Recursive Output Only Subspace Identification for In-flight Flutter Monitoring

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Abstract

In this paper a new recursive output-only identification algorithm is proposed based on stochastic realization, a classical covariance driven subspace identification technique. The recursive algorithm yields results that are in close agreement with those of its non-recursive counterpart. Furthermore, the relatively low complexity of the algorithm makes it an excellent kandidate for use during in-flight flutter tests.

1 INTRODUCTION

In-flight flutter tests require a continuous monitoring of vibrational modes and their evolution. Traditionally this is accomplished by applying various types of artificial excitation during flight tests and analyzing the vibro-acoustic response of the plane using semi-online input-output system identification techniques. Such tests are expensive, due to their time consuming nature, and the necessity of the excitation devices. Current research efforts therefore focus on the possibility of the use of output-only identification techniques for flight-tests and the replacement of semi-online identification techniques by fully recursive online versions. In general such identification techniques need to satisfy two goals:

1. Allow for the continuous online monitoring of modal frequencies and dampings with a reasonably fast response to changes in the response of the airplane, at least in the order of minutes.
2. Provide a good nominal model for fault detection techniques, which look for sudden discrepancies between incomming data and the nominal model. As such sudden changes may signal the onset of flutter, the quality of the fault detection algorithm, and hence also of the nominal model that it uses is of critical importance.

In this paper a recursive subspace identification algorithm is proposed to reach these goals. The algorithm will be shown to be suited for online applications and capable of both the tracking of modal parameters and the generation of a good nominal model for fault detection algorithms.

The outline of the paper is as follows; In Section 2 subspace algorithms will be introduced and their use in modal analysis discussed. In Section 3 a brief overview of the stochastic realization algorithm for offline use will be given. In Section 4, the basic ingredients of the IV-PAST algorithm will be discussed and applied to the stochastic realization algorithm. In Section 5 the new recursive algorithm will be tested by means of some examples. In Section 6, finally, the conclusions will be drawn.

Throughout this paper, we will make extensive use of vector- and matrix-notations. In general, lowercase symbols (u,v,..) denote column vectors. Uppercase symbols (A,B,..) will be used for matrices. Submatrices will be refered to in matlab-style, eg. $A(:,1)$ denotes the first column of A, $B(1:2,1:2)$ is the uppermost 2 by 2 submatrix of B.

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2 SUBSPACE ALGORITHMS

Subspace identification algorithms have gained increasing attention in the modal analysis community over the last few years [1, 2]. This due to their inherent robustess and their ability to deal with large numbers of inputs and outputs. In contrast to classical predictor error methods where a costly, often nonlinear, optimization has to be performed, subspace identification techniques derive models for linear systems solely by applying well conditioned operations like projections and QR- and SVD-decompositions on structured data matrices.

Subspace algorithms can roughly be divided in two groups, based on the information that is available in the structured data matrices. A first group, the so-called data-driven methods are directly applied to input-output or output-only data. Various such subspace algorithms (see eg. [3, 4]) have been proposed , and are actively used in many industrial domains, such as the chemical industry [5] and wireless communication [6]. For the input-output case, recursive versions have recently been studied to extend the applicability of subspace algorithms from offline to online applications. Especially the use of fast approximate subspace tracking algorithms like PAST (projection approximation subspace tracking) and its instrumental variable extension IV-PAST, as reported in [7], seems promising. A second group of subspace algorithms, the covariance-driven methods, are applied to auto- and cross-covariance measurements of inputs and outputs. Especially in their output-only setting, the so-called stochastic realization algorithm [8], they have become the primal choice for modal analysis on ambiently excited structures in many research groups [1, 2]. In the following sections, we will show that also for the stochastic realization algorithm, a recursive version can be derived using the IV-PAST algorithm. Given the fact that the stochastic realization algorithm has been shown to be well suited for the identification of vibrational modes from in-flight test data [9, 10], such a recursive algorithm is an excellent candidate for mode-tracking during in-flight flutter tests.

