TENSORIZATION

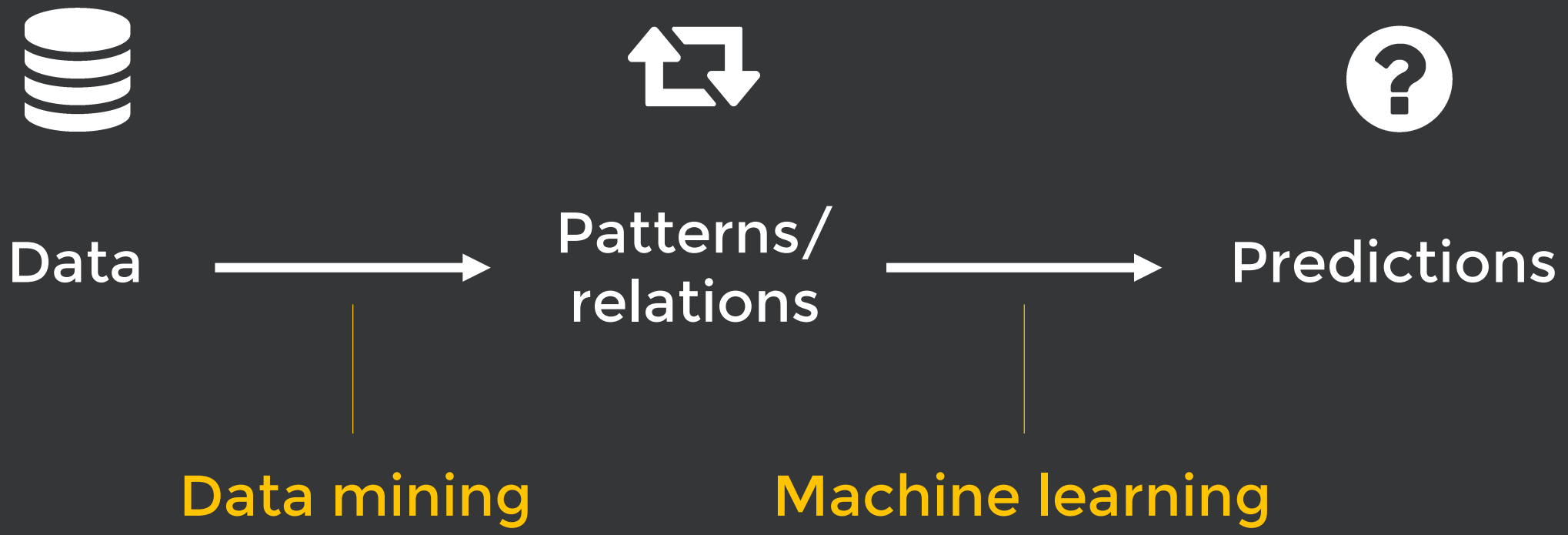
AND APPLICATIONS IN BLIND SIGNAL SEPARATION

