^l)^H \mathbf{p}_k^j|^2)}\right), \tag{2}$$

where $\Gamma$ denotes the capacity gap for practical QAM implementations, and $w_l$ is the weight for line $l$. We assume w.l.o.g. $w_1 \geq \cdots \geq w_L$. Throughout the paper, we always refer to the case of zero capacity gap ($\Gamma = 0$ dB) when making a statement about "optimality". However, we remark that even for nonzero capacity gap ($\Gamma \neq 0$ dB) the DPC transmission structure can be used to obtain a set of achievable line rates (not necessarily capacity-achieving) and to outperform ZF schemes as will be shown in the simulation section.

### B. Linear Precoding

Although LP is generally outperformed by NLP, like the capacity-achieving DPC, it requires less complexity at the transmitter and receiver, and therefore offers a valuable alternative. For LP based transmission, the weighted line sum-rate is

$$C^{DS} = \sum_k \sum_l w_l \log_2 \left(1 + \frac{s_k^{l,DS} |(\mathbf{h}_k^l)^H \mathbf{p}_k^l|^2}{\Gamma(1 + \sum_{j\neq l} s_k^{j,DS} |(\mathbf{h}_k^l)^H \mathbf{p}_k^j|^2)}\right). \tag{3}$$

While the algorithm developed in this paper is applicable to both LP and NLP, we focus on the nonlinear case, and adopt the weighted line sum-rate definition given in (2) throughout the paper unless stated otherwise.

### C. Problem Statement

We focus on finding the downstream precoding vectors $\mathbf{P}_k$ and symbol transmit powers $\mathbf{S}_k^{DS}$ that maximize the weighted line sum-rate under per-line total power and per-tone spectral mask constraints

$$\max_{\{\mathbf{P}_k, \mathbf{S}_k^{DS} | \forall k\}} C^{DS} \quad \text{s.t.} \quad \sum_k \left[\mathbf{P}_k \mathbf{S}_k^{DS} \mathbf{P}_k^H\right]_{l,l} \leq P_{\text{line}}^l \quad \forall l$$
$$\left[\mathbf{P}_k \mathbf{S}_k^{DS} \mathbf{P}_k^H\right]_{l,l} \leq P_{k,\text{mask}}^l \quad \forall l, k \tag{4}$$

where $[\cdot]_{l,l}$ denotes the $(l,l)$-entry of a matrix. These power constraints are particularly important for G.fast transmission due to the lack of diagonal dominance structure of the channel matrix. The strong crosstalk then results in large precompensation signals which (especially for LP) may increase the per-line total power and per-tone transmit power, which should therefore be accounted for by means of DSM.

## III. BC-DSB

As the downstream weighted sum-rate functions (2) and (3) are neither convex nor concave in $\mathbf{P}_k$ or $\mathbf{S}_k^{DS}$, (4) is a difficult problem to solve [9]. To overcome this difficulty, we solve this downstream problem by solving its dual upstream problem using US-DS duality theory. Conventional US-DS duality states that the same set of data rates can be achieved in both domains under the same sum total transmit power constraint across all lines [9]. More realistic per-line total

power constraints can be considered by transforming the downstream Lagrange dual function based on the Lagrange multipliers into an equivalent version with a virtual sum power constraint before exploiting US-DS duality [7]. In this section, we extend the approach of [7] by adding spectral mask constraints and considering both LP and NLP. Furthermore, we propose a low-complexity iterative fixed-point formula to solve the resulting dual upstream problems, and SINR duality transformations to convert the resulting upstream solutions back into the downstream domain.

### A. Nonlinear Precoding

Firstly, using the "zero duality gap"-result for multi-carrier systems [11], we formulate a dual decomposition approach with standard subgradient based updating of the Lagrange multipliers. The Lagrangian of (4) is given as

$$\begin{aligned}\mathcal{L}^{DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k, \mathbf{P}_k, \mathbf{S}_k^{DS}|\forall k) = \\ \left(C^{DS} - \sum_k \sum_l s_k^{l,DS} \mathrm{Tr}\{(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)\mathbf{p}_k^l(\mathbf{p}_k^l)^H\}\right) \\ + \mathrm{Tr}\{\boldsymbol{\Omega}\mathrm{diag}\{P_{\mathrm{line}}^1, \cdots, P_{\mathrm{line}}^L\}\} \\ + \sum_k \mathrm{Tr}\{\boldsymbol{\Lambda}_k \mathrm{diag}\{P_{k,\mathrm{mask}}^1, \cdots, P_{k,\mathrm{mask}}^L\}\}, \end{aligned} \quad (5)$$