3 THE STOCHASTIC REALIZATION ALGORITHM

Consider an n$^{\text{th}}$ order linear output-only system of the form:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k, \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

with $y_k \in \mathbb{R}^{l\times 1}$ the output of the system at time k, $x_k \in \mathbb{R}^{n\times 1}$ the internal state, and $w_k \in \mathbb{R}^{n\times 1}$ and $v_k \in \mathbb{R}^{l\times 1}$ the process and measurement noise at time k which are assumed to be white, stationary, gaussian zero mean with covariance matrix:

$$\mathcal{E}\left\{\begin{bmatrix} w_k \\ v_k \end{bmatrix}\begin{bmatrix} w_l^T & v_l^T \end{bmatrix}\right\} = \begin{bmatrix} Q_w & 0 \\ 0 & Q_v \end{bmatrix}\delta_{kl}. \tag{2}$$

Systems of the form (1) have been extensively used to model large vibrating structures like bridges, cars and airplanes [1, 9]. Following a system identification procedure on measured output only data, estimates for the resonance frequencies f_i, dampings d_i, and modal vectors ν_i of the structure are easily found as:

$$f_i = \arg\left(\lambda_i \frac{T_s}{2\pi}\right), \tag{3}$$

$$d_i = \frac{\ln\left(|\lambda_i|\right)}{\sqrt{\ln\left(|\lambda_i|\right)^2 + \arg\left(\lambda_i\right)^2}}, \tag{4}$$

$$\nu_i = \text{Real}\left(CV(:,i)\right), \tag{5}$$

where $A = V\Lambda V^{-1}$ is the eigenvalue decomposition of A and T_s is the sampling frequency. Defining $G \equiv \mathcal{E}\{x_k y_k^T\}$ and $R_i \equiv \mathcal{E}\{y_k y_{k-i}^T\}, R_i \geq 0$ it can be proven that $R_i = CA^iG$. As a result, with $\mathcal{O}$ and $\mathcal{C}$ the infinity observability and controllability matrices of a system (A, G, C), we have:

$$\mathcal{H} = \begin{bmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix}\begin{bmatrix} G & AG & A^2G & \dots \end{bmatrix} = \mathcal{O}C. \tag{6}$$

Given a finite set of measured covariances $\hat{R}_0 \dots \hat{R}_{2i}$, and defining

$$\hat{\mathcal{H}} = \begin{bmatrix} \hat{R}_0 & \hat{R}_1 & \dots & \hat{R}_i \\ \hat{R}_1 & \hat{R}_2 & & \hat{R}_{i+1} \\ \vdots & & \ddots & \vdots \\ \hat{R}_i & \hat{R}_{i+1} & \dots & \hat{R}_{2i} \end{bmatrix}, \tag{7}$$

a finite dimensional estimate $\hat{\mathcal{O}}$ for $\mathcal{O}$ can now be found by taking the singular value decomposition of $\hat{\mathcal{H}}$ and retaining the left singular vectors corresponding to the n highest singular values. If the exact model order is not known, as is most often the case, a good estimate for n can for instance be found by looking for a sudden drop in the ordered set of singular values. From $\hat{\mathcal{O}}$, estimates for A and C can be derived as:

$$\hat{C} = \hat{\mathcal{O}}(1:l,:), \tag{8}$$

$$\hat{A} = \underline{\hat{\mathcal{O}}}^{\dagger}\overline{\hat{\mathcal{O}}}, \tag{9}$$

with $\underline{\hat{\mathcal{O}}}$ and $\overline{\hat{\mathcal{O}}}$ shorthand notations for $\hat{\mathcal{O}}$ with its last, respectively first l lines removed. It is important to note that in practical applications $\hat{\mathcal{H}}$ can easily be calculated starting from data by defining:

$$y_k^+ \equiv \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+i} \end{bmatrix}, \qquad y_k^- \equiv \begin{bmatrix} y_k \\ y_{k-1} \\ \vdots \\ y_{k-i} \end{bmatrix}, \tag{10}$$

and calculating

$$\hat{\mathcal{H}} = \frac{1}{p}\sum_k y_k^+ {y_k^-}^T, \tag{11}$$

where k is ranging over the entire set of available data and p is an optional normalization parameter. As multiplying $\hat{\mathcal{H}}$ by a constant does not affect the obtained matrices $\hat{A}$ and $\hat{C}$, p can be set to 1 without further influence on the obtained models. Equation (11) will play a leading role in the derivation of the recursive identification algorithm in the following section.