OTTO DEBALS

Supervisors:
Prof. Lieven De Lathauwer
Prof. Marc Van Barel

iwt
KU LEUVEN

Slides and manuscript can be found on www.defense.debals.com


Data
Patterns/
relations
Predictions
Data mining
Machine learning

Self-driving car algorithms

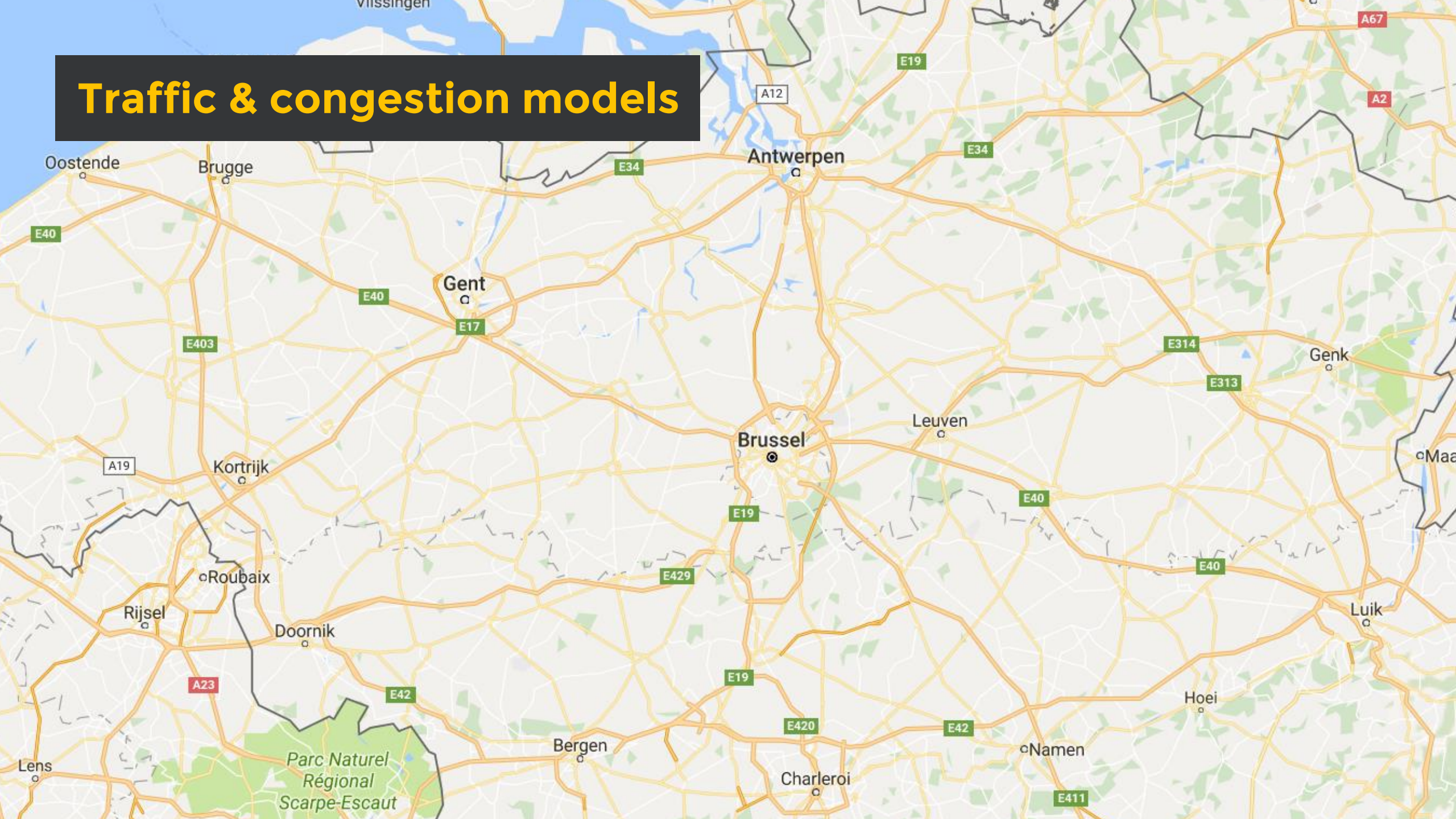
Patterns: car & passengers

Predictions: behaviors

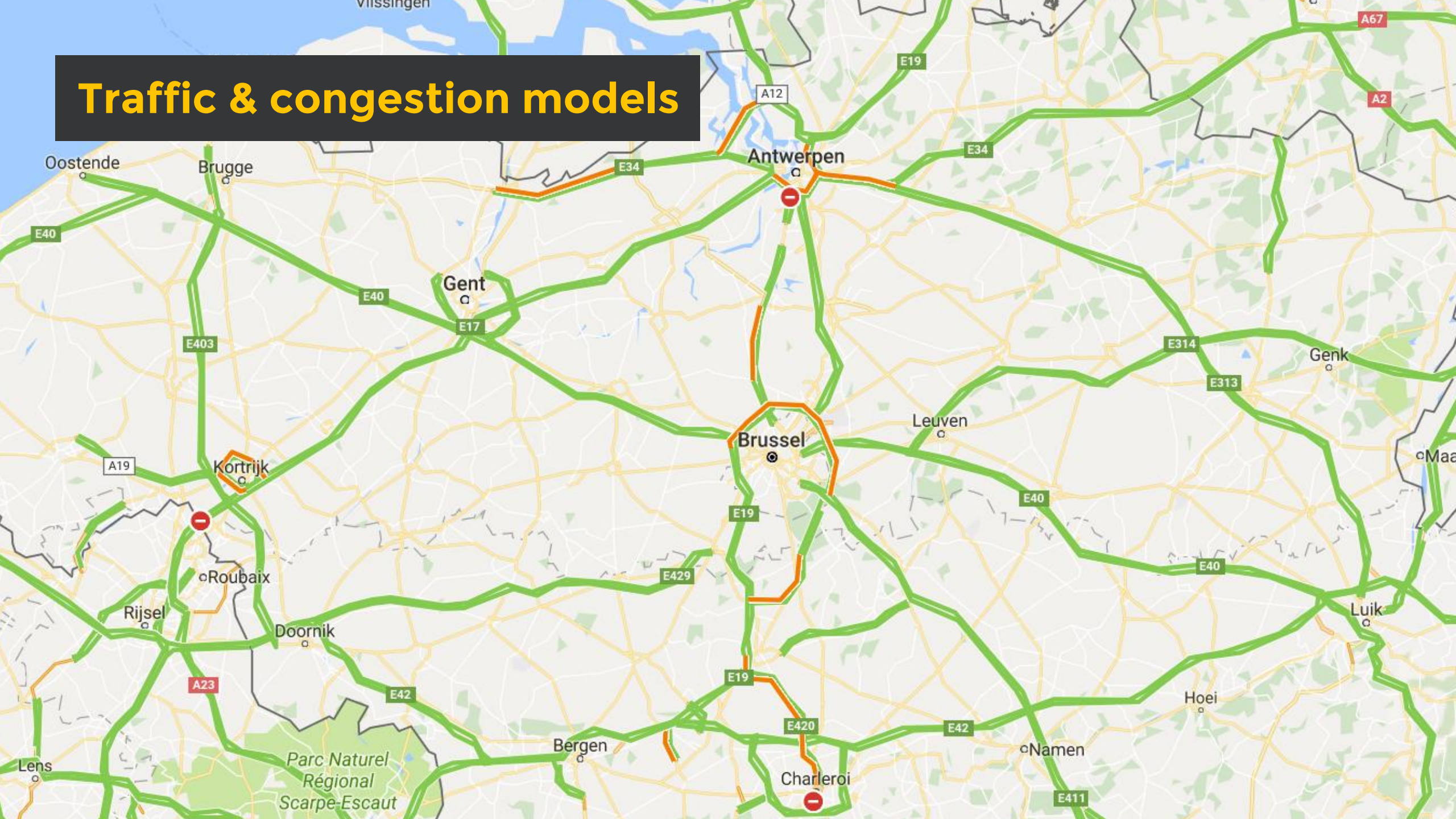


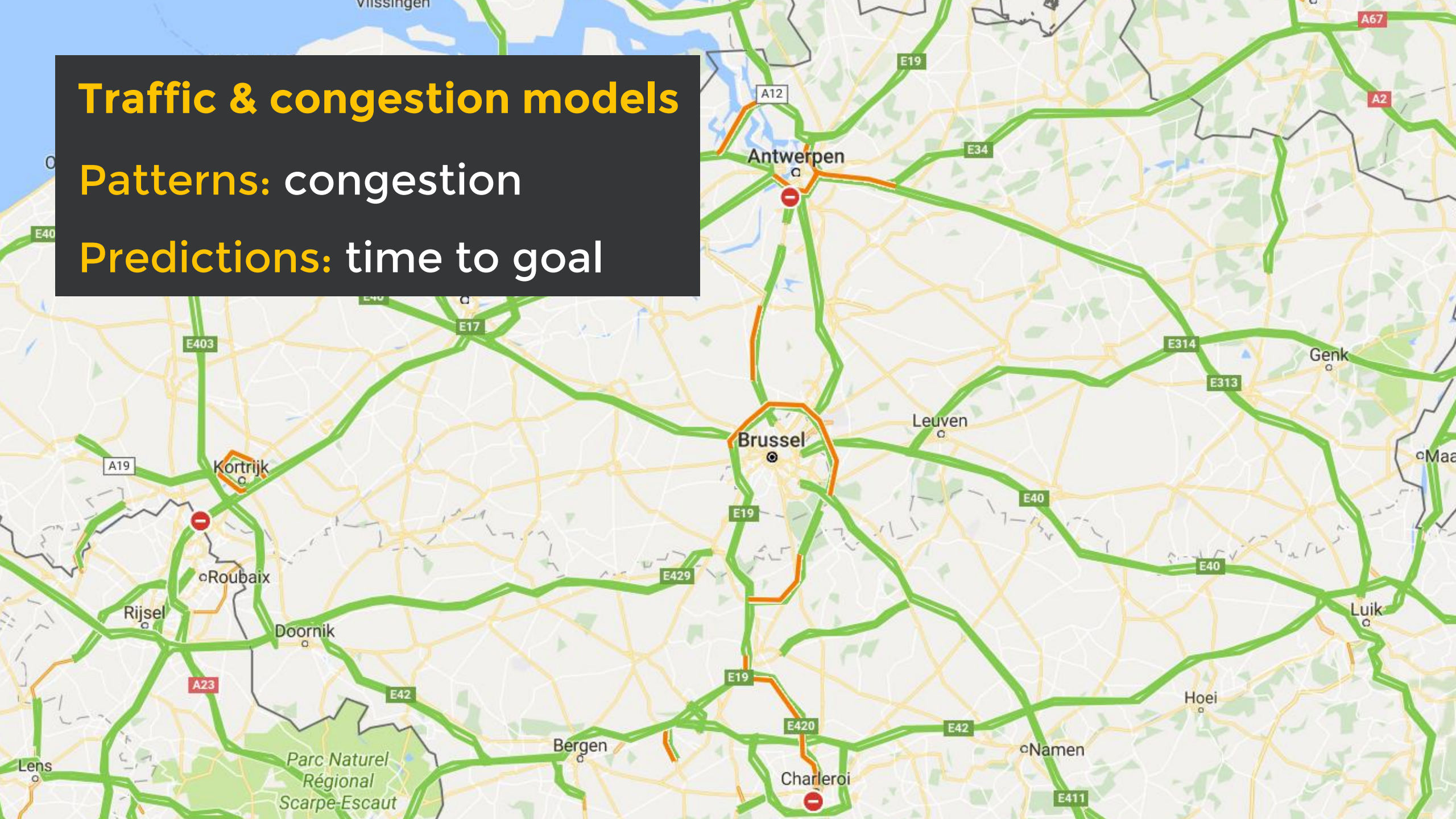


Traffic & congestion models



Traffic & congestion models

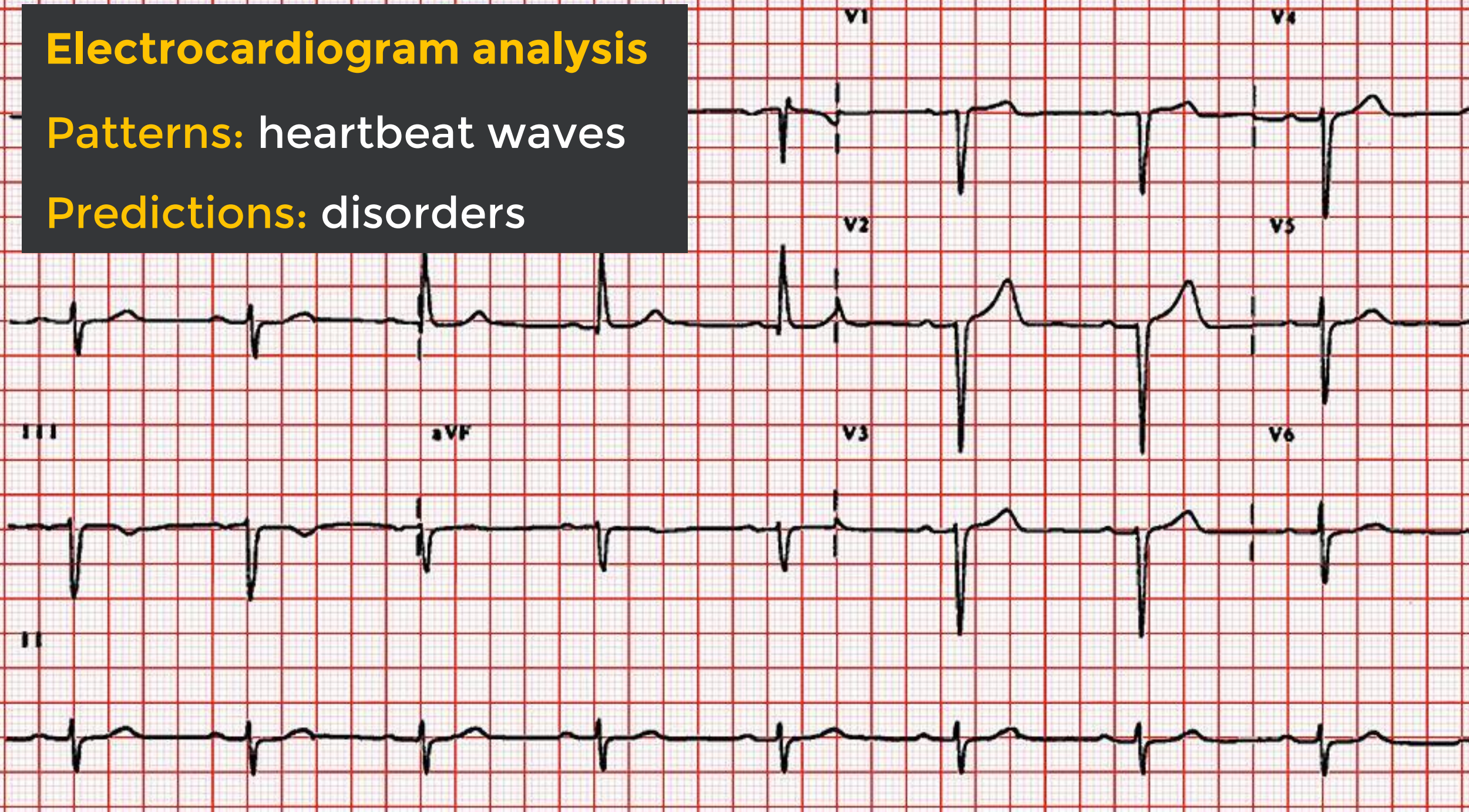


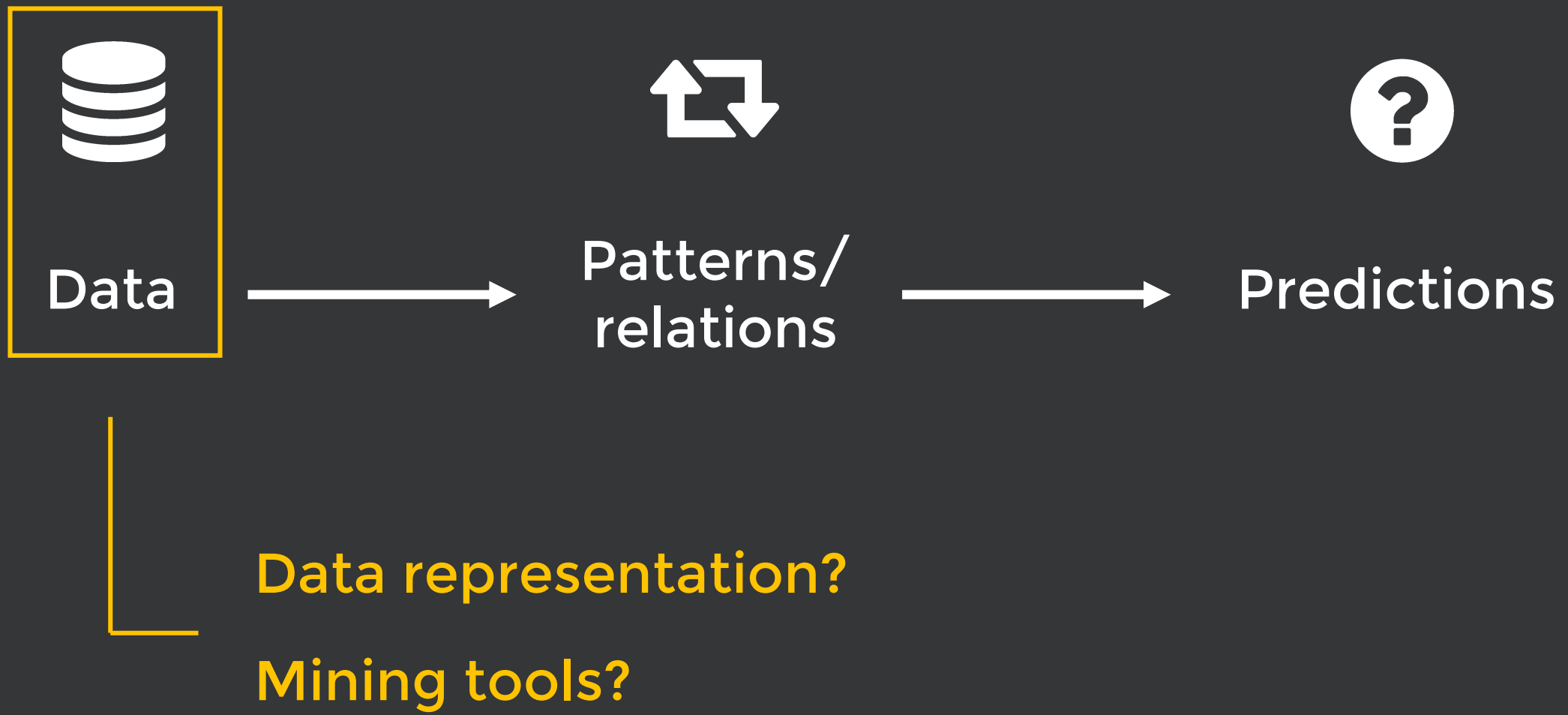

Traffic & congestion models
Patterns: congestion
Predictions: time to goal
Vlissingen
A12
E19
A67
A2
Antwerpen
E34
E40
E17
E403
E314
E313
Genk
Leuven
Brussel
A19
Kortrijk
E40
E19
E429
Roubaix
Rijsel
Doornik
E40
Luik
A23
E42
E19
E420
E42
Hoei
Bergen
Namen
Lens
Parc Naturel Régional Scarpe-Escaut
Charleroi
E411

Electrocardiogram analysis

Patterns: heartbeat waves

Predictions: disorders




Data
Patterns/
relations
Predictions
Data representation?
Mining tools?



