DISTRIBUTED GEVD-BASED SIGNAL SUBSPACE ESTIMATION IN A FULLY-CONNECTED WIRELESS SENSOR NETWORK

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ABSTRACT

In this paper, we present a distributed algorithm for network-wide signal subspace estimation in a fully-connected wireless sensor network with multi-sensor nodes. We consider scenarios where the noise field is spatially correlated between the nodes. Therefore, rather than an eigenvalue decomposition (EVD-) based approach, we apply a generalized EVD (GEVD-) based approach which allows to directly incorporate the (estimated) noise covariance. Furthermore, the GEVD is also immune to unknown per-channel scalings. We first use a distributed algorithm to estimate the principal generalized eigenvectors (GEVCs) of a pair of network-wide sensor signal covariance matrices, without explicitly constructing these matrices, as this would inherently require data centralization. We then apply a transformation at each node to extract the actual signal subspace estimate from the principal GEVCs. The resulting distributed algorithm can reduce the per-node communication and computational cost. We demonstrate the effectiveness of the algorithm by means of numerical simulations.

Index Terms— Wireless sensor network (WSN), distributed estimation, signal subspace estimation, generalized eigenvalue decomposition (GEVD)

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1. INTRODUCTION

Signal subspace estimation plays an important role in array processing algorithms [1]. For instance the direction-of-arrival (DOA) estimation performance of algorithms like MUSIC [2] or ESPRIT [3] strongly depends upon a signal subspace estimation. Moreover, in the field of adaptive beamforming , an imprecise signal subspace estimation often results in a significant performance degradation [4].

We consider the problem of network-wide signal subspace estimation in a fully-connected wireless sensor network (WSN), with multi-sensor nodes, and where the noise field is possibly spatially correlated. The per-node signal subspace can be estimated from the local sensor signal covariance matrix without signal exchange between nodes. However, if also the relative geometry between the nodes has to be captured in the signal subspace, the network-wide signal subspace should be estimated from the network-wide sensor signal covariance matrix. Furthermore, even where this relative geometry is irrelevant, the computation of the network-wide signal subspace may provide better estimates of the per-node signal subspaces, because more correlation structure can be exploited (as demonstrated in [5], and in the simulations in this paper). To estimate the network-wide signal subspace, one option is to let each node transmit its sensor observations to a fusion center (FC) where the data is then processed in a centralized fashion. However this centralization of data requires the availability of a sufficiently powerful FC and demands a significant per-node communication cost. In this paper we propose an alternative distributed algorithm to obtain the centralized estimation performance, without explicitly constructing a network-wide sensor signal covariance matrix.

To estimate the network-wide signal subspace in a distributed fashion, an eigenvalue decomposition- (EVD-) based approach was proposed in [6], [7]. However, the GEVD is better suited for scenarios with spatially correlated noise, assuming that the noise covariance is known a-priori or can be

estimated as explained in this paper, e.g., based on "noise-only" signal segments. Furthermore, the GEVD is immune to a scaling of the individual sensor observations, e.g., if the sensor gain is not calibrated between nodes. Hence, we first estimate the S network-wide principal generalized eigenvectors (GEVCs) using the distributed adaptive covariance-matrix generalized eigenvector estimation (DACGEE) algorithm [8], where S is the a-priori defined dimension of the signal subspace. However, while the eigenvectors of a sensor signal covariance matrix may directly correspond to the underlying signal subspace, this is not the case for the GEVCs of a pair of covariance matrices, i.e., the GEVCs cannot directly be used as a signal subspace estimate. The actual signal subspace can be extracted by the inversion of a matrix containing all the GEVCs. However, as the DACGEE algorithm only extracts S principal GEVCs, the latter is not possible in a distributed fashion. Therefore, we propose a technique that allows to transform the estimated principal GEVCs into a set of basis vectors that span the actual signal subspace, i.e., without the need to also compute the other GEVCs.