4 A RECURSIVE VERSION FOR THE STOCHASTIC REALIZATION ALGORITHM

By far the most time-consuming step in the stochastic realization as outlined in Section 3 is the singular value decomposition. It will therefore come as no surprise that efforts to deduce a recursive version of the stochastic realization algorithm will focus on updating the singular value decomposition of the matrix $\hat{\mathcal{H}}$ when new data becomes available. Suppose we have an initial measured output sequence $\{y_0, \ldots, y_{N-1}\} \in \mathbb{R}^{l\times N}$, $\hat{H}$ has been formed according to (11) with $p = 1$, and an initial estimate for the model order n and the matrices $\mathcal{O}$ and $\mathcal{C}$ has been obtained using the singular value decomposition. if a new data point y_{N+1} is measured, y^+_{N+1-i}, and y^-_{N+1-i} can be formed and the hankel matrix $\hat{\mathcal{H}}$ can be updated in one of the three following ways.

1. **Exponential forgetting:** Old data is exponentially forgotten by multiplying the existing $\hat{\mathcal{H}}$ by a factor β with β in general close to one. The new data is added as follows:

$$\hat{\mathcal{H}}_{N+1} = \beta\hat{\mathcal{H}}_N + y^+_{N+1-i}(y^-_{N+1-i})^T. \tag{12}$$

2. **Sliding window:** The recursive algorithm forms a sliding window over the data. New data is incorporated into $\hat{\mathcal{H}}$ at the same rate as old data is removed. A typical updating rule for $\hat{\mathcal{H}}$ might look as follows.

$$\hat{\mathcal{H}}_{N+1} = \hat{\mathcal{H}}_N + y^+_{N+1-i}(y^-_{N+1-i})^T - y_i^+ {y_i^-}^T. \tag{13}$$

   It is important to note that the length of the output sequence used for the initial estimate of the model N, does not necessarily have to be equal to the window-size of the recursive algorithm. One might decide to start with a small or empty initial sequence, gradually extend it while updating $\hat{\mathcal{H}}$, and only start removing data from $\hat{\mathcal{H}}$ once a suitable window length has been reached.

3. **Combined approach:** The recursive algorithm forms a sliding window over the data. Inside this sliding window, exponential forgetting is applied. Some book-keeping is required when using this approach to make sure that old-data is removed with a weight equal to the exponential forgetting that the data has undergone while inside the window. A typical update rule might look like:

$$\hat{\mathcal{H}}_{N+1} = \beta\hat{\mathcal{H}}_N + y^+_{N+1-i}(y^-_{N+1-i})^T - \beta^{N+1-2i} y_i^+ {y_i^-}^T. \tag{14}$$

It is clear that all manipulations of $\hat{\mathcal{H}}$ amount to updates of the form $\hat{\mathcal{H}}_t = \mu\hat{\mathcal{H}}_{t-1} + y_t^+ {y_t^-}^T$. As the influence of a multiplication with μ is obvious, the problem of updating the stochastic realization reduces to the problem of updating the singular value

decomposition of $\hat{\mathcal{H}}_{t-1}$, given a rank 1 update. An approximate algorithm to do this has been described in [11]. It was shown that

$$V[W(t)] = \|\hat{\mathcal{H}}_t - W(t)W^T(t)\hat{\mathcal{H}}_t\|_F^2 \tag{15}$$

with $W(t) \in \mathbb{R}^{(i+1)\times n}$ reaches its global minimum for $W(t) = \hat{U}_s T$ with T an arbitrary unitary matrix and U_s the n dominant singular vectors in the singular value decomposition of $\hat{\mathcal{H}}_t$.

$$\hat{\mathcal{H}}_t = \begin{bmatrix}\hat{U}_s & \hat{U}_n\end{bmatrix} \begin{bmatrix}\hat{S}_s & 0 \\ 0 & \hat{S}_n\end{bmatrix} \begin{bmatrix}\hat{V}_s^T \\ \hat{V}_n^T\end{bmatrix}. \tag{16}$$

Hence, in stead of calculating the singular value decomposition of $\hat{\mathcal{H}}_t$, an estimate for $\hat{\mathcal{O}}_t$ can be found by minimizing the criterium (15). A recursive algorithm to do so can be found by replacing $W^T(t)$ by $W^T(t-1)$, which reduces the problem to a least squares problem with the following update rule.