Vectors Matrices Tensors

VECTOR = ONEWAY ARRAY

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temperature in ℃	22	24	29	23	25	25	26

	R0	E40	R1	E19	E17	E314	R4
# traffic jam	5.1	2.7	9.4	8.8	1.7	2.2	3.4

MATRIX = TWOWAY ARRAY

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Leuven	22	24	29	23	25	25	26

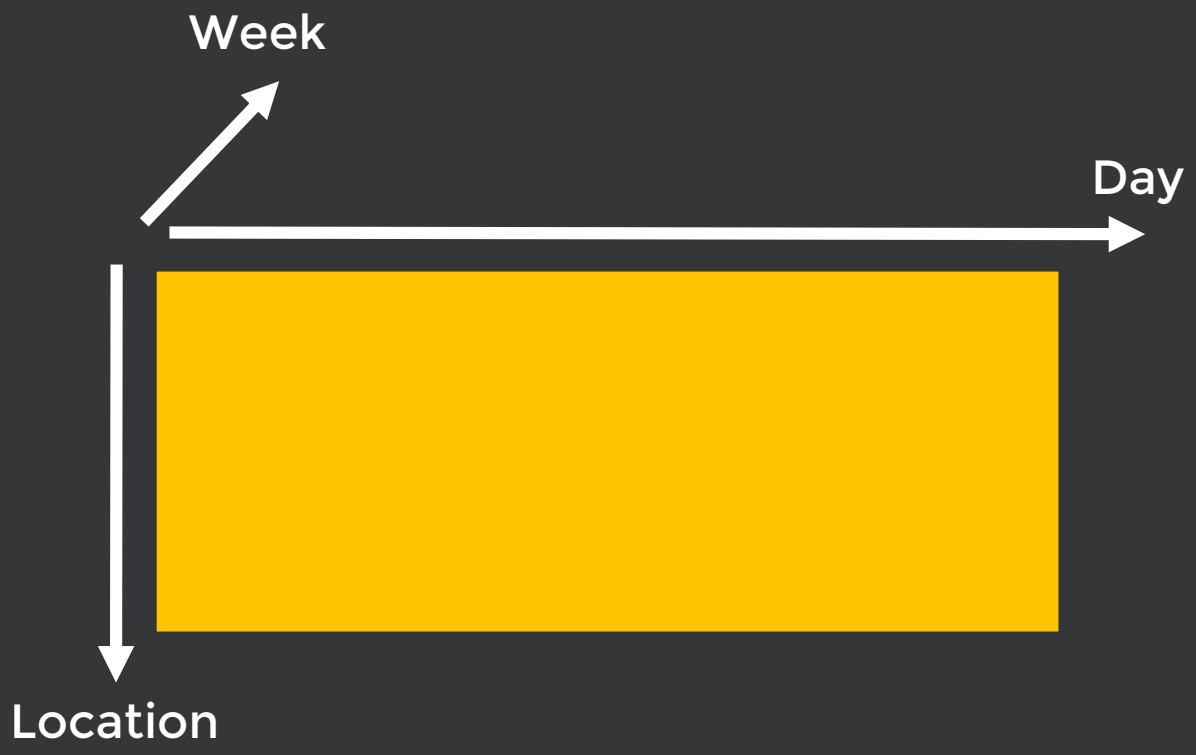
MATRIX = TWOWAY ARRAY

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Leuven	22	24	29	23	25	25	26

MATRIX = TWOWAY ARRAY

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Leuven	22	24	29	23	25	25	26
London	21	23	22	21	23	23	22
Paris	27	29	28	26	26	27	26
Athens	30	30	31	32	32	31	32


Day
Location


Week
Day
Location

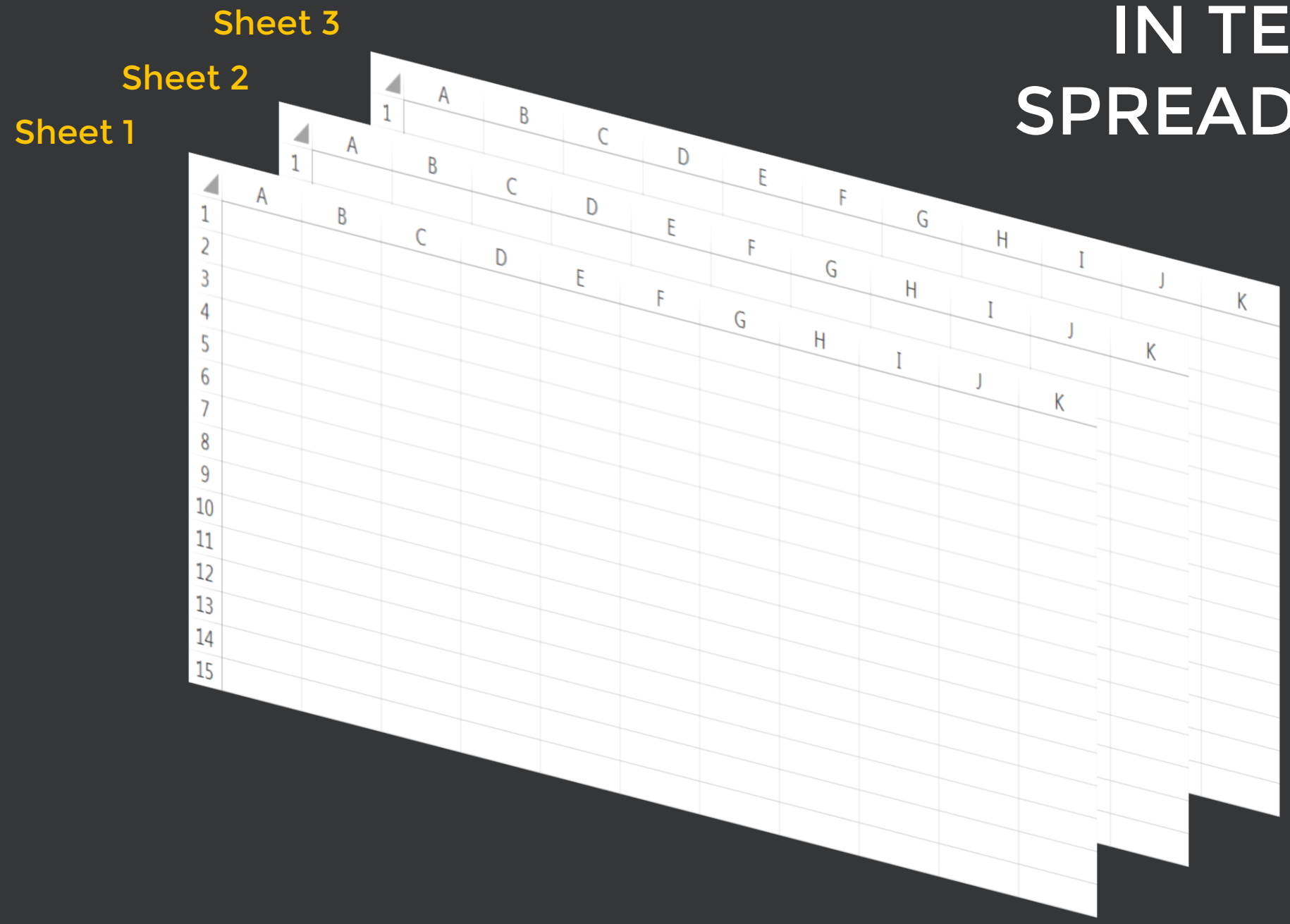

Week


Week

TENSOR = MULTIWAY ARRAY



IN TERMS OF SPREADSHEETS



DATA MINING TOOLS



Matrix tools

Information



VERY POWERFUL!

Tensor tools

Information

WHAT EXACTLY IS TENSORIZATION?

WHERE DO WE FIND TENSORS?

Data

Application of tensor tools

WHERE DO WE FIND TENSORS?

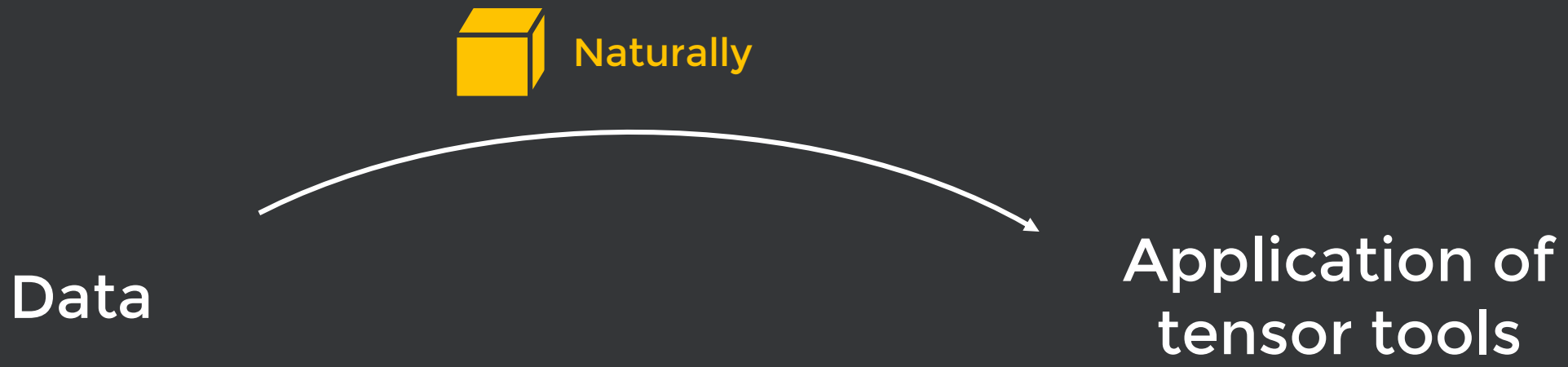


Naturally

Data

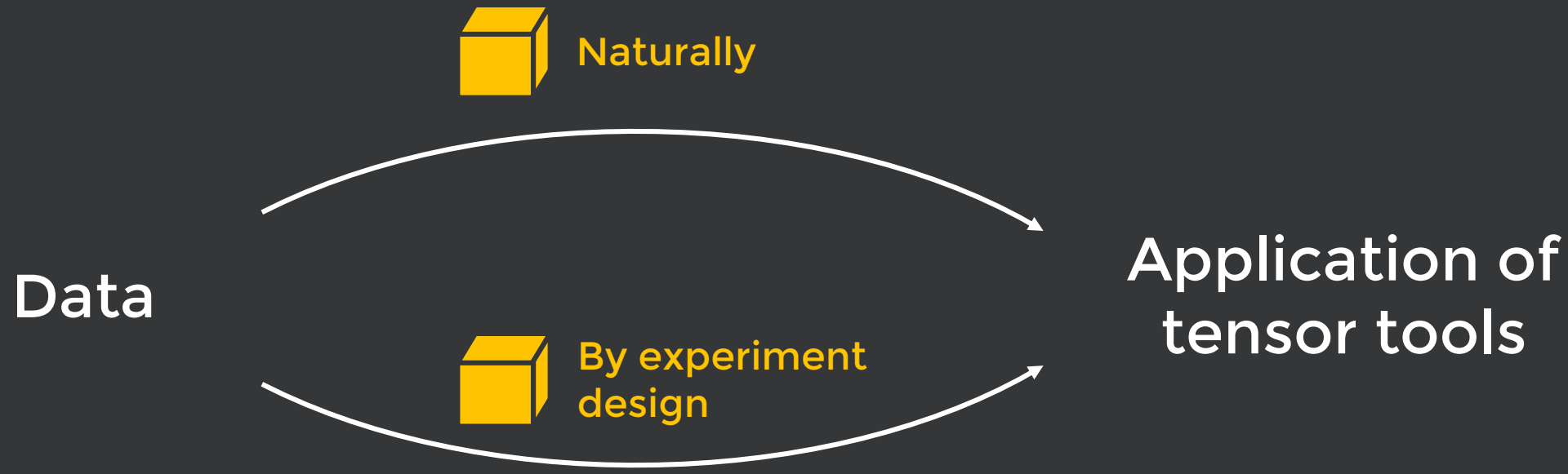
Application of tensor tools

WHERE DO WE FIND TENSORS?