where $\boldsymbol{\Omega} = \mathrm{diag}\{\mu^1, \cdots, \mu^L\}$ and $\boldsymbol{\Lambda}_k = \mathrm{diag}\{\lambda_k^1, \cdots, \lambda_k^L\}$ are diagonal matrices containing the Lagrange multipliers corresponding to the per-line total power and spectral masks contraints for tone $k$, respectively. The Lagrange dual function is then defined as an unconstrained maximization of the Lagrangian for a given set of Lagrange multipliers

$$g^{DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k|\forall k) = \max_{\{\mathbf{P}_k, \mathbf{S}_k^{DS}|\forall k\}} \mathcal{L}^{DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k, \mathbf{P}_k, \mathbf{S}_k^{DS}|\forall k) \quad (6)$$

The downstream dual optimization problem is

$$\min_{\{\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k|\forall k\}} g^{DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k|\forall k) \quad \text{s.t.} \quad [\boldsymbol{\Omega}]_{l,l} \geq 0 \quad \forall l \qquad [\boldsymbol{\Lambda}_k]_{l,l} \geq 0 \quad \forall l, k \quad (7)$$

Secondly, we transform the downstream Lagrangian (5) into an equivalent Lagrangian corresponding to a sum-power constraint, before exploiting US-DS duality. This transformation is necessary as US-DS duality transformations do not preserve the per-line total power and spectral mask constraints when converting directly the upstream solution of (7) to the downstream domain. In particular, this consists in incorporating a virtual precoding matrix based on the Lagrange multipliers rescaling the channel matrix [7]. The resulting channel of tone $k$ is then described by

$$\mathbf{y}_k^{DS} = \underbrace{\mathbf{H}_k^H(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)^{-1/2}}_{\mathbf{H}_k^{'H}} \mathbf{P}_k' \mathbf{S}_k^{DS} \mathbf{x}_k^{DS} + \mathbf{z}_k, \quad (8)$$

where $\mathbf{P}_k' = (\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)^{1/2}\mathbf{P}_k$. Now, the downstream Lagrange dual function (6) is rewritten as

$$g^{DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k|\forall k) = \max_{\{\mathbf{P}_k', \mathbf{S}_k^{DS}|\forall k\}} \mathcal{L}^{'DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k, \mathbf{P}_k', \mathbf{S}_k^{DS}|\forall k) \quad (9)$$

where the transformed Lagrangian is

$$\begin{aligned}\mathcal{L}^{'DS}(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k, \mathbf{P}_k', \mathbf{S}_k^{DS}|\forall k) = \\ \sum_k \sum_l \left[\log_2\left(1 + \frac{s_k^{l,DS}|(\mathbf{h}_k^{'l})^H \mathbf{p}_k^{'l}|^2}{\Gamma(1+\sum_{j>l} s_k^{j,DS}|(\mathbf{h}_k^{'l})^H \mathbf{p}_k^{'j}|^2)}\right) \right. \\ \left. - s_k^{l,DS}\mathrm{Tr}\{\mathbf{p}_k^{'l}(\mathbf{p}_k^{'l})^H\}\right] + P_{\mathrm{v}}. \end{aligned} \quad (10)$$

