A new input-output model for multidimensional linear shift invariant dynamical systems

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1 Introduction

In recent year the next big revolution in system theory has been initiated, the step to a multidimensional system description, or nD systems. Multidimensional systems are systems characterised by signals who depend on several independent variables. These variables could be for example time and space. In recent years a number of results have been developed around this idea. A handful of nD state space models can be found in the literature such as those proposed by Roesser, Attasi, Fornasini and Marchesini. A summary and generalization of these models is presented by Kurek [3]. All of the previous models differ in the way that the inputs are introduced.

2 Problem statement

In this project we approach the modelling of nD systems in an other way. The need for a new multidimensional model class has been initiated by looking at the flaws from previous attempts. To include the inputs, Green's functions [1] were used. Green's functions are a direct generalization of the impulse response of 1D systems to nD systems. By using the reasoning behind Green's functions and an extra natural constraint that the model solution should be path invariant, we were able to construct a new model class satisfying the imposed constraints.

3 Model Class

To derive a new model for nD dynamical systems we started from the Dreesen model, first presented in [2], this is an nD autonomous system. To include the inputs we notice that every discrete impulse generates a corresponding Green's function. In this way the 2D extension of Dreesen model is given by,

$$
\begin{aligned}
x_{k+1,l} &= A_1 x_{k,l} + A_1 \sum_{i=0}^{N_x} A_2^{l-i} B u_{k,i} \\
x_{k,l+1} &= A_2 x_{k,l} \\
y_{k,l} &= C x_{k,l}.
\end{aligned}
$$

Here A_1 and A_2 are commuting matrices. The first equation updates the states in the time direction, the second one in the spatial direction. The inputs enter via a convolution in the time-update equation. This convolution is the convolution of the Green's function and the input at some time instance. For some specific equations, for example the heat equation, the interpretation of this convolution can be directly linked to a Fourier expansion of the input signal. For the moment the input-output behaviour for separable partial differential equations can be modelled using this model class.
One of the applications that have been worked out is the simulation of the dynamics of a cantilever beam. In classical 1D system theory this problem is treated as a MIMO system, where the discrete spatial correlation is not explicitly modelled. Using the correct nD model representation a SISO model that captures the dynamics in space and time can be used. This is a natural representation for lumped parameter model.

4 Future research

In this paper only the first steps of our current research on nD systems is presented, there are still a lot of challenges to overcome. The model presented can only handle homogeneous boundary conditions. The next step is to include a general set of boundary conditions in to the equations. After this extension, corresponding realisation algorithms have to be developed to estimate the model from a given dataset. At the moment the model estimation is only possible in the autonomous case. Research for subspace methods to estimate the full model are being developed.

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References

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