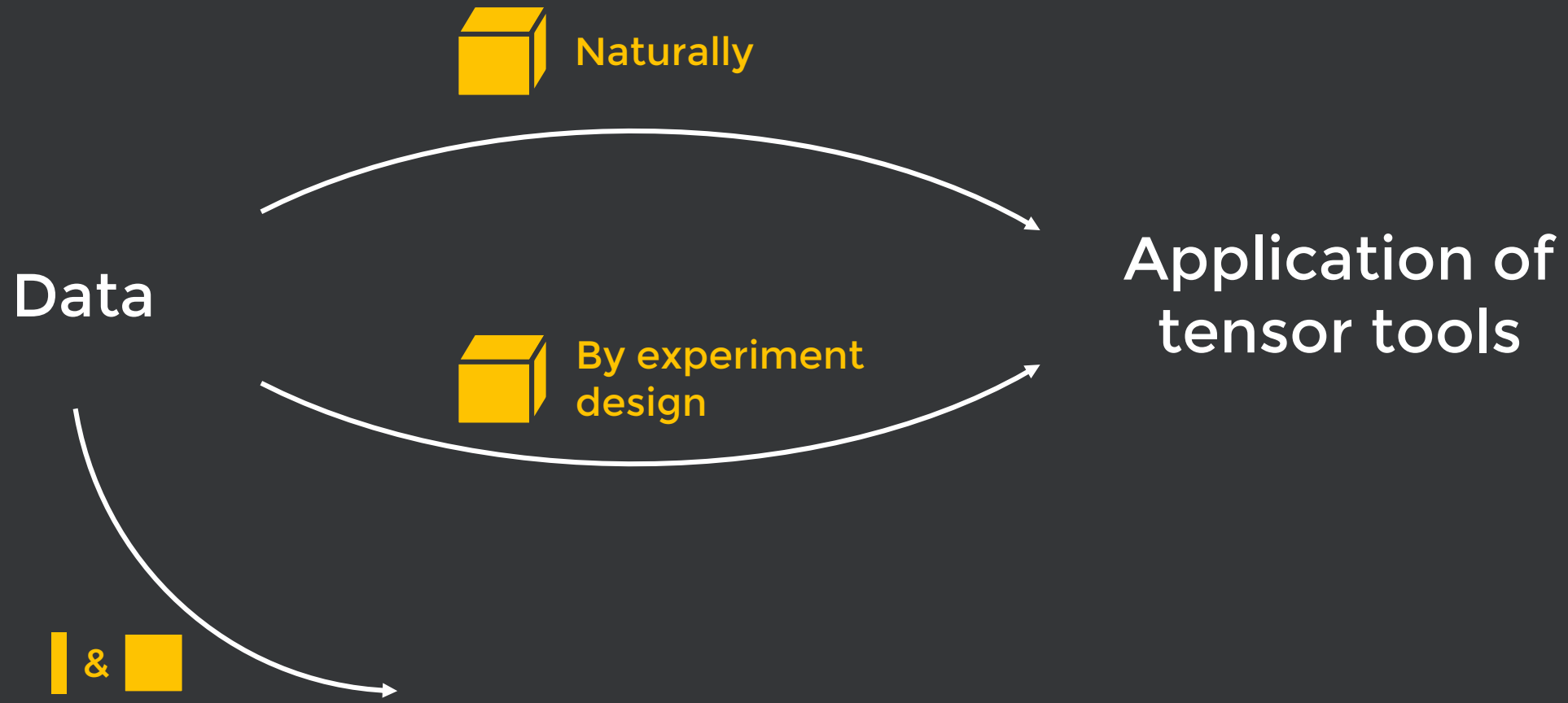
- **Physical quantities** in function of height x width x depth
- **Video data:** height x width x time
- **Color image data, hyperspectral data, …**

WHERE DO WE FIND TENSORS?

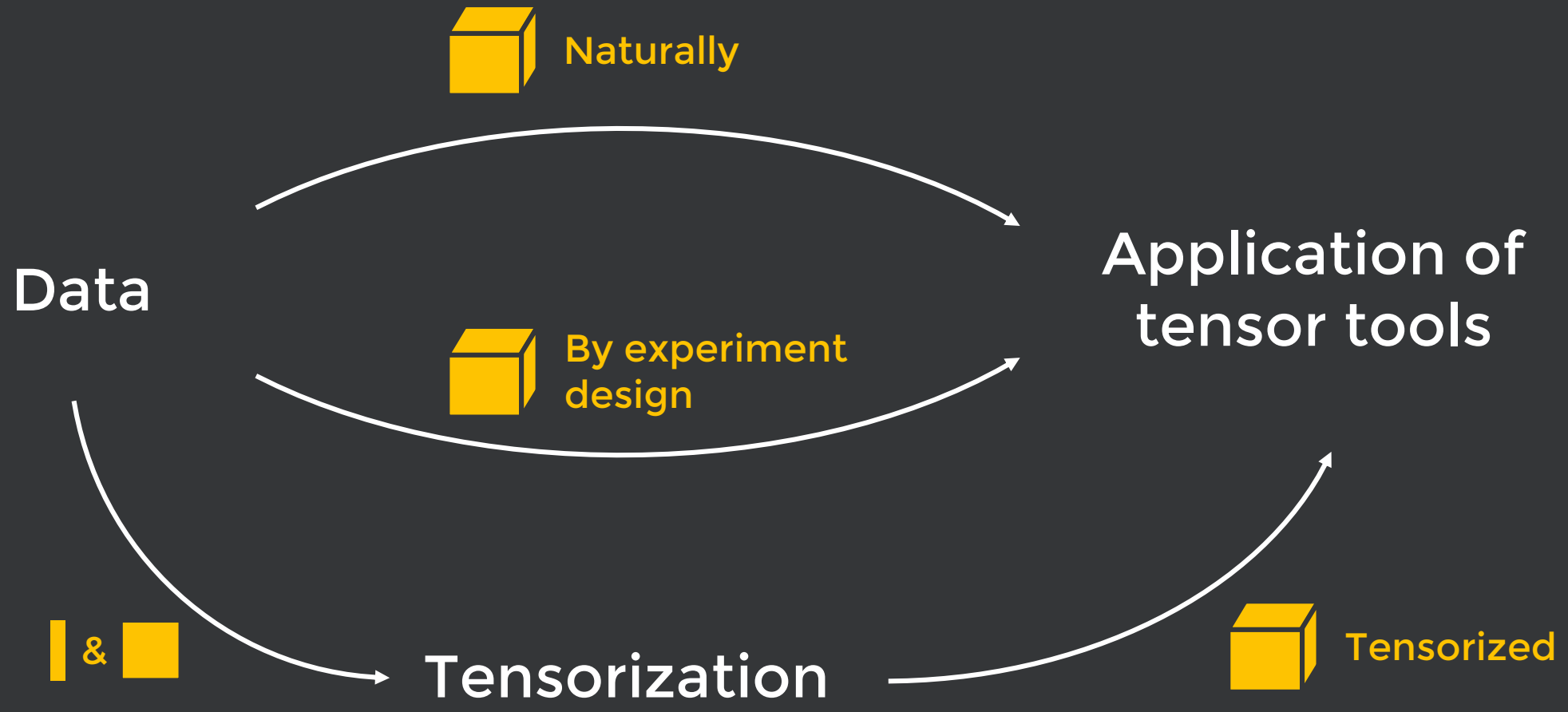


- **Recommendation systems:** user x movie → user x movie x month x topic x ...
- **Biomedical signal processing:** electrode x time → electrode x time x subject x ...
- **Face databases:** height x width → height x width x person x emoticon x ...

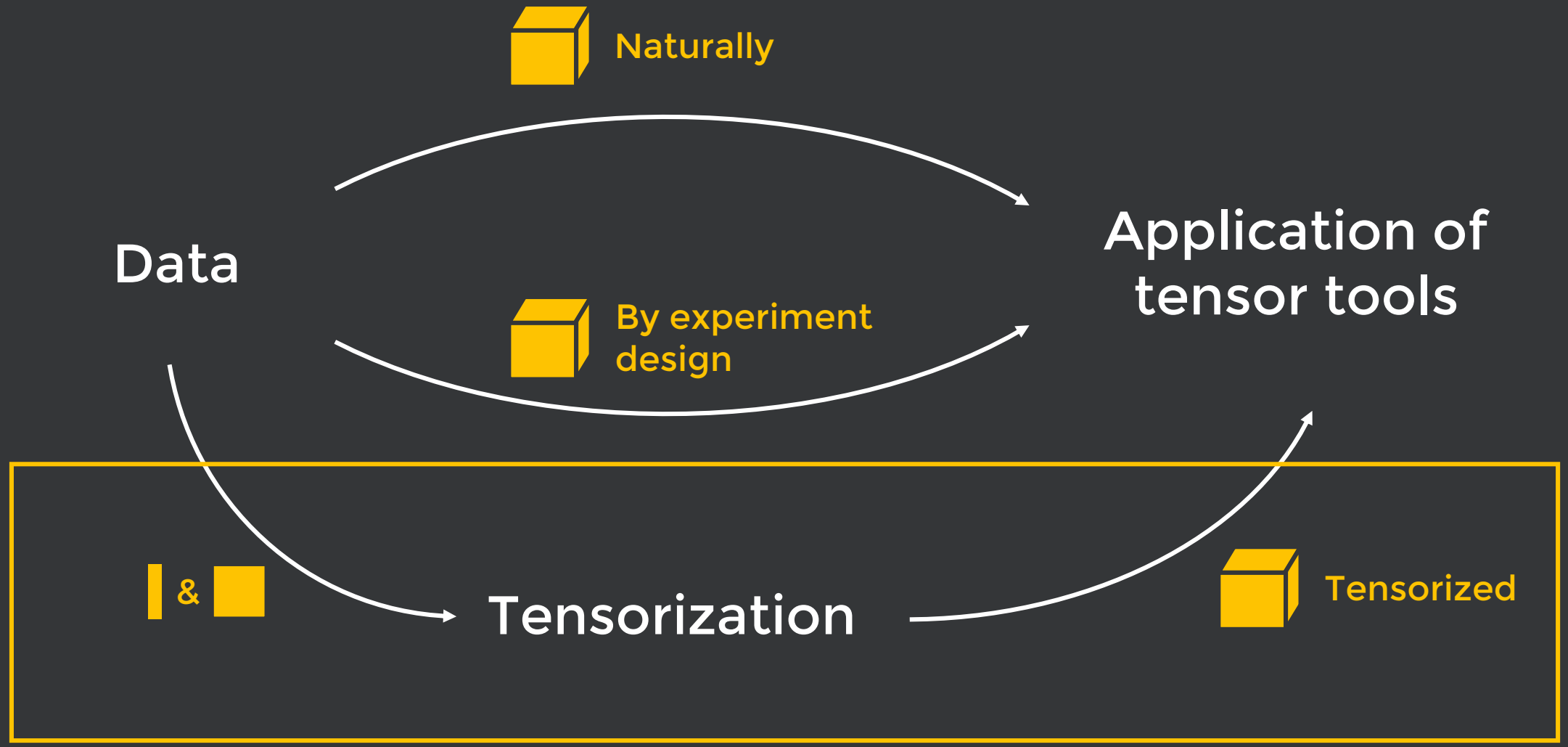
WHERE DO WE FIND TENSORS?



WHERE DO WE FIND TENSORS?



WHERE DO WE FIND TENSORS?