$\mathbf{h}_k^{'l} = (\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)^{-1/2}\mathbf{h}_k^l$ are the rescaled channel vectors and $\mathbf{p}_k^{'l} = (\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)^{1/2}\mathbf{p}_k^l$ are the rescaled precoding vectors. Furthermore, for a given set of Lagrange multipliers, the per-line power and spectral mask constraints appear as a virtual sum power constraint $P_{\mathrm{v}} = \mathrm{Tr}\{\boldsymbol{\Omega}\mathrm{diag}\{P_{\mathrm{line}}^1, \cdots, P_{\mathrm{line}}^L\}\} + \sum_k \mathrm{Tr}\{\boldsymbol{\Lambda}_k \mathrm{diag}\{P_{k,\mathrm{mask}}^1, \cdots, P_{k,\mathrm{mask}}^L\}\}$. This transformation hides the Lagrange multipliers into the channel $\mathbf{h}_k^{'l}$ and the precoding $\mathbf{p}_k^{'l}$ vectors. As a result, the transformed Lagrangian (10) can be interpreted as the Lagrangian of a downstream problem with a virtual sum power constraint and with its virtual Lagrange multipliers equal to one. Therefore, we can now apply US-DS duality theory to transform (9) into an equivalent upstream Lagrange dual function for a fixed set of Lagrange multipliers $\{\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k|\forall k\}$ and under the same virtual sum power constraint.

In order to use this US-DS duality, we first introduce the dual upstream transmission system of (8) with only receiver coordination between the $L$ users (right-hand side of Fig. 1). This scenario is referred to as a Multiple Access Channel (MAC). This upstream model is obtained by switching the role of precoding and equalizer vectors and by Hermitian transposing the channel matrix, i.e.

$$\mathbf{y}_k^{US} = \mathbf{Q}_k\mathbf{H}_k(\mathbf{S}_k^{US})^{1/2}\mathbf{x}_k^{US} + \mathbf{Q}_k \underbrace{(\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)^{1/2}\mathbf{z}_k}_{\mathbf{z}_k'}. \quad (11)$$

We observe that the downstream virtual precoder results in an upstream noise covariance matrix equal to $E\{\mathbf{z}_k'(\mathbf{z}_k')^H\} = (\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k)$, as in [6]. We remark that this formulation is preferred over scaling the channel and equalizer matrices since it is numerically more robust whenever some of the Lagrange multipliers converge to zero. $(\mathbf{S}_k^{US})^{1/2} \triangleq \mathrm{diag}\{\sqrt{s_k^{1,US}}, \cdots, \sqrt{s_k^{L,US}}\}$ is the diagonal precoder matrix on tone $k$, where $s_k^{l,US}$ represents the symbol transmit power of line $l$. $\mathbf{Q}_k$ is the $L \times L$ matrix with the equalizer vectors on tone $k$. We emphasize that in the upstream domain it is easy to compute the optimal equalizer vectors and power allocation that maximize the weighted line sum-rate. Given the line transmit powers $s_k^{l,US}$, the theoretically optimal receiver with successive interference cancellation is the MMSE-GDFE [10], and is calculated using a simple closed form expression

$$\bar{\mathbf{q}}_k^l = \alpha_k^l \sqrt{s_k^{l,US}}(\mathbf{h}_k^l)^H \left((\boldsymbol{\Omega}+\boldsymbol{\Lambda}_k) + \sum_{j \leq l} s_k^{j,US}\mathbf{h}_k^j \mathbf{h}_k^{j\,H}\right)^{-1}, \forall l, \quad (12)$$

where we define $\bar{\mathbf{q}}_k^l$ as the $l$-th row of $\mathbf{Q}_k$ with a unity $l_2$-norm by setting $\alpha_k^l$ appropriately. The decoding order is the reverse

of the encoding order (i.e. line 1 is decoded last, line $L$ is decoded first) as US-DS duality dictates. Then, the weighted line sum-rate of this upstream system is

$$C'^{US} = \sum_k \sum_l w_l \log_2 \left(1 + \frac{1}{\Gamma} s_k^{l,US} (\mathbf{h}_k^l)^H ((\mathbf{\Omega} + \mathbf{\Lambda}_k) + \sum_{j<l} s_k^{j,US} \mathbf{h}_k^j (\mathbf{h}_k^j)^H)^{-1} \mathbf{h}_k^l\right). \quad (13)$$

We remark that this weighted upstream sum-rate function is concave in $\mathbf{S}_k^{US}$, meaning that the power allocation in this dual upstream system can be globally optimal computed [9].

