A discussion on the canonical decomposition of two dimensional descriptor systems

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1 Introduction

The linear autonomous descriptor systems[2][3] in two dimensions that we will consider are described by

$$\begin{aligned} Ex[k+1,l] &= Ax[k,l] \\ Fx[k,l+1] &= Bx[k,l] \\ y[k,l] &= Cx[k,l], \end{aligned} \tag{1}$$

where $y[k,l] \in \mathbb{R}^p$ and $x[k,l] \in \mathbb{R}^n$, the output and the state vector of the system respectively. The system equations are characterized by four square matrices A, B, E and F. A natural question to ask is, *under what conditions is this model well-posed and does there exist a canonical form*? One dimensional descriptor systems have been analyzed in [4][1].

2 Two dimensional descriptor systems

We propose the following: The two-dimensional system described in Eqn. (1), with (E,A) and (F,B), two regular pencils, is well-posed if and only if there exist square matrices P, Q, and U of full rank, such that the equivalent model

$$\begin{aligned} PEQx[k+1,l] &= PAQx[k,l] \\ UFQx[k,l+1] &= UBQx[k,l] \\ y[k,l] &= CQx[k,l], \end{aligned}$$

exists with

$$PEQ = \begin{bmatrix} \mathbb{1} & & & \\ & \mathbb{1} & & \\ & & E_1 & \\ & & & E_2 \end{bmatrix}, PAQ = \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \mathbb{1} & \\ & & & \mathbb{1} \end{bmatrix},$$

$$UFQ = \begin{bmatrix} \mathbb{1} & & & \\ & F_1 & & \\ & & \mathbb{1} & \\ & & & F_2 \end{bmatrix}, UBQ = \begin{bmatrix} B_1 & & & \\ & \mathbb{1} & & \\ & & B_2 & \\ & & & \mathbb{1} \end{bmatrix},$$

and

$$CQ = \begin{bmatrix} C_{RR} & C_{RS} & C_{SR} & C_{SS} \end{bmatrix},$$

where the matrices E_1, E_2, F_1, and F_2 are nilpotent. In this basis, the state vector has the form

$$x[k,l] = \begin{bmatrix} x^{RR}[k,l]^T & x^{RS}[k,l]^T & x^{SR}[k,l]^T & x^{SS}[k,l]^T \end{bmatrix}^T$$

and the following additional equations must hold

$$A_1B_1 = B_1A_1, A_2F_1 = F_1A_2, E_1B_2 = B_2E_1, E_2F_2 = F_2E_2.$$

When the two dimensional descriptor system is transformed to this form, all four system matrices E, A, F, B commute. As a consequence, the state sequence $x[k,l]$ satisfies

$$E^nF^mx[n,m] = A^nB^mx[0,0].$$

3 Conclusion

In this presentation, some necessary conditions for the existence of a non-trivial state sequence $x[k,l]$ of a two-dimensional descriptor system has been derived. As a consequence of this it is shown that a well-posed descriptor system can always be transformed to a form where the system matrices commute.

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