- Without using additional data
- In a meaningful way

SIMPLE EXAMPLE: RESHAPING



SIMPLE EXAMPLE: RESHAPING



SIMPLE EXAMPLE: RESHAPING



SIMPLE EXAMPLE: RESHAPING



MORE ADVANCED EXAMPLE: HANKELIZATION

$$\begin{bmatrix} a & b & c & d & e \end{bmatrix}$$



Hankel matrix

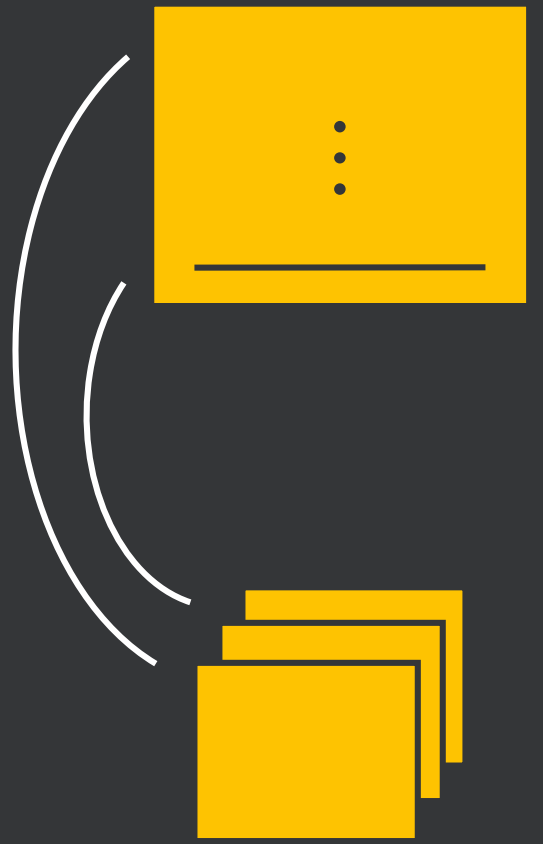
$$\begin{bmatrix} a & b & c \\ b & c & d \\ c & d & e \end{bmatrix}$$