$$W(t) = \hat{\mathcal{H}}_t \left[W^T(t-1)\hat{\mathcal{H}}_t\right]^\dagger, \tag{17}$$

Calculating (17) is numerically less expensive than re-calculating the entire singular value decomposition, but still too complex to be used for online applications as mode tracking for in-flight flutter tests. A faster implementations based on the multiplication of $W^T(t-1)$ with every incomming data vector y_t^+ rather than the entire covariance matrix $\hat{\mathcal{H}}_t$, and employing the inversion theorem to speed up the calculation of the pseudo inverse has been outlined in [11]. From a practical point of view, the algorithm can be summarized as follows:

1. Suppose estimates for $W(t-1)$, $P(t-1)$, $\hat{\mathcal{H}}_{t-1}$, and $\hat{\mathcal{W}}_{t-1}$ are available at a certain iteration step t, where:

$$\hat{\mathcal{W}}_{t-1} = W(t-1)\hat{\mathcal{H}}_{t-1}, \tag{18}$$

$$P(t-1) = \left[\hat{\mathcal{W}}_{t-1}\hat{\mathcal{W}}_{t-1}^T\right]^{-1}. \tag{19}$$

2. Given a rank one update $y_t^+ y_t^{-T}$ of $\hat{\mathcal{H}}_{t-1}$, calculate new estimates $W(t)$, $P(t)$, $\hat{\mathcal{H}}_t$, and $\hat{\mathcal{W}}_t$ using the following algorithm:

$$h(t) = W^T(t-1)y_t^+, \tag{20}$$

$$w(t) = \hat{\mathcal{W}}_{t-1}y_t^-, \tag{21}$$

$$v(t) = \begin{bmatrix}\hat{\mathcal{H}}_{t-1}y_t^- & y_t^+\end{bmatrix}, \tag{22}$$

$$\psi(t) = \begin{bmatrix}w(t) & h(t)\end{bmatrix}, \tag{23}$$

$$\Lambda(t) = \frac{1}{\mu^2}\begin{bmatrix}-y_t^{-T}y_t^- & \mu \\ \mu & 0\end{bmatrix}, \tag{24}$$

$$K(t) = [\mu^2\Lambda(t) + \psi(t)^T P(t-1)\psi(t)]^{-1}\psi(t)^T P(t-1), \tag{25}$$

$$W(t) = W(t-1) + [v(t) - W(t-1)\psi(t)]K(t), \tag{26}$$

$$P(t) = \frac{1}{\mu^2}[P(t-1) - P(t-1)\psi(t)K(t)], \tag{27}$$

$$\hat{\mathcal{W}}_t = \mu\hat{\mathcal{W}}_{t-1} + h(t)y_t^{-T}, \tag{28}$$

$$\hat{\mathcal{H}}_t = \mu\hat{\mathcal{H}}_{t-1} + y_t^+ y_t^{-T}. \tag{29}$$

$$\tag{30}$$

3. Continue the iteration in step 2 until all data has been processed.

Note that the iteration step t in the algorithm is not necessarily a measure for the number of data-samples that has been processed. Recursive algorithms with a sliding window may need two iterations for each data-sample, one updating, and one downdating. The complexity of the algorithm is $3n(i+1) + \mathcal{O}(mn)$ for each iteration as opposed to $\mathcal{O}(i+1)^3$ for the full singular value decomposition. Hence, a fast recursive stochastic realization technique is effectively obtained when applying this algorithm to the update rules (12-14). In the following section, the tracking capabilities of the algorithm will be studied on a set of examples.