Thirdly, based on this upstream model, we define the equivalent upstream Lagrange dual function of (9) as

$$g'^{US}(\mathbf{\Omega} + \mathbf{\Lambda}_k | \forall k) = \max_{\{\mathbf{S}_k^{US} | \forall k\}} \mathcal{L}'^{US}(\mathbf{\Omega} + \mathbf{\Lambda}_k, \mathbf{S}_k^{US} | \forall k) \quad (14)$$

where the Lagrangian is

$$\mathcal{L}'^{US}(\mathbf{\Omega} + \mathbf{\Lambda}_k, \mathbf{S}_k^{US} | \forall k) = \left[C'^{US} - \sum_k \sum_l s_k^{l,US}\right] + P_{\mathrm{v}}. \quad (15)$$

This means that for a given set of Lagrange multipliers $\{\mathbf{\Omega} + \mathbf{\Lambda}_k | \forall k\}$, the original downstream dual Lagrange function (6) can be solved by solving the equivalent upstream function (14), resulting in the optimal upstream transmit powers and MMSE equalizer vectors, and that then the corresponding downstream transmit powers and precoding vectors can be obtained using SINR duality transformations. Hence, updating the Lagrange multipliers such that the downstream per-line total power and spectral mask constraints are enforced, will eventually solve (4) optimally.

Fourthly, we propose a low-complexity iterative method to solve the upstream Lagrange dual function (14), based on a fixed-point reformulation of the KKT stationary condition of (14), similar to [12], [13], which is both necessary and sufficient. The fixed-point upstream transmit power update formula for line $l$ and tone $k$ is then given as

$$s_k^{l,US} = \left[\frac{w_l / \log(2)}{1 + \sum_{j>l} \frac{(w_j s_k^{j,US}) / \log(2)}{\Gamma + s_k^{j,US} (\mathbf{h}_k^j)^H (\mathbf{X}_k^j)^{-1} \mathbf{h}_k^j} |(\mathbf{h}_k^j)^H (\mathbf{X}_k^j)^{-1} \mathbf{h}_k^l|^2} - \frac{\Gamma}{(\mathbf{h}_k^l)^H (\mathbf{X}_k^l)^{-1} \mathbf{h}_k^l}\right]^+ \quad (16)$$

where $[x]^+$ means max(0, x) and

$$\mathbf{X}_k^l = \left((\mathbf{\Omega} + \mathbf{\Lambda}_k) + \sum_{j<l} s_k^{j,US} \mathbf{h}_k^j (\mathbf{h}_k^j)^H\right).$$

We observe that this formula is equivalent to the formula in [13] with the noise covariance matrix replaced by the Lagrange multipliers matrix. This iterative fixed-point approach has been shown in [12], [13] to work well with low complexity, and is amenable to distributed implementation. It converges to good accuracy in only 2-3 iterations over all lines in a Gauss-Seidel manner for each tone. Furthermore, we highlight that (16) corresponds to a continuous transmit power and bit loading, unlike the BC-OSB algorithm that can only do exhaustive searches over a discrete power and bit loading.

Finally, each time that, for given $\{\mathbf{\Omega} + \mathbf{\Lambda}_k | \forall k\}$, the dual Lagrange function is solved using the iterative fixed-point approach, we transform the resulting upstream transmit powers and MMSE equalizer vectors into downstream transmit powers and dual MMSE precoding vectors based on the SINR duality between the transformed upstream and original downstream channel [14]. It ensures that each line SINR, and therefore also line rate, is preserved between upstream and downstream on each tone $k$. The upstream SINR for line $l$ and tone $k$ is

$$\mathrm{SINR}_k^{l,US} = \frac{s_k^{l,US} |\bar{\mathbf{q}}_k^l \mathbf{h}_k^l|^2}{\bar{\mathbf{q}}_k^l (\mathbf{\Omega} + \mathbf{\Lambda}_k) (\bar{\mathbf{q}}_k^l)^H + \sum_{j<l} s_k^{j,US} |\bar{\mathbf{q}}_k^l \mathbf{h}_k^j|^2}. \quad (17)$$

Defining the relation between the upstream equalizing and downstream precoding vectors as

$$\mathbf{p}_k^l = (\bar{\mathbf{q}}_k^l)^H, \quad \forall l, k, \quad (18)$$

the downstream SINR is

$$\mathrm{SINR}_k^{l,DS} = \frac{s_k^{l,DS} |\bar{\mathbf{q}}_k^l \mathbf{h}_k^l|^2}{1 + \sum_{j>l} s_k^{j,DS} |\bar{\mathbf{q}}_k^j \mathbf{h}_k^l|^2}. \quad (19)$$