The paper is organized as follows. The problem statement and data model are presented in Section 2. Centralized GEVD-based signal subspace estimation is described in Section 3. The proposed distributed algorithm is presented in Section 4. The simulation results are presented in Section 5. Finally, conclusions are drawn in Section 6.

2. PROBLEM STATEMENT AND DATA MODEL

We consider a WSN with K multi-sensor nodes in which each node $k \in \mathcal{K} = \{1, \ldots, K\}$ collects observations of a complex-valued M_k-channel sensor signal $\mathbf{u}_k$. Note that this also allows for a hierarchical WSN where K master nodes receive sensor observations from M_k slave nodes with a single sensor. The topology of the network is assumed to be fully-connected which means that data broadcast by a node can be received by all other $K-1$ nodes in the network. The sensor signal $\mathbf{u}_k$ consists of a mixture of S target source signals and additive noise, which may be spatially correlated between nodes. Hence $\mathbf{u}_k$ can be modeled as

$$\mathbf{u}_k = \mathbf{A}_k\mathbf{s} + \mathbf{n}_k \tag{1}$$

where $\mathbf{s}$ is an S-channel signal containing S target source signals, $\mathbf{A}_k = [\mathbf{a}_{k1} \cdots \mathbf{a}_{kS}]$ is a static (or slowly varying) $M_k \times S$ steering matrix where $\mathbf{a}_{ks}$ $(s = 1, \cdots, S)$ is the so-called steering vector (SV) from source s to the sensors of node k, and $\mathbf{n}_k$ is the additive noise. The sensor signal $\mathbf{u}_k$ is assumed to satisfy short-term stationarity and ergodicity conditions. By stacking all $\mathbf{u}_k$'s and $\mathbf{n}_k$'s, we obtain the network-wide M-channel sensor signal $\mathbf{u}$ and $\mathbf{n}$, respectively. Likewise, we define the $M \times S$ matrix $\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_S]$ as the stacked version of all $\mathbf{A}_k$'s such that

$$\mathbf{u} = \mathbf{A}\mathbf{s} + \mathbf{n}. \tag{2}$$

In this paper we consider the problem of estimating an S-dimensional basis that for the so-called signal subspace, i.e., the column space of the network-wide steering matrix $\mathbf{A}$, based on a GEVD of the covariance matrices of $\mathbf{u}$ and $\mathbf{n}$. The signal subspace is estimated without explicitly constructing these covariance matrices, as this would require centralization of all the sensor observations. Instead the nodes will only exchange S-channel sensor observations, which results in a compression factor of M_k/S at node k (assuming $M_k \geq S$). We assume that S is known or estimated a-priori (as in [2], [6], [7]). It is noted that, if $S = 1$, the problem reduces to an SV estimation problem, where we estimate $\mathbf{a}_1$ up to a scaling ambiguity.

3. CENTRALIZED GEVD-BASED SIGNAL SUBSPACE ESTIMATION

In this section, we first explain how the signal subspace can be estimated by means of the GEVD of the covariance matrices of $\mathbf{u}$ and $\mathbf{n}$. Without loss of generality (w.l.o.g.), we assume that $\mathbf{u}$ is zero-mean which possibly requires a mean subtraction preprocessing step. The network-wide sensor signal correlation matrix is then defined as

$$\mathbf{R}_{uu} = E\{\mathbf{u}\mathbf{u}^H\} \tag{3}$$

where $E\{\cdot\}$ denotes the expected value operator, and the superscript H denotes the conjugate transpose operator. The exact sensor signal covariance matrix as defined in (3) is often not available in practice, but can be estimated via sample averaging. To this end, we define the $M \times N$ observation matrix $\mathbf{U}$, where each column corresponds to an observation of $\mathbf{u}$ at a certain time instant, such that $\mathbf{R}_{uu}$ can be approximated as

$$\mathbf{R}_{uu} \approx \frac{1}{N}\mathbf{U}\mathbf{U}^H \tag{4}$$

and when having an infinitely long observation window we can write $\mathbf{R}_{uu} = \lim_{N\to\infty} \frac{1}{N}\mathbf{U}\mathbf{U}^H$.