MORE ADVANCED EXAMPLE: HANKELIZATION

$$\begin{bmatrix} a & b & c & d & e \end{bmatrix}$$

Hankel matrix

$$\begin{bmatrix} a & b & c \\ b & c & d \\ c & d & e \end{bmatrix}$$

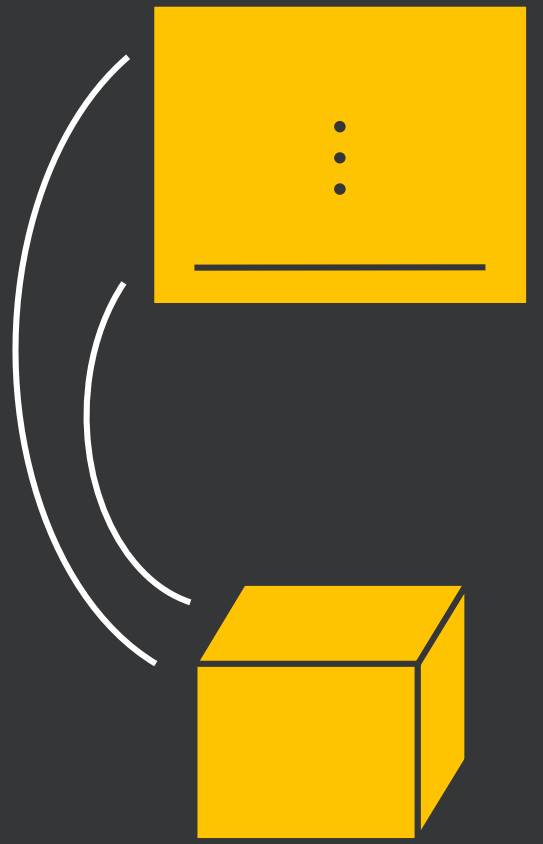


MORE ADVANCED EXAMPLE: HANKELIZATION

$$\begin{bmatrix} a & b & c & d & e \end{bmatrix}$$

Hankel matrix

$$\begin{bmatrix} a & b & c \\ b & c & d \\ c & d & e \end{bmatrix}$$



VARIOUS TENSORIZATION TECHNIQUES EXIST

Monomial relations

Piecewise outer product

Covariance matrices

Hessian & Jacobian matrices

Time-frequency & time-scale

Single vector ____ Tensorization overview ____ Sets of vectors

Löwnerization

Score functions

Segmentation

Moments & cumulants

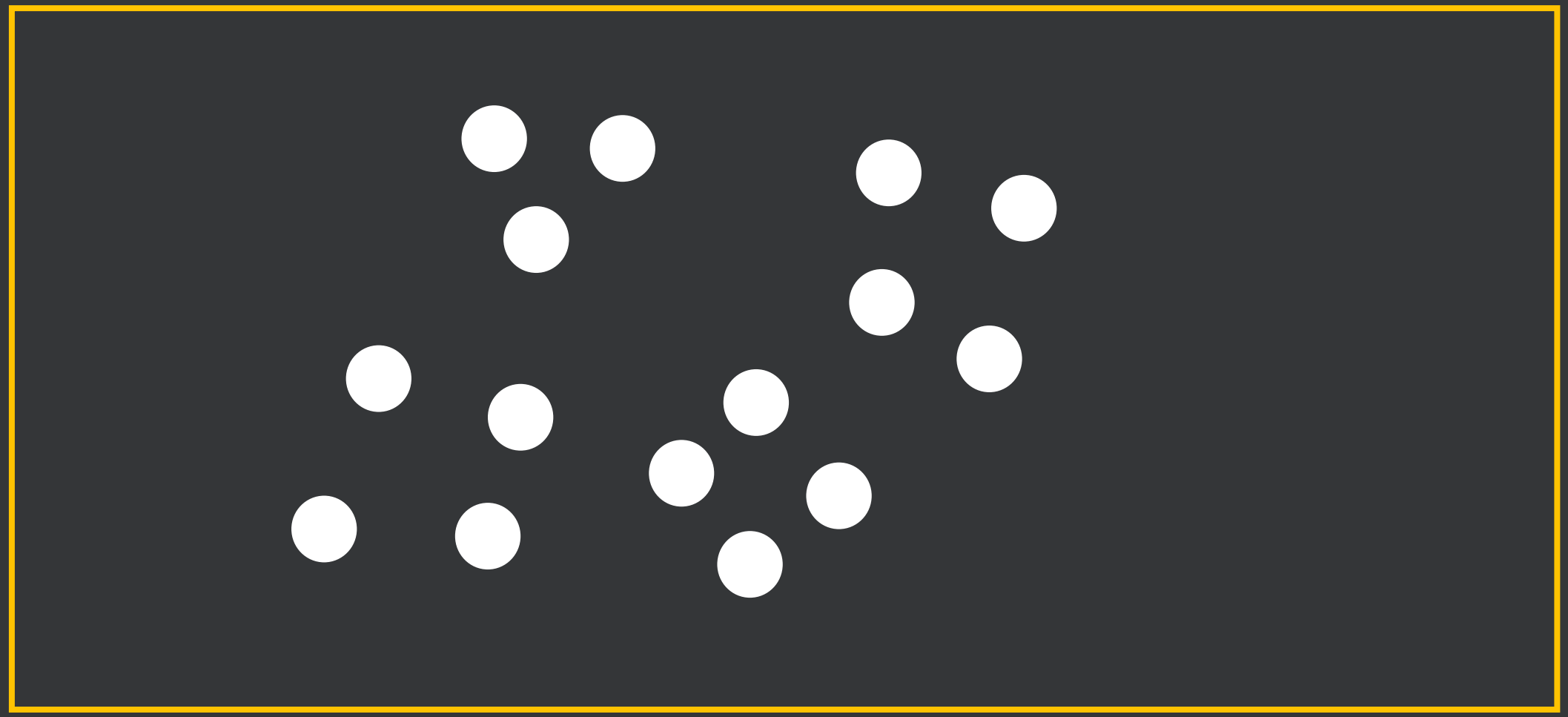
Hankelization

And many more

Adjacency tensors

BLIND SIGNAL SEPARATION

COCKTAIL PARTY PROBLEM



COCKTAIL PARTY PROBLEM