Equating the upstream and downstream SINR for all lines and tone $k$, generates a linear system of equations

$$\mathbf{Z}_k \cdot [s_k^{1,DS}, \cdots, s_k^{L,DS}]^T = [s_k^{1,US}, \cdots, s_k^{L,US}]^T, \forall k \quad (20)$$

with $[\mathbf{Z}_k]_{l,j}$ described by

$$\begin{cases} \bar{\mathbf{q}}_k^l (\mathbf{\Omega} + \mathbf{\Lambda}_k) (\bar{\mathbf{q}}_k^l)^H + \sum_{j<l} s_k^{j,US} |\bar{\mathbf{q}}_k^l \mathbf{h}_k^j|^2 & j = l \\ -s_k^{l,US} |\bar{\mathbf{q}}_k^j \mathbf{h}_k^l|^2 & j > l \\ 0 & j < l \end{cases} \quad (21)$$

The linear system of equations can always be solved yielding valid solutions $s_k^{l,DS} \geq 0$, because $\mathbf{Z}_k$ has non-negative diagonal and non-positive off-diagonal entries, and is column diagonally dominant for each tone $k$ [15], [16]. We remark that $s_k^{l,US} = 0$ results in a zero row and column in $\mathbf{Z}_k$ that have to be removed, together with the respective $s_k^{l,DS}$ and $s_k^{l,US}$, before solving (20), and then $\mathbf{p}_k^l = \mathbf{0}$ and $s_k^{l,DS} = 0$ are chosen. Furthermore, summing up the rows of (20), we obtain

$$\sum_l s_k^{l,DS} \underbrace{\bar{\mathbf{q}}_k^l (\mathbf{\Omega} + \mathbf{\Lambda}_k) (\bar{\mathbf{q}}_k^l)^H}_{\|\mathbf{p}_k'^l\|^2} = \sum_l s_k^{l,US}, \quad (22)$$

meaning that the same virtual sum power is consumed both in the downstream and upstream domain.

A complete algorithm description is given in **Algorithm 1**. This algorithm will be referred to as BC-DSB.

### *B. Linear Precoding*

LP based DSM for downstream transmission can also be handled by BC-DSB. This is in contrast with BC-OSB where the per-tone discrete exhaustive searches become computationally infeasible for LP. The only change when using LP instead

**Algorithm 1** BC-DSB

initialize $\mu_l, \lambda_k^l, s_k^{l,US}, \forall l,k$
**repeat**
  **repeat**
    **for** $l = 1 \dots L$ **do**
      $s_k^{l,US} =$ (16) or (23) $\forall k$
    **end for**
  **until** convergence
  Calculate $\mathbf{Q}_k$ with (12) or (24), $\forall k$
  Calculate $\mathbf{S}_k^{DS}$ via (20) and (21) or (25), $\forall k$
  Calculate $\mathbf{P}_k$ with (18), $\forall k$
  $\forall l,k : \lambda_k^l = \left[\lambda_k^l + \epsilon\big([\mathbf{P}_k\mathbf{S}_k^{DS}\mathbf{P}_k^H]_{l,l} - P_{k,\text{mask}}^l\big)\right]^+$
  $\forall l : \mu_l = \left[\mu^l + \epsilon\big(\sum_k [\mathbf{P}_k\mathbf{S}_k^{DS}\mathbf{P}_k^H]_{l,l} - P_{\text{line}}^l\big)\right]^+$
**until** convergence

of NLP is that (3) is used for $C^{DS}$ instead of (2), which results in a different fixed-point update formula

$$s_k^{l,US} = \left[\frac{w_l/\log(2)}{1+\sum_{j\neq l}\frac{(w_j s_k^{j,US})/\log(2)}{\Gamma+s_k^{j,US}(\mathbf{h}_k^j)^H(\mathbf{X}_k^j)^{-1}\mathbf{h}_k^j}|(\mathbf{h}_k^j)^H(\mathbf{X}_k^j)^{-1}\mathbf{h}_k^l|^2} - \frac{\Gamma}{(\mathbf{h}_k^l)^H(\mathbf{X}_k^l)^{-1}\mathbf{h}_k^l}\right]^+ \quad (23)$$