We also define the network-wide sensor noise covariance matrix $\mathbf{R}_{nn} = E\{\mathbf{n}\mathbf{n}^H\}$ where it is assumed that $\mathbf{R}_{nn}$ is either known a-priori or can be estimated from noise-only segments in the sensor observations (similar to (4)). The latter can be performed in applications such as speech enhancement where $\mathbf{R}_{uu}$ and $\mathbf{R}_{nn}$ can be estimated during "speech-and-noise" and "noise-only" segments, respectively, which can be distinguished by means of a voice activity detection (VAD) mechanism [9].

In order to perform a GEVD of the ordered matrix pair $(\mathbf{R}_{uu}, \mathbf{R}_{nn})$, each GEVC and its corresponding generalized eigenvalue (GEVL), $\mathbf{x}_m$ and λ_m $(m = 1 \cdots M)$, respectively, must be computed such that $\mathbf{R}_{uu}\mathbf{x}_m = \lambda_m\mathbf{R}_{nn}\mathbf{x}_m$ [10], or equivalently

$$\mathbf{R}_{uu}\mathbf{X} = \mathbf{R}_{nn}\mathbf{X}\mathbf{\Lambda} \tag{5}$$

where $\mathbf{X} = [\mathbf{x}_1 ... \mathbf{x}_M]$ and $\mathbf{\Lambda} = diag\{\lambda_1 \cdots \lambda_M\}$. Note that this can be written as a non-symmetric EVD as

$$\mathbf{R}_{nn}^{-1}\mathbf{R}_{uu} = \mathbf{X}\mathbf{\Lambda}\mathbf{X}^{-1} \tag{6}$$

if $\mathbf{R}_{nn}$ is invertible. In the sequel, we assume w.l.o.g. that the GEVLs in $\mathbf{\Lambda}$ are sorted in descending order. Since the

GEVCs are defined up to a scaling, we assume w.l.o.g. that all $\mathbf{x}_m$'s are scaled such that $\mathbf{x}_m^H \mathbf{R}_{nn} \mathbf{x}_m = 1$.

Note that the GEVD is equivalent to a joint diagonalization of $\mathbf{R}_{uu}$ and $\mathbf{R}_{nn}$, i.e., it can be verified from (6) that

$$\mathbf{R}_{uu} = \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^H \tag{7}$$

$$\mathbf{R}_{nn} = \mathbf{Q}\mathbf{\Gamma}\mathbf{Q}^H \tag{8}$$

where $\mathbf{Q} = \mathbf{X}^{-H}$ is a full-rank $M \times M$ matrix (not necessarily orthogonal), and where $\mathbf{\Sigma} = diag\{\sigma_1 \cdots \sigma_M\}$ and $\mathbf{\Gamma} = diag\{\gamma_1 \cdots \gamma_M\}$ are diagonal matrices. Note that (6) then implies that the GEVLs are equal to $\mathbf{\Lambda} = diag\{\frac{\sigma_1}{\gamma_1} \cdots \frac{\sigma_M}{\gamma_M}\}$. From (2) and (8), it follows that

$$\mathbf{R}_{uu} = \mathbf{A}\mathbf{\Pi}\mathbf{A}^H + \mathbf{R}_{nn} = \mathbf{A}\mathbf{\Pi}\mathbf{A}^H + \mathbf{Q}\mathbf{\Gamma}\mathbf{Q}^H \tag{9}$$

where $\mathbf{\Pi} = diag\{P_1, ..., P_S\}$ with P_s the power of target source signal s. With (7), it follows that

$$\mathbf{A}\mathbf{\Pi}\mathbf{A}^H = \mathbf{Q}\big(\mathbf{\Sigma} - \mathbf{\Gamma}\big)\mathbf{Q}^H. \tag{10}$$

Since $\mathbf{Q}$ is full rank, and since the left-hand side of (10) consists of a positive semi-definite matrix with rank S, we see that $\mathbf{\Sigma} - \mathbf{\Gamma}$ contains only S non-zero diagonal entries. Therefore, the first S GEVLs are larger than one ($\sigma_m > \gamma_m$), and others are all equal to 1 ($\sigma_m = \gamma_m$). The first S columns of Q must then span the same S-dimensional subspace as the columns of $\mathbf{A}$, i.e., define the signal subspace.