COCKTAIL PARTY PROBLEM



LINEAR INSTANTANEOUS MIXTURE

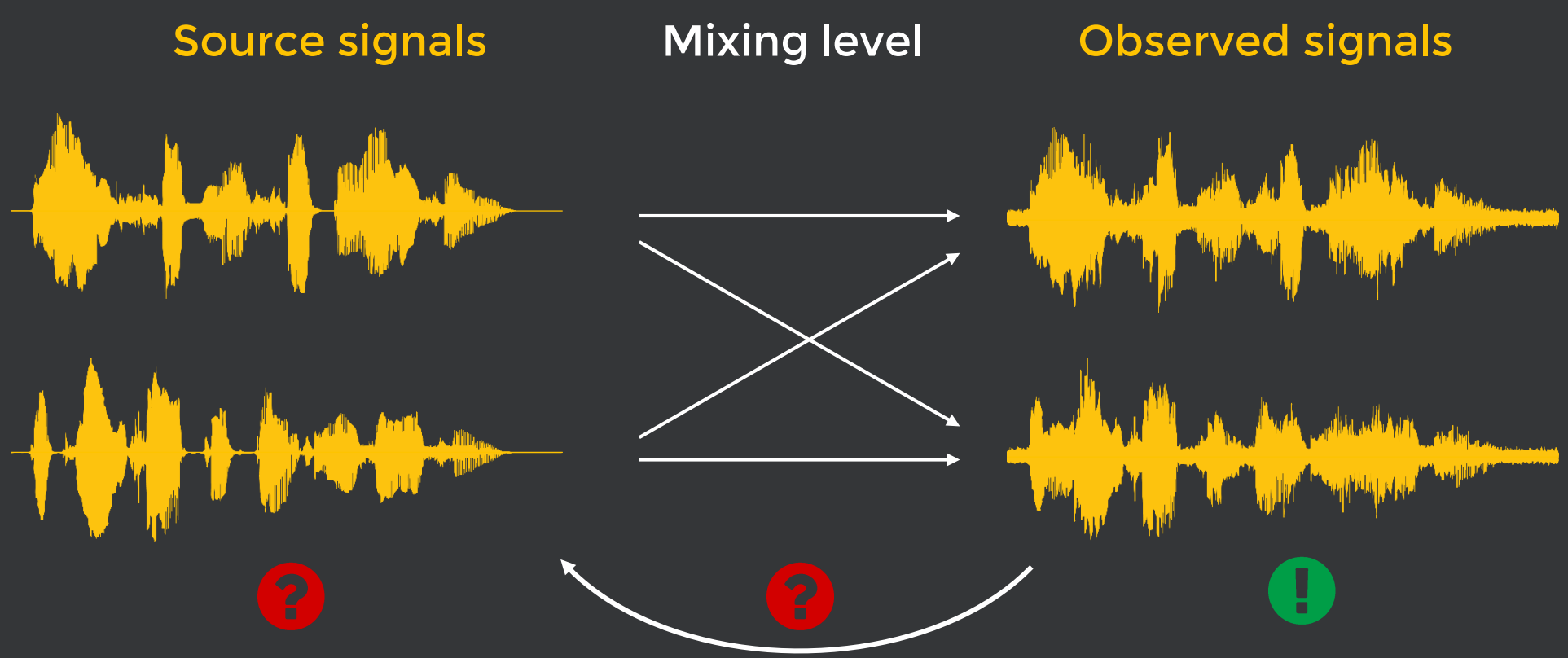
Source signals

Mixing level

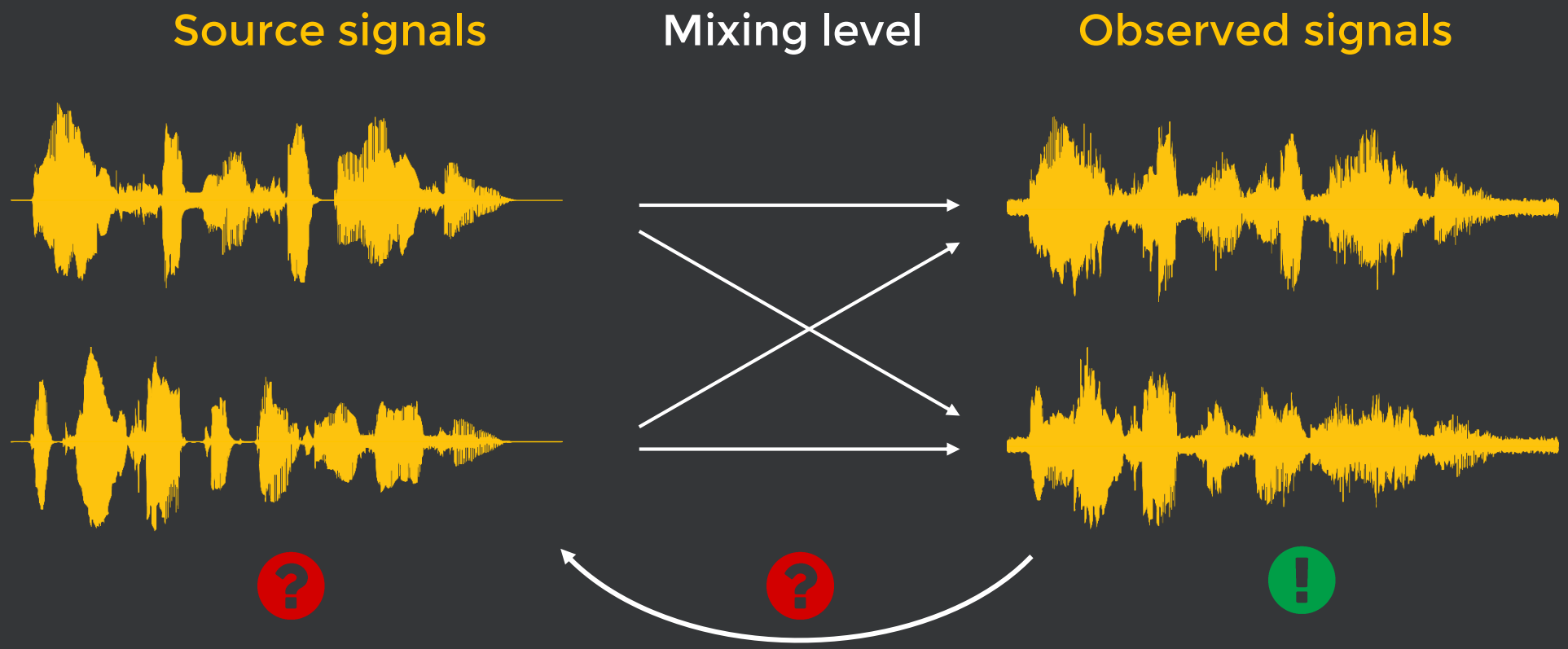
Observed signals



LINEAR INSTANTANEOUS MIXTURE

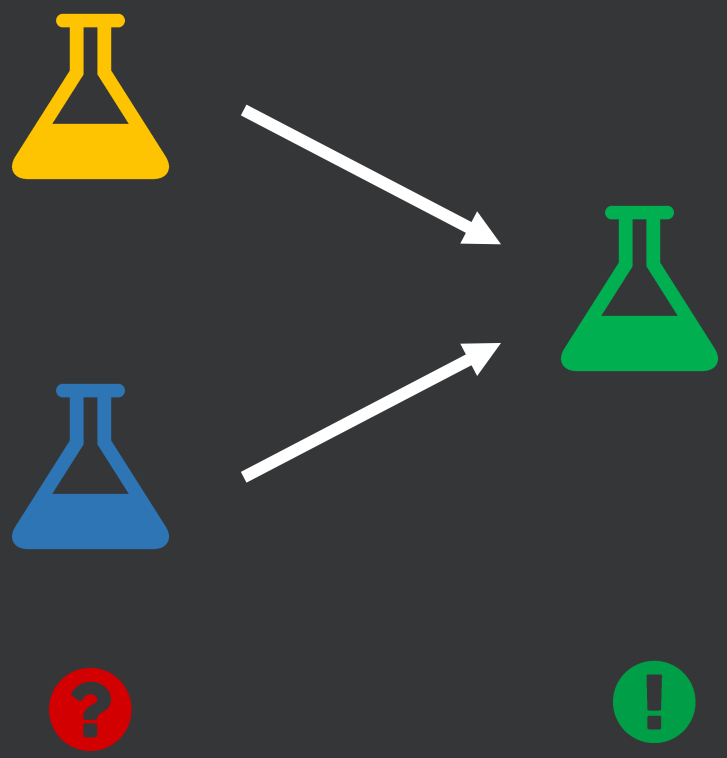


LINEAR INSTANTANEOUS MIXTURE

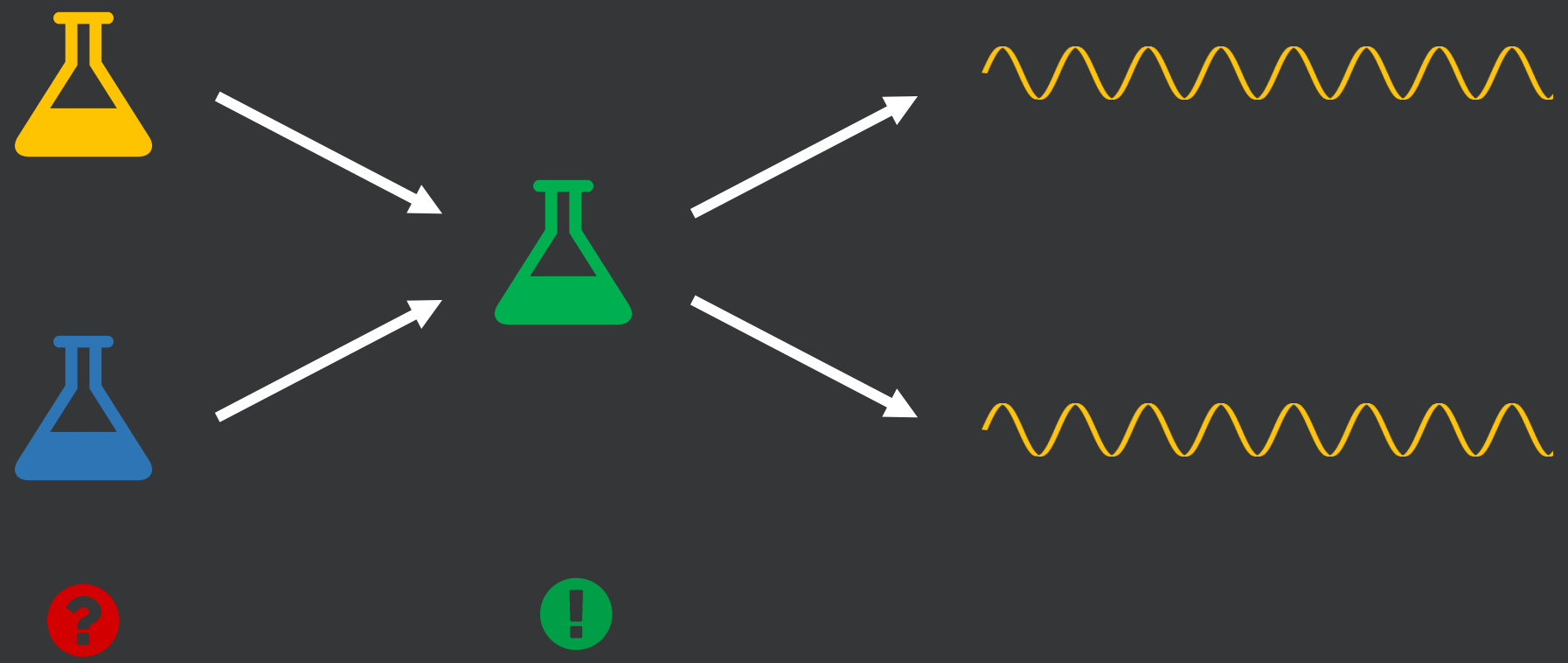


- Assumption of independence allows the recovery of underlying components

FLUORESCENCE SPECTROSCOPY



FLUORESCENCE SPECTROSCOPY

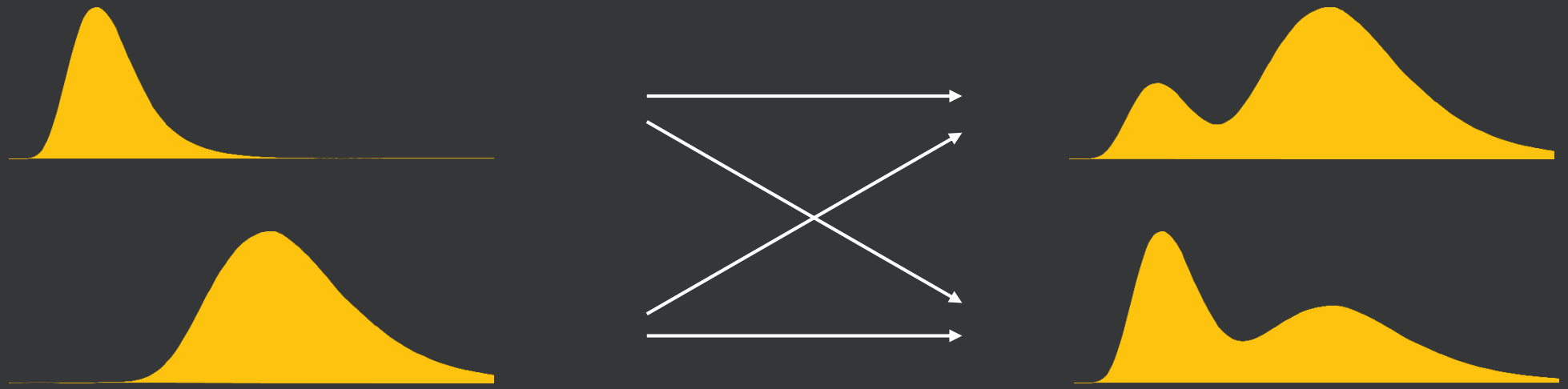


FLUORESCENCE SPECTROSCOPY

Emission spectra of individual analytes

Influence of excitation and concentrations

Observed emission spectra of the compound at different excitation levels

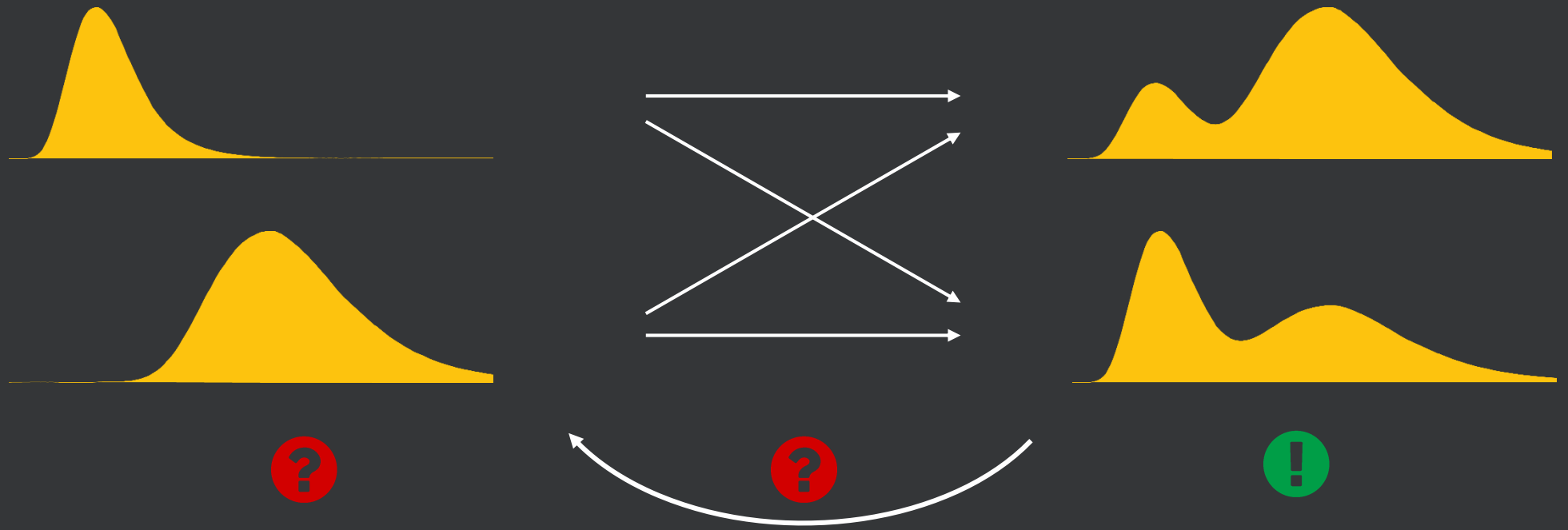


FLUORESCENCE SPECTROSCOPY

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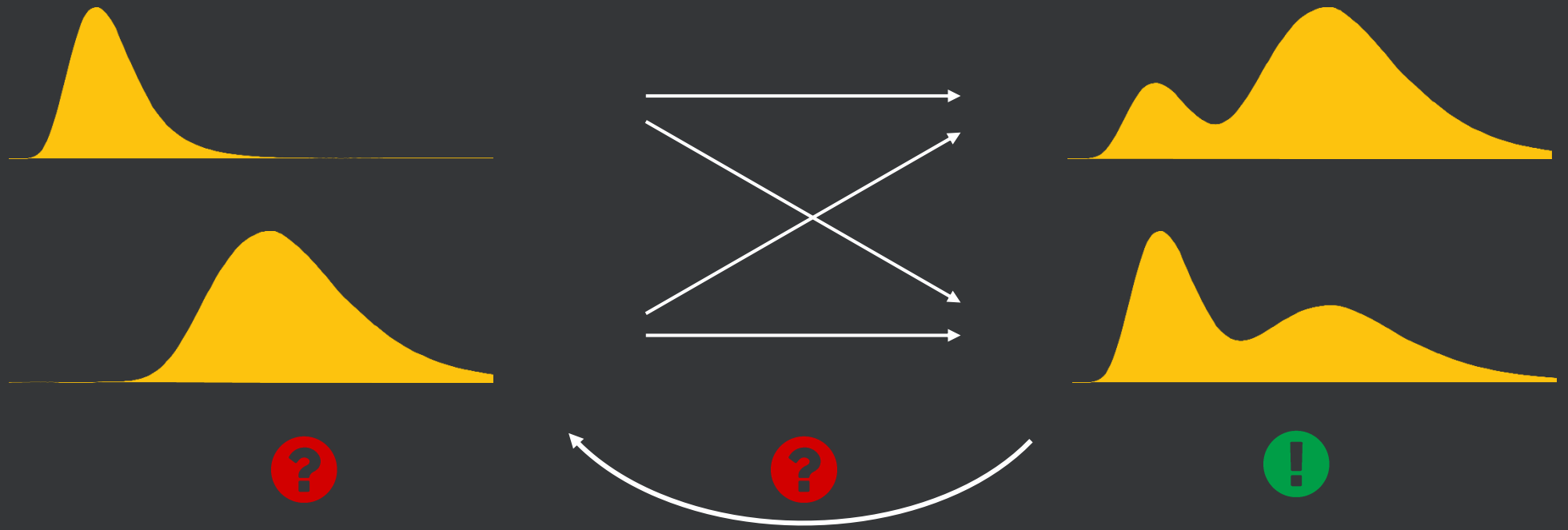