where $\mathbf{X}_k^l = \left((\mathbf{\Omega}+\mathbf{\Lambda}_k) + \sum_{j\neq l} s_k^{j,US}\mathbf{h}_k^j(\mathbf{h}_k^j)^H\right)$,

and a linear MMSE equalizer closed form expression

$$\bar{\mathbf{q}}_k^l = \alpha_k^l\sqrt{s_k^{l,US}}(\mathbf{h}_k^l)^H\left((\mathbf{\Omega}+\mathbf{\Lambda}_k) + \sum_j s_k^{j,US}\mathbf{h}_k^j{\mathbf{h}_k^j}^H\right)^{-1}, \forall l. \quad (24)$$

Furthermore, when using the duality transformations, $[\mathbf{Z}_k]_{l,j}$ is given by

$$\begin{cases} \bar{\mathbf{q}}_k^l(\mathbf{\Omega}+\mathbf{\Lambda}_k)(\bar{\mathbf{q}}_k^l)^H + \sum_{j\neq l} s_k^{j,US}|\bar{\mathbf{q}}_k^l\mathbf{h}_k^j|^2 & j=l \\ -s_k^{l,US}|\bar{\mathbf{q}}_k^j\mathbf{h}_k^l|^2 & j\neq l \end{cases} \quad (25)$$

We notice that the weighted upstream sum-rate function with linear MMSE equalizers is a nonconcave function in $\mathbf{S}_k^{US}$. As a result, the KKT stationary condition is now only necessary and not sufficient anymore, and therefore (23) will only converge to a locally optimal stationary point.

## IV. Performance Comparisons

In this section, we evaluate the performance of BC-DSB for the downstream G.fast 106 MHz and 212 MHz profile in a scenario consisting of 10 lines of 80 m. We consider for both profiles standard compliant G.fast parameters. The per-line total transmit power is 4 dBm. The capacity gap $\Gamma$ is set to 10.25 dB and the noise PSD to -140 dBm/Hz. The tone spacing is 51.75 kHz and the symbol rate is 48 KHz. The bandwidth of the G.fast profiles starts from 2.2 MHz and ends at 106 MHz and 212 MHz corresponding to a total of about $K = 2000$ and $K = 4000$ tones respectively. Spectral masks are obtained from [17]. The channel matrices have been

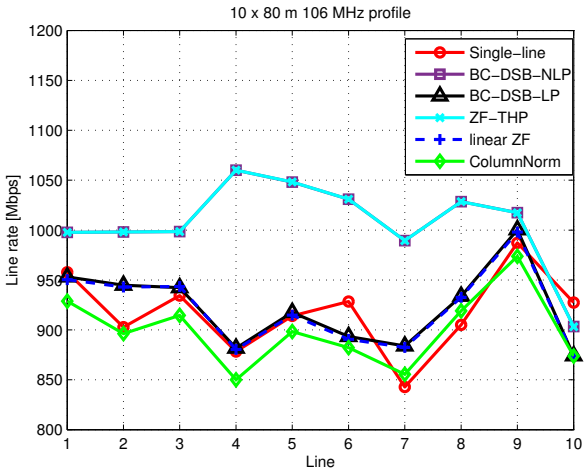


Fig. 2. The achieved line rates of all lines for the different precoding schemes and 106 MHz G.fast profile.

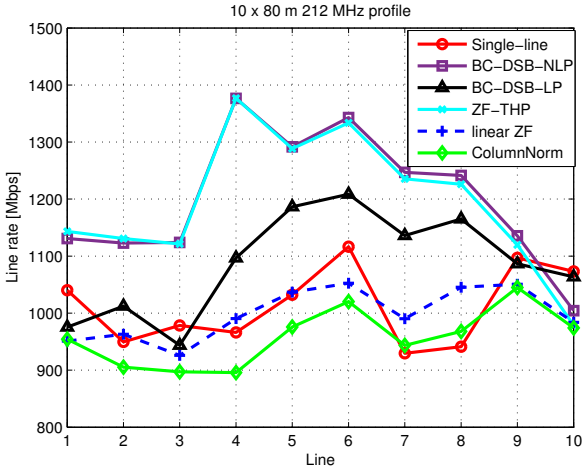


Fig. 3. The achieved line rates of all lines for the different precoding schemes and 212 MHz G.fast profile.