We define $\hat{\mathbf{X}} = [\mathbf{x}_1 \cdots \mathbf{x}_S]$ as an $M \times S$ matrix where the columns are the principal GEVCs corresponding to the S largest GEVLs of $(\mathbf{R}_{uu}, \mathbf{R}_{nn})$, i.e., the first S columns of $\mathbf{X}$. Similarly, we define $\hat{\mathbf{Q}}$ as the $M \times S$ matrix containing the first S columns of $\mathbf{Q}$, which span the signal subspace. In the sequel, we explain how the columns of $\hat{\mathbf{Q}}$ can be estimated in a distributed fashion.

It is reiterated that the GEVD-based signal subspace estimation allows to directly incorporate the (estimated) noise covariance matrix, which is not the case in an EVD-based approach. In addition, the GEVD is also immune to unknown per-channel scalings (e.g., due to lack of sensor calibration between nodes), which is explained as follows. Applying a random scaling to (some) channels of $\mathbf{u}_k$ at node k results in a similar scaling of the corresponding rows and columns of the network-wide sensor signal correlation matrix $\mathbf{R}_{uu}$. This scaling has an influence on the entire eigenstructure of $\mathbf{R}_{uu}$, i.e., all coefficients of all its eigenvectors are affected. This is an undesirable effect if the eigenvectors are used to estimate the signal subspace or SV. Indeed, a simple scaling of the channels in one node should not affect the signal subspace or SV estimate in other nodes. However, it can be shown that the GEVD of $(\mathbf{R}_{uu}, \mathbf{R}_{nn})$ does not have this effect, i.e., the same scaling will only affect the coefficients in the GEVCs corresponding to the scaled channels at the node k, i.e., the scaling remains localized and does not spread out to other GEVC coefficients. As a result, the signal subspace or SV estimate at others nodes will not be affected.

4. DISTRIBUTED GEVD-BASED SIGNAL SUBSPACE ESTIMATION

In a WSN, a node k has only access to its own M_k-channel sensor signal $\mathbf{u}_k$ corresponding to M_k rows of the observation matrix $\mathbf{U}$ in (4) and hence can only estimate an $M_k \times M_k$ submatrix of $\mathbf{R}_{uu}$ and $\mathbf{R}_{nn}$. This seems to hamper the computation of $\hat{\mathbf{Q}}$, unless all the sensor observations are centralized to estimate the network-wide $\mathbf{R}_{uu}$ and $\mathbf{R}_{nn}$. In this section, we explain how $\hat{\mathbf{Q}}$ can be estimated and updated (in a block-adaptive fashion) while reducing the per-node communication cost by a factor M_k/S (assuming $M_k \geq S$). To this end, we use the DACGEE algorithm [8] to first estimate $\hat{\mathbf{X}}$ in a distributed fashion. We then explain how the subspace spanned by the columns of $\hat{\mathbf{Q}}$ can be computed from $\hat{\mathbf{X}}$, without performing the explicit matrix inversion $\mathbf{Q} = \mathbf{X}^{-H}$, which would otherwise also require the other GEVCs to construct the full matrix $\mathbf{X}$.

4.1. DACGEE algorithm

The goal of the DACGEE algorithm is to estimate and update $\hat{\mathbf{X}}$ in a distributed fashion. Here we review the DACGEE algorithm only briefly, yet the reader can find the details of the algorithm derivation and convergence proofs in [8].

