Order estimation of multidimensional transfer function by calculating Hankel intersections

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1 Introduction

This presentation introduces a data-driven method for determining the order of the transfer function representation of a multidimensional (nD) linear system. A Hankel matrix (referred to as recursive Hankel matrix) is constructed from the available multidimensional data in a recursive way. This work extends the concept of past and future data to nD systems and introduces the concept of mode$-k$ left and right data. The intersection between two mode$-k$ left and right matrices reveals the order of the system in dimension k.

2 Problem statement

In this work the class of nD models is restricted to linear difference equations [1], referred to as PdEs. All equations are defined on a rectangular domain in $n-$dimensions. For a two-dimensional model, the class of linear PdEs is given by,

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\beta_{i,j}y[k_1+i,k_2+j]=\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\alpha_{i,j}u[k_1+i,k_2+j], \quad (1)$$

where k_1 and k_2 are two independent variables. $u[\cdot,\cdot]$ and $y[\cdot,\cdot]$ are the input and output variables respectively, $\beta_{i,j}$ and $\alpha_{i,j}$ are the coefficients of the PdE and N_1 and N_2 determine the order of the PdE. Note that the order of the PdE is a tuple: for every dimension the order of the PdE is equal to the highest order of the shift operator.
At the basis of the identification algorithm presented in this work lies the concept of the recursive Hankel matrix. This matrix is a block Hankel matrix where all the blocks are Hankel themselves.

3 Intersections between past, future, left and right.

The intersection algorithm presented in [2] calculates the intersection between past and future Hankel matrices. For a two-dimensional dataset, past and future is extended with left and right. Graphically the concept of past and future, left and right is shown in Fig. 1. The data matrix is first Hankelized, and afterwards split up in four matrices, past, future, left and right. The intersections between these matrices reveals the order of the PdE.

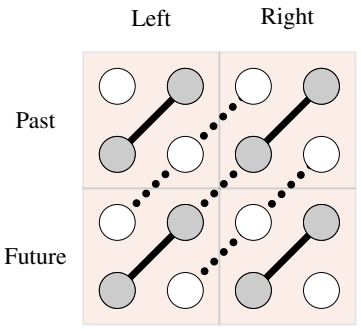

Fig. 1: Multidimensional transfer functions can graphically be represented by stencils. The dots in this figure represent data points distributed in two dimensions. The solid and dashed lines connecting two points represent linear relations between adjacent data points. By splitting up the data in left-right, past-future some relations are removed, these linear relations are denoted by the dashed lines.

4 Results

The main result of this presentation is an algorithm to estimate the order of a PdE on a rectangular grid with a uniform sampling time/distance. The data is Hankelized and split up in different Hankel matrices. Based on the rank of these matrices the order of the PdE is estimated. The presented method for estimating the order of a PdE is demonstrated on a numerical simulation example.

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References

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[2] Marc Moonen, Bart De Moor, Lieven Vandenberghe, and Joos Vandewalle. On-and off-line identification of linear state-space models. *International Journal of Control*, 49(1):219–232, 1989.