5 EXAMPLES

Two examples will be given in this section to evaluate the tracking capabilities of the recursive stochastic realization as discussed in Section 4. In a first example, a single output, simulated dataset with a length of 7 minutes, sampled at 32 Hz was used. Three

modes are present in the dataset, of whom two modes are kept constant during the experiment at 5.1 and 10.7 Hz with damping coefficients 0.8 $\%$ and 0.5 $\%$ respectively. The damping of a third mode at 6.6 Hz was made to vary between 0.2 and 1.9 $\%$ with a sudden drop in damping at about 4 minutes into the dataset. A phenomenon, not uncommon, during in-flight flutter tests and usually a reason for concern. The dataset was identified with the recursive stochastic realization algorithm as outlined in Section 4 with a forgetting factor of 1 (no forgetting) and a moving window of 1.5 minutes. The identification parameters were set to $n = 6$ and $i = 40$. The results are shown in Figure 1. The dashed lines are the known frequencies and damping coefficients. The stars symbolize the estimated modal parameters in a time window, 1.5 minutes long, centered around the displayed estimate. From the figure, it is clear that, although on a time-scale of 1.5 minutes damping estimates using output only techniques can be expected to be quite noisy [12], the most important development during the experiment, which is the change of the damping of the mode at 6.6 Hz is nicely tracked.

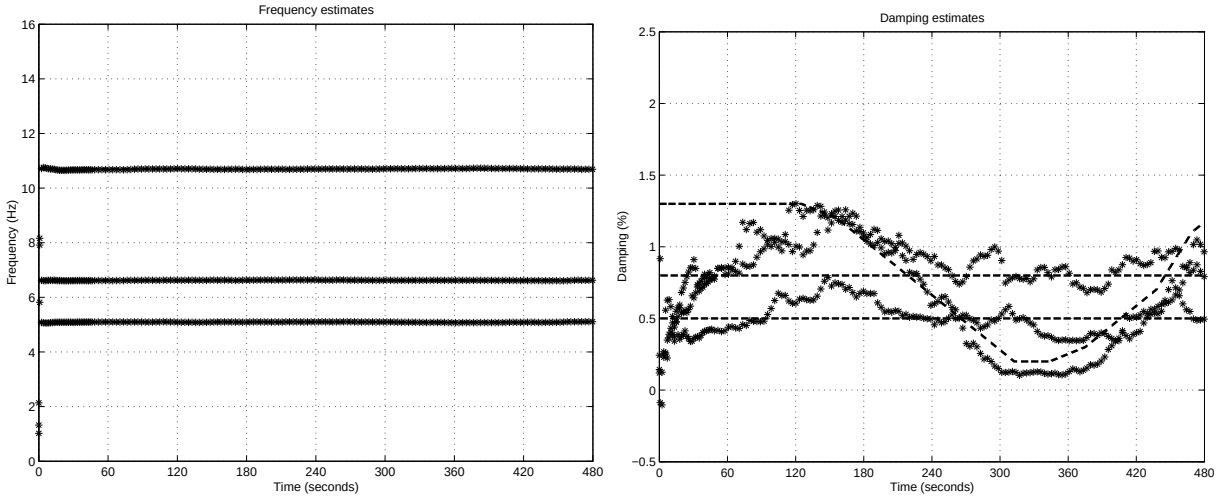


Figure 1: Extracted frequencies and damping coefficients from a simulated dataset containing three modes. True values are depicted with a dashed line. Estimated values with a '*'. Estimated values are placed with the time-window in which they are derived centered around the symbols. Simulations where run with an empty initial dataset. Note the fast convergence of the frequencies (in the order of seconds) and the changing damping which is correctly tracked

Furthermore, a comparisson of the models obtained in a certain window using the IV-PAST algorithm and a classical stochastic realization algorithm reveales that the recursive algorithm yields models that are almost of the same quality as its non-recursive counterpart (Figure 2).

In Figures 1 and 2, the plotted damping coefficients are the ones estimated in a time window centered around the displayed estimate. Although this is a logical way to display data for offline analysis, in online applications, only past measurements are available. In Figure 3, experimental results are displayed as they would appear during an online application. Note the obvious delay between true damping coefficients and their estimates, about half the length of the time-window. This situation could be improved by adding a forgetting factor so that more recent data gets weighted more heavily in the estimation procedure. For the tracking of very fast variations one has to revert to fault detection techniques as reported in [10, 13], in which a recursively updated nominal model based on recent data as discussed in Section 4 is a powerfull asset.