Defining i as the iteration index, we define $\hat{\mathbf{X}}^i$ as the estimate of $\hat{\mathbf{X}}$ at iteration i (or in the i-th block of N sensor observations). We also define the partitioning $\hat{\mathbf{X}}^i = [\hat{\mathbf{X}}_1^{i\,T} \cdots \hat{\mathbf{X}}_K^{i\,T}]^T$ in which $\hat{\mathbf{X}}_k^i$ is the part that corresponds to node k. Hence we can also write $\hat{\mathbf{X}}^{i\,H}\mathbf{u} = \sum_{k \in \mathcal{K}} \hat{\mathbf{X}}_k^{i\,H}\mathbf{u}_k$. Each node k only updates the submatrix $\hat{\mathbf{X}}_k^i$, and then uses it to compress its M_k-channel sensor signal into the S-channel signal

$$\overline{\mathbf{u}}_k^i = \hat{\mathbf{X}}_k^{i\,H}\mathbf{u}_k\,. \tag{11}$$

We assume for the sake of an easy exposition that $S < M_k$, $\forall k \in \mathcal{K}$, yet if at a node k, $S \geq M_k$, node k merely broadcasts its sensor observations $\mathbf{u}_k$, in which case no compression is achieved at node k. A node k broadcasts N observations of $\overline{\mathbf{u}}_k^i$ in iteration i where all other nodes can collect them (fully-connected topology). Therefore a node k has access to the following signal and its corresponding covariance matrix:

$$\widetilde{\mathbf{u}}_k^i = \begin{bmatrix} \mathbf{u}_k \\ \overline{\mathbf{u}}_{-k}^i \end{bmatrix} \Longrightarrow \mathbf{R}_{\tilde{u}_k \tilde{u}_k}^i = E\{\widetilde{\mathbf{u}}_k^i \widetilde{\mathbf{u}}_k^{i\,H}\} \tag{12}$$

where $\overline{\mathbf{u}}_{-k}^i = [\overline{\mathbf{u}}_1^{i\,T} \ \ldots \ \overline{\mathbf{u}}_{k-1}^{i\,T}\ \overline{\mathbf{u}}_{k+1}^{i\,T} \ \ldots \ \overline{\mathbf{u}}_K^{i\,T}]^T$. In a similar way, we can define $\mathbf{R}_{\tilde{n}_k \tilde{n}_k}^i$, which can be estimated from $\widetilde{\mathbf{u}}_k^i$ during "noise-only" segments[1]. The nodes then sequentially compute the reduced-dimension GEVD of $(\mathbf{R}_{\tilde{u}_k \tilde{u}_k}^i, \mathbf{R}_{\tilde{n}_k \tilde{n}_k}^i)$ and use the result to update $\hat{\mathbf{X}}_k^i$. The DACGEE algorithm is summarized in Table 1 (ignore step 5 for the time being).

[1] If the network-wide $\mathbf{R}_{nn}$ is known a-priori , one can also compute $\mathbf{R}_{\tilde{n}_k \tilde{n}_k}^i$ directly by means of the compression matrices $\mathbf{X}_k^i$ from the other nodes.

Note that the token assigning the updating node q is moving in a round-robin fashion and that the updates happen in a block-adaptive fashion, in blocks of N observations. In [8], it has been shown that the DACGEE algorithm converges to the centralized solution, i.e., $\lim_{i\to\infty} \hat{\mathbf{X}}^i = \hat{\mathbf{X}}$. It is noted that this only holds perfectly if iterations are performed with one block of N observations. In practice, iterations are spread out over different blocks, in which case the convergence and optimality is only approximately satisfied due to discrepancies in the sensor signal and noise covariance matrix estimates in the different blocks.