obtained by measurements. Performance is compared with: single-line activation without precoding, linear ZF with DSM by means of a heuristic low-complexity algorithm referred as 'ColumnNorm' [5], linear ZF [2] and ZF-THP [18] with DSM by means of gain scaling optimization such that all per-line total power and per-tone spectral mask constraints are satisfied. The bit loadings resulting from the different algorithms are capped at 12 bit in order to remain compliant with G.fast when calculating the line rates. Note that these line rates include the coding overhead meaning that the net data rates are lower.

The line rates achieved by the different lines in the 106 MHz (Fig. 2) and 212 MHz scenario (Fig. 3) for the different precoding schemes and equal weights demonstrate an improved performance of NLP over LP. For instance, BC-DSB-NLP outperforms BC-DSB-LP by roughly 10% in both profiles. However, these NLP results are slightly optimistic for G.fast transmission because the power penalties due to the modulo operations resulting in an increased transmit power

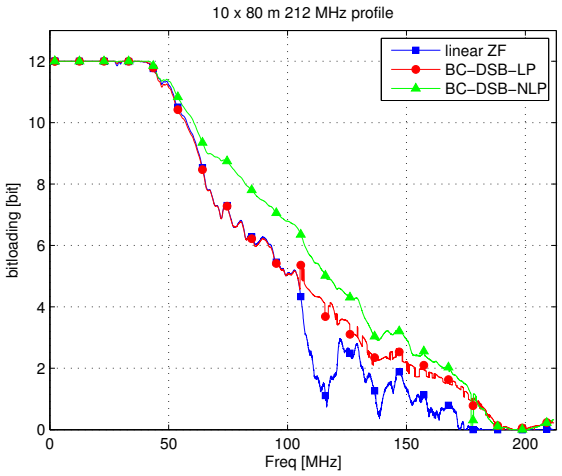


Fig. 4. The average bit loading, capped at 12 bit, of all lines across all tones clearly shows both BC-DSB-LP and -NLP outperforming linear ZF at high frequencies due to the strong crosstalk at these frequencies.

and capacity gap have been ignored [19]. We also highlight the performance variation across lines, which is due to the successive interference cancellation of NLP. Lines encoded last achieve lower rate gains than lines encoded first. Furthermore, this successive interference cancellation results only in a small performance improvement of BC-DSB-NLP over ZF-THP since it enhances the crosstalk environment and SINR, which then reduces the difference between ZF and MMSE filtering. For LP, on the other hand, the results show BC-DSB outperforming its linear ZF counterpart significantly (by about 8.5%) for the 212 MHz profile due the very strong crosstalk and high channel attenuation at high frequencies resulting in low SINR, such that MMSE outperforms ZF filtering (Fig. 4). However, due to very high direct channel attenuations at frequencies above 175 MHz, even BC-DSB-LP and -NLP bit loadings drop almost to zero (Fig. 4). We highlight the difference with the VDSL2 scenario where linear ZF-precoding is near-optimal due to the diagonal dominance structure of the channel matrix at low (i.e. below 30 MHz) frequencies [2]. Finally, we remark that the performance of the low-complexity heuristic 'ColumnNorm' algorithm is remarkably close (about 2% and 4% resp.) to the performance of ZF with optimal gain scaling for both profiles.

## V. Conclusion

Motivated by the strong crosstalk at high frequencies making linear ZF precoding no longer near-optimal, we have investigated both linear and nonlinear precoding based DSM for line sum-rate maximization in the context of downstream vectored G.fast transmission under realistic per-line total power and per-tone spectral mask constraints. For this problem, we have developed a novel algorithm called BC-DSB that transforms the downstream dual decomposition approach formulation before exploiting US-DS duality. Furthermore, a low-complexity iterative fix-point formula has been proposed to solve the resulting upstream problems, and finally SINR duality transformations have been presented to convert the upstream solutions back to the downstream domain. A performance comparison with measured channel data up to both 106 and 212 MHz has been provided revealing a significantly improved performance of BC-DSB over linear ZF precoding.

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