In a second example, the proposed algorithm was applied, to a reallife dataset available within the Eureka project Flite (see acknowledgments). A data sample from a test-flight was recursively identified with a forgetting factor β set to 1 (no forgetting) and a window length of 24 seconds. The identification parameters were set to $i = 64$ and $n = 32$. In Figure 4 the evolution of the obtained frequency and damping for one of the dominant modes in an arbitrary time-slice with a length of slightly more than one minute is displayed. For obvious reasons the actual values for the frequency and damping coefficient are not displayed. Yet, an increase in frequency and damping over the slice are clearly observed in the Figure. To accurately track modes which are not dominant in the dataset (due to weak excitations or an unfortunate placement of the measurement sensors), the window length will have to be increased beyond the 24 seconds. Again, in this case one while have to rely on fault-detection techniques.

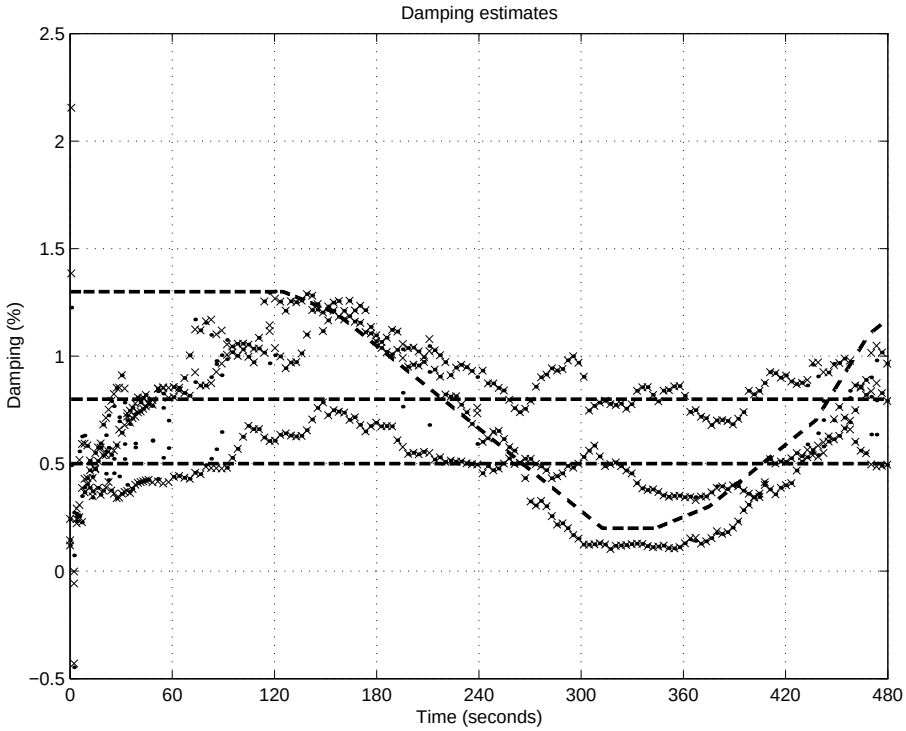


Figure 2: Comparisson of Extracted damping coefficients from a simulated dataset using a recursive algorithm '.' and the classical stochastic realization 'x'. True values are depicted with a dashed line. Note that both identification algorithms generate about the same results

6 CONCLUSIONS

We have proposed a recursive algorithm for the online output-only identification of the vibro-acoustic behavior of airplanes, based on the classical stochastic realization algorithm and the IV-PAST subspace tracking algorithm. The proposed algorithm was seen to be able to provide accurate tracking of damping coefficients and frequencies, in simulations as well as on reallife datasets. The algorithm can furthermore conveniently be used to generate nominal models for use in online fault detection techniques to detect the onset of flutter.