4.2. Signal subspace estimation

In this Section we propose a technique to transform $\hat{\mathbf{X}}$ to $\hat{\mathbf{Q}}$ (up to a column scaling), i.e., to estimate $\hat{\mathbf{Q}}$ from $\hat{\mathbf{X}}$ without explicitly relying on the full $\mathbf{X}$ and computing $\mathbf{Q} = \mathbf{X}^{-H}$. We define the S-channel signal

$$\overline{\mathbf{u}} = \hat{\mathbf{X}}^H \mathbf{u}\,. \tag{13}$$

When considering the covariance between $\mathbf{u}$ and $\overline{\mathbf{u}}$, we have that

$$\mathbf{R}_{u\bar{u}} = E\{\mathbf{u}\overline{\mathbf{u}}^H\} = E\{\mathbf{u}\mathbf{u}^H\hat{\mathbf{X}}\} = \mathbf{R}_{uu}\hat{\mathbf{X}}\,. \tag{14}$$

Using this with (7) and the fact that $\mathbf{Q} = \mathbf{X}^{-H}$, we find

$$\mathbf{R}_{u\bar{u}} = \mathbf{Q}\mathbf{\Sigma}\mathbf{Q}^H\mathbf{Q}^{-H}\mathbf{E}_S = \hat{\mathbf{Q}}\hat{\mathbf{\Sigma}} \tag{15}$$

with $\hat{\mathbf{\Sigma}} = diag\{\sigma_1 \cdots \sigma_S\}$ and with $\mathbf{E}_S = [\mathbf{I}_s\ \mathbf{0}]$, where $\mathbf{I}_s$ denotes the $S \times S$ identity matrix and $\mathbf{0}$ is an all-zero matrix. The diagonal matrix $\hat{\mathbf{\Sigma}}$ only scales the columns of $\hat{\mathbf{Q}}$ and hence does not affect its column space. This shows that $\mathbf{R}_{u\bar{u}}$ defines a matrix for which the columns span the same subspace as the columns of $\hat{\mathbf{Q}}$, which is sufficient for our actual goal, i.e., estimating a basis for the column space of $\mathbf{A}$ in (2).

$\mathbf{R}_{u\bar{u}}$ can be estimated based on per-node operations without any additional data exchange. Indeed, $\bar{\mathbf{u}}$ can be constructed at each node as

$$\overline{\mathbf{u}}^i = \sum_{k\in\mathcal{K}} \overline{\mathbf{u}}_k^i. \tag{16}$$

Node k can then estimate its part of $\mathbf{R}_{u\bar{u}} = E\{\mathbf{u}\overline{\mathbf{u}}^H\}$ as

$$\mathbf{R}_{u_k\bar{u}}^i = E\{\mathbf{u}_k\overline{\mathbf{u}}^{i\,H}\} \approx \frac{1}{N}\mathbf{U}_k\overline{\mathbf{U}}^{i\,H} \tag{17}$$

where $\mathbf{U}_k$ and $\overline{\mathbf{U}}^i$ are $M_k \times N$ and $S \times N$ matrices containing N observations of $\mathbf{u}_k$ and $\overline{\mathbf{u}}^i$ in their columns, respectively. Stacking all $\mathbf{R}_{u_k\bar{u}}^i$'s yields an estimate $\mathbf{R}_{u\bar{u}}^i$ for $\mathbf{R}_{u\bar{u}}$, i.e., an estimate of the signal subspace. Note that, due to the fact that $\lim_{i\to\infty} \hat{\mathbf{X}}^i = \hat{\mathbf{X}}$, we also have that

$$\lim_{i\to\infty} \mathbf{R}_{u\bar{u}}^i = \mathbf{R}_{u\bar{u}}. \tag{18}$$

The resulting algorithm is described in Table 1.

Table 1. Distributed GEVD-based signal subspace estimation

1. Set $i \leftarrow 0$, $q \leftarrow 1$, and initialize all $\hat{\mathbf{X}}_k^0$, $\forall\, k \in \mathcal{K}$, with random entries.
2. Each node $k \in \mathcal{K}$ broadcasts N new compressed observations $\overline{\mathbf{u}}_k^i[j] = \hat{\mathbf{X}}_k^{i\,H}\mathbf{u}_k[iN+j]$ (where $j = 1 \ldots N$).
3. At node q:
   - Estimate $\mathbf{R}_{\tilde{u}_q\tilde{u}_q}^i$ and $\mathbf{R}_{\tilde{n}_q\tilde{n}_q}^i$ similar to (4).
   - Compute the columns of $\widetilde{\mathbf{X}}_q^{i+1}$ as the S principal GEVCs of $(\mathbf{R}_{\tilde{u}_q\tilde{u}_q}^i, \mathbf{R}_{\tilde{n}_q\tilde{n}_q}^i)$.
   - Define $P = S(K-1)$ and partition $\widetilde{\mathbf{X}}_q^{i+1}$ as
   $$\hat{\mathbf{X}}_q^{i+1} = \left[\mathbf{I}_{M_k}\ \mathbf{O}_{M_k\times P}\right]\widetilde{\mathbf{X}}_q^{i+1} \tag{19}$$
   $$\mathbf{G}_{-q} = \left[\mathbf{O}_{P\times M_k}\ \mathbf{I}_{S(K-1)}\right]\widetilde{\mathbf{X}}_q^{i+1} \tag{20}$$
   and broadcast $\mathbf{G}_{-q}$ and $\overline{\mathbf{u}}_q^{i,new}[j] = \hat{\mathbf{X}}_q^{i+1\,H}\mathbf{u}_q[iN+j]$ to all the other nodes.
4. Each node $k \in \mathcal{K}\backslash\{q\}$ updates
   $$\hat{\mathbf{X}}_k^{i+1} = \hat{\mathbf{X}}_k^i\mathbf{G}_k \tag{21}$$
   where $\mathbf{G}_{-q} = \left[\mathbf{G}_1^T\ \ldots\ \mathbf{G}_{q-1}^T\,\mathbf{G}_{q+1}^T\ \ldots\ \mathbf{G}_K^T\right]^T$.
5. Each node $k \in \mathcal{K}$ computes $\overline{\mathbf{u}}^i = \sum_{k\in\mathcal{K}\backslash\{q\}} \overline{\mathbf{u}}_k^i + \overline{\mathbf{u}}_q^{i,new}$ locally and updates $\mathbf{R}_{u_k\bar{u}}^i$ as in (17).
6. $i \leftarrow i+1$ and $q \leftarrow (q \mod K) + 1$.
7. Return to step 2.

5. SIMULATION RESULTS

In this Section, we demonstrate the performance of the proposed distributed signal subspace estimation via numerical Monte-Carlo (MC) simulations, and compare it with the "centralized" and the "isolated" approach. The latter approach corresponds to each node only having access to its own M_k-channel sensor signal and hence there is no cooperation.

A different simulation scenario with $K = 10$ nodes is created in each MC run where the data model described in (1)-(2) is considered. Each node k observes a 15-channel ($M_k = 15$) stochastic sensor signal $\mathbf{u}_k$, $\forall k \in \mathcal{K}$. In total 10 localized sources are assumed in each MC scenario, from which S are considered as the target sources and the remaining $10 - S$ sources are treated as noise sources (we simulate for different values of S). The S target sources have an on-off behavior, while the other $10 - S$ sources are continuously active. The network-wide noise signal $\mathbf{n}$ can be described as $\mathbf{n} = \mathbf{B}\mathbf{z} + \mathbf{v}$ where $\mathbf{B}$ is the steering matrix corresponding to the noise sources, $\mathbf{z}$ contains the $10 - S$ noise source signals, and $\mathbf{v}$ models the spatially uncorrelated noise signals. The network-wide steering matrices $\mathbf{A}$ and $\mathbf{B}$ are static matrices with dimensions $150 \times S$ and $150 \times (10 - S)$, respectively, in which the entries are drawn from a uniform distribution over the interval $[-0.5; 0.5]$. $\mathbf{s}$ and $\mathbf{z}$ are S-channel and $(10 - S)$-channel stochastic source signals from which the observations are independently drawn from a uniform distribution over the interval $[-0.5; 0.5]$. Moreover, $\mathbf{v}$ is a 150-channel stochastic signal from which the observations are in-