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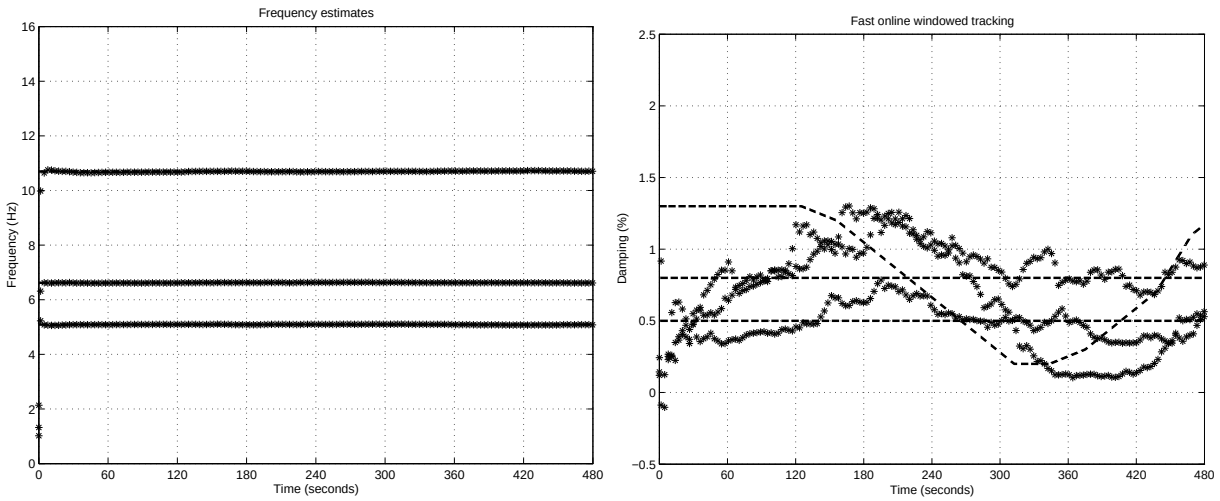


Figure 3: Extracted frequencies and damping coefficients from a simulated dataset containing three modes. True values are depicted with a dashed line. Estimated values with a '*'. The time-window in which the estimates are derived is located at the left side of the symbols, in accordance with the information as it is available in true online applications.

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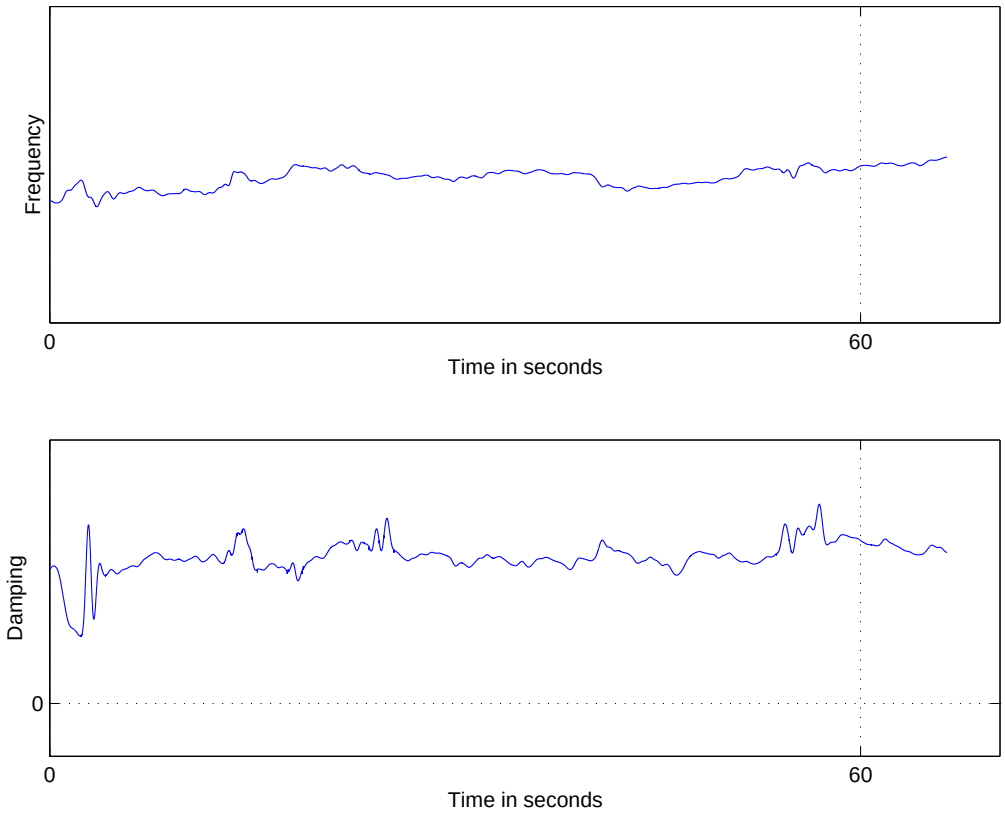


Figure 4: Tracking of a dominant mode in a reallife flight-test dataset