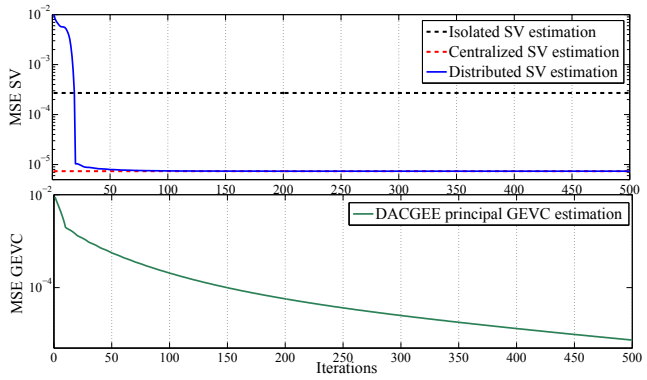


Fig. 1. Single target source scenario (SV estimation)

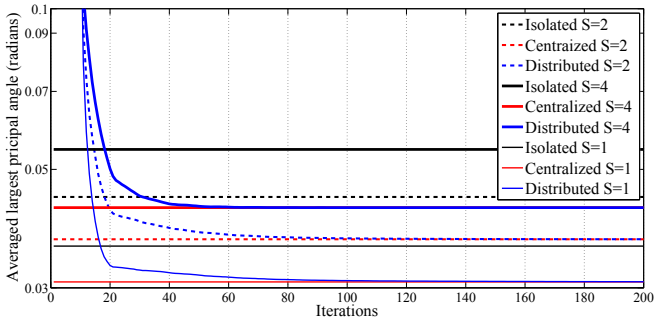


Fig. 2. Multiple target sources (signal subspace estimation)

dependently drawn from a uniform distribution over the interval $[-\sqrt{0.1}/2; \sqrt{0.1}/2]$.

Fig. 1 illustrates the results for the case where $S = 1$, i.e., a single target source scenario, averaged over 200 MC runs. In the upper part of this figure the mean squared errors (MSEs) between the entries of the exact SV $\mathbf{a}_1$ and the SV estimate in the isolated approach, centralized approach and the proposed distributed algorithm are shown over the different iterations of the algorithm. Note that a normalization is performed, as well as a compensation for the sign ambiguity, before computing the MSE. As can be seen, the SV estimation obtained with the distributed algorithm converges to the SV estimation obtained by the centralized approach, which is significantly better than the isolated approach. The bottom part of Fig. 1 illustrates the MSE between the entries of the centralized GEVC $\hat{\mathbf{X}}$ and its DACGEE-based estimate $\hat{\mathbf{X}}^i$. Comparing these figures demonstrates that the distributed SV estimation converges faster than the principal GEVC estimation in the DACGEE algorithm.

To evaluate the performance when $S \geq 1$, we compute the largest canonical angle (principal angle) between the true steering matrix $\mathbf{A}_k$ and its corresponding signal subspace estimate $\mathbf{R}^i_{u_k \bar{u}}$ at each node k. Fig. 2 shows the averaged principal angles for different values of S over the different iterations of the distributed algorithm. It is again observed that cooperation (either centralized or distributed) improves the signal subspace estimate. It is also observed that the estimate obtained with the distributed algorithm converges to the estimate obtained with the centralized approach, where the convergence speed improves when S increases. This fact relates to the convergence speed of the DACGEE algorithm which as disscussed in [8], is faster with a larger S.

6. CONCLUSION

In this paper, we have proposed a distributed algorithm for network-wide signal subspace in a fully-connected WSN. Rather than a standard EVD-based approach, we have applied a GEVD-based approach which not only allows us to directly incorporate the (estimated) noise covariance matrix, but which is also immune to unknown per-channel scalings. We have used the DACGEE algorithm to first compute the principal GEVCs, from which a basis estimate for the signal subspace is then extracted (without relying on the full set of GEVCs). We have shown that the estimates obtained with the proposed distributed algorithm converges to the estimates obtained with the centralized approach. The effectiveness of our algorithm has been demonstrated via numerical MC simulations.

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