FLUORESCENCE SPECTROSCOPY

Emission spectra of individual analytes

Influence of excitation and concentrations

Observed emission spectra of the compound at different excitation levels



- Assumption of rational functions allows the recovery of underlying components

HEARTBEAT SEPARATION



HEARTBEAT SEPARATION



HEARTBEAT SEPARATION

Maternal and fetal ECG signals

Mixing level

Observed ECG signals

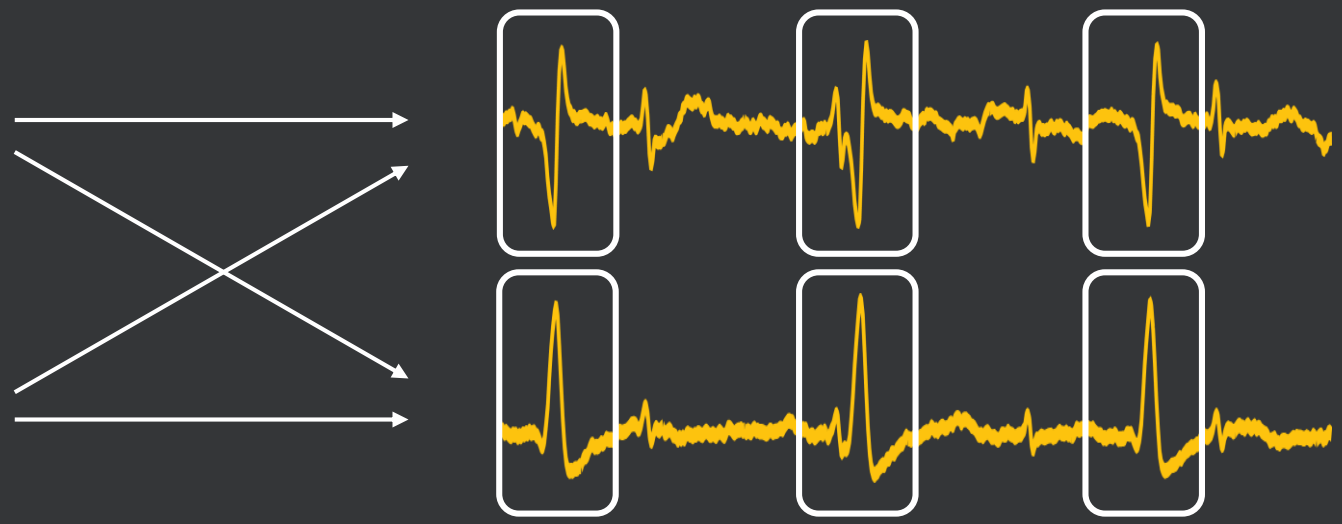


HEARTBEAT SEPARATION

Maternal and fetal ECG signals

Mixing level

Observed ECG signals



HEARTBEAT SEPARATION

Maternal and fetal ECG signals

Mixing level

Observed ECG signals

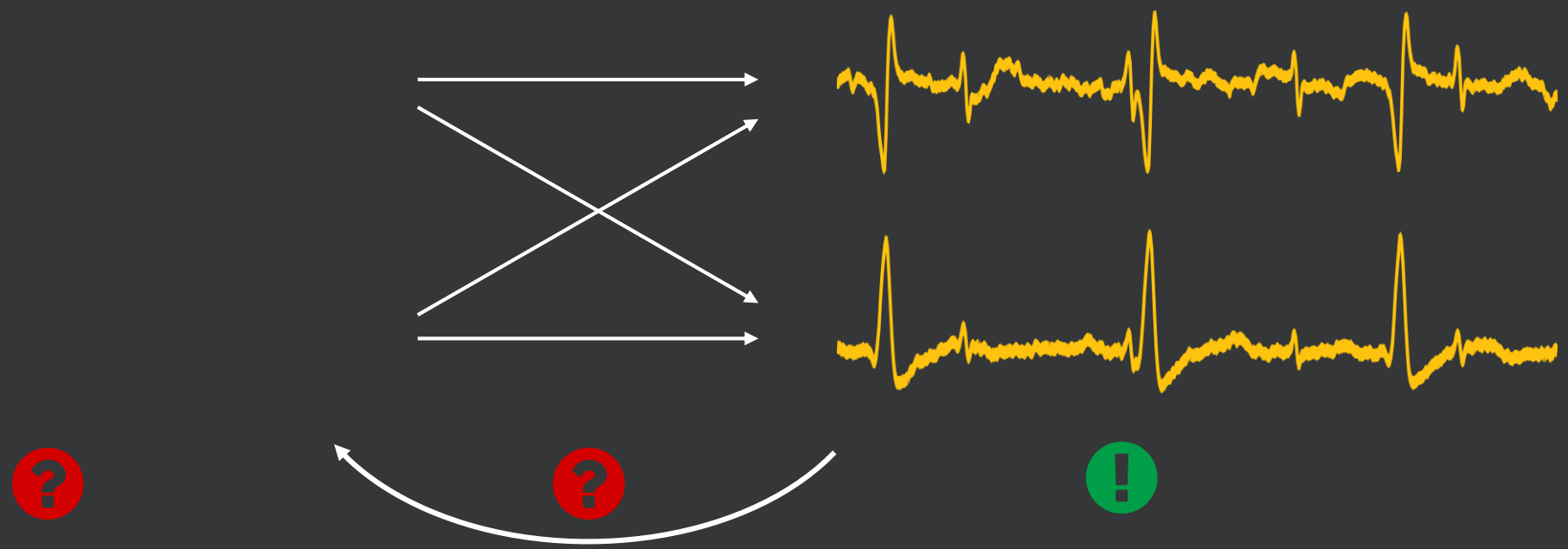


HEARTBEAT SEPARATION

Maternal and fetal ECG signals

Mixing level

Observed ECG signals

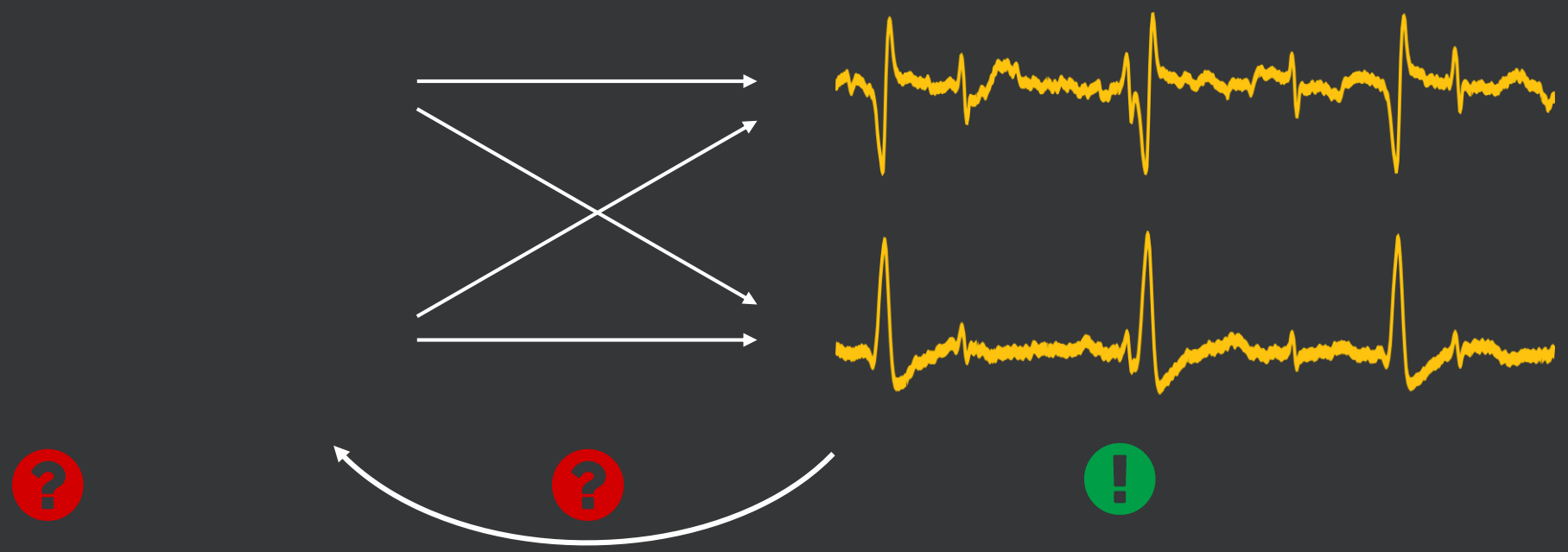


HEARTBEAT SEPARATION

Maternal and fetal ECG signals

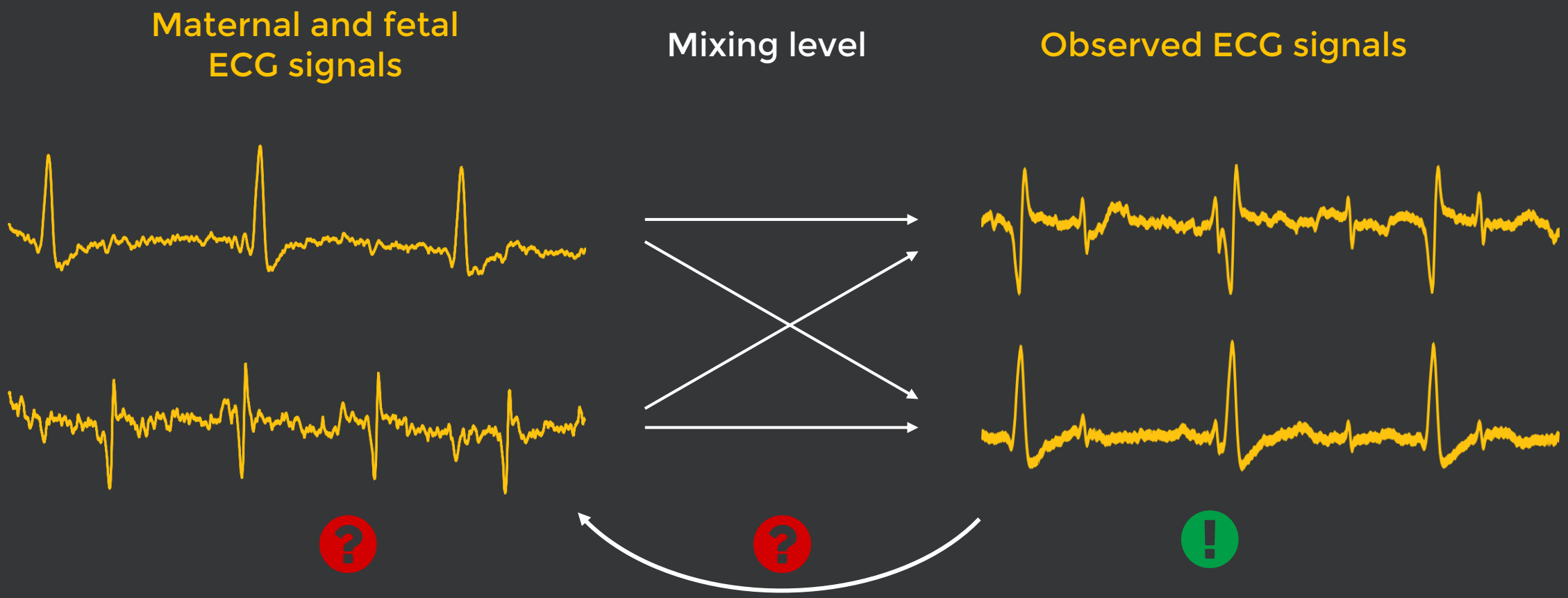
Mixing level

Observed ECG signals



- Assumption of independence / rational functions / sums-of-Kronecker-products allows the recovery of underlying components

HEARTBEAT SEPARATION



- Assumption of independence / rational functions / sums-of-Kronecker-products allows the recovery of underlying components

HOW CAN WE USE TENSORS TO SOLVE THE PROBLEM OF BLIND SIGNAL SEPARATION?

TWO THINGS ARE NEEDED.

1 Translate assumptions → TENSORIZATION

Different assumptions require different tensorization techniques:

- Independence → STATISTICS
- Rational functions → LÖWNERIZATION
- Exponential polynomials → HANKELIZATION
- Etc.

TWO THINGS ARE NEEDED.

1. Translate assumptions → TENSORIZATION

Different assumptions require different tensorization techniques:

- Independence → STATISTICS
- Rational functions → LÖWNERIZATION
- Exponential polynomials → HANKELIZATION
- Etc.

2. Identify underlying components → TENSOR TOOLS

Ring around Brussels

R0

8h	80
12h	40
16h	120
20h	20

Measure of traffic

Major Belgian highway

	R0	E40
8h	80	20
12h	40	10
16h	120	30
20h	20	5

Ring around Antwerp

	R0	E40	R1
8h	80	20	40
12h	40	10	20
16h	120	30	60
20h	20	5	10

RANK-1 MATRIX

	R0	E40	R1	Church Str.
8h	80	20	40	4
12h	40	10	20	2
16h	120	30	60	6
20h	20	5	10	1

RANK-1 MATRIX

	R0	E40	R1	Church Str.
8h	80	20	40	4
12h	40	10	20	2
16h	120	30	60	6
20h	20	5	10	1

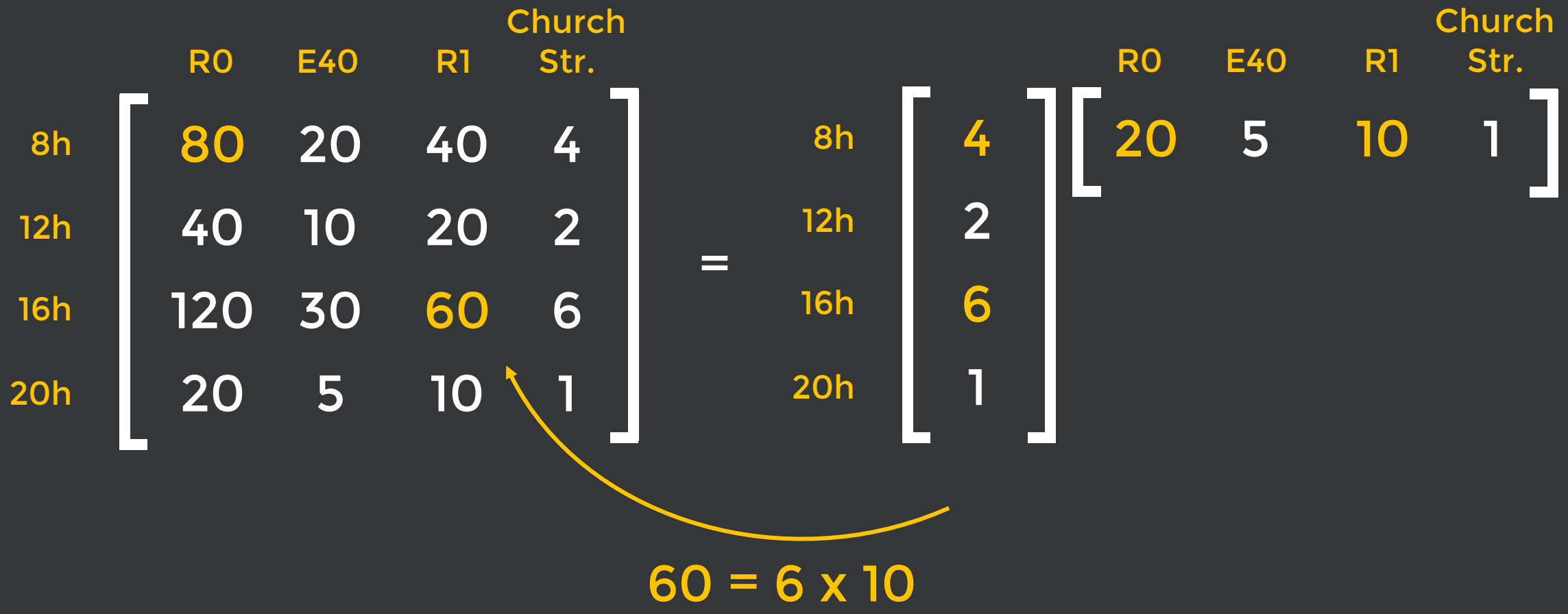
RANK-1 MATRIX

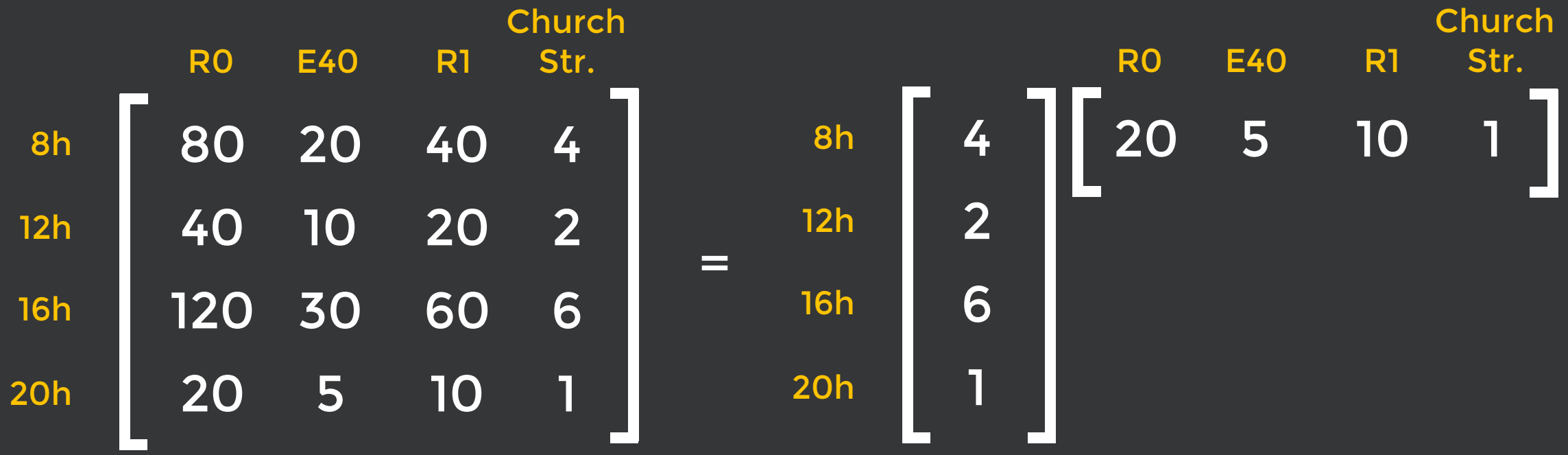


RANK-1 MATRIX

$$
\begin{array}{c}
 \\
\text{8h} \\
\text{12h} \\
\text{16h} \\
\text{20h}
\end{array}
\begin{array}{c}
\begin{array}{cccc} \text{R0} & \text{E40} & \text{R1} & \text{Church Str.} \end{array} \\
\begin{bmatrix}
80 & 20 & 40 & 4 \\
40 & 10 & 20 & 2 \\
120 & 30 & 60 & 6 \\
20 & 5 & 10 & 1
\end{bmatrix}
\end{array}
=
\begin{array}{c}
 \\
\text{8h} \\
\text{12h} \\
\text{16h} \\
\text{20h}
\end{array}
\begin{bmatrix} 4 \\ 2 \\ 6 \\ 1 \end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{R0} & \text{E40} & \text{R1} & \text{Church Str.} \end{array} \\
\begin{bmatrix} 20 & 5 & 10 & 1 \end{bmatrix}
\end{array}
$$

RANK-1 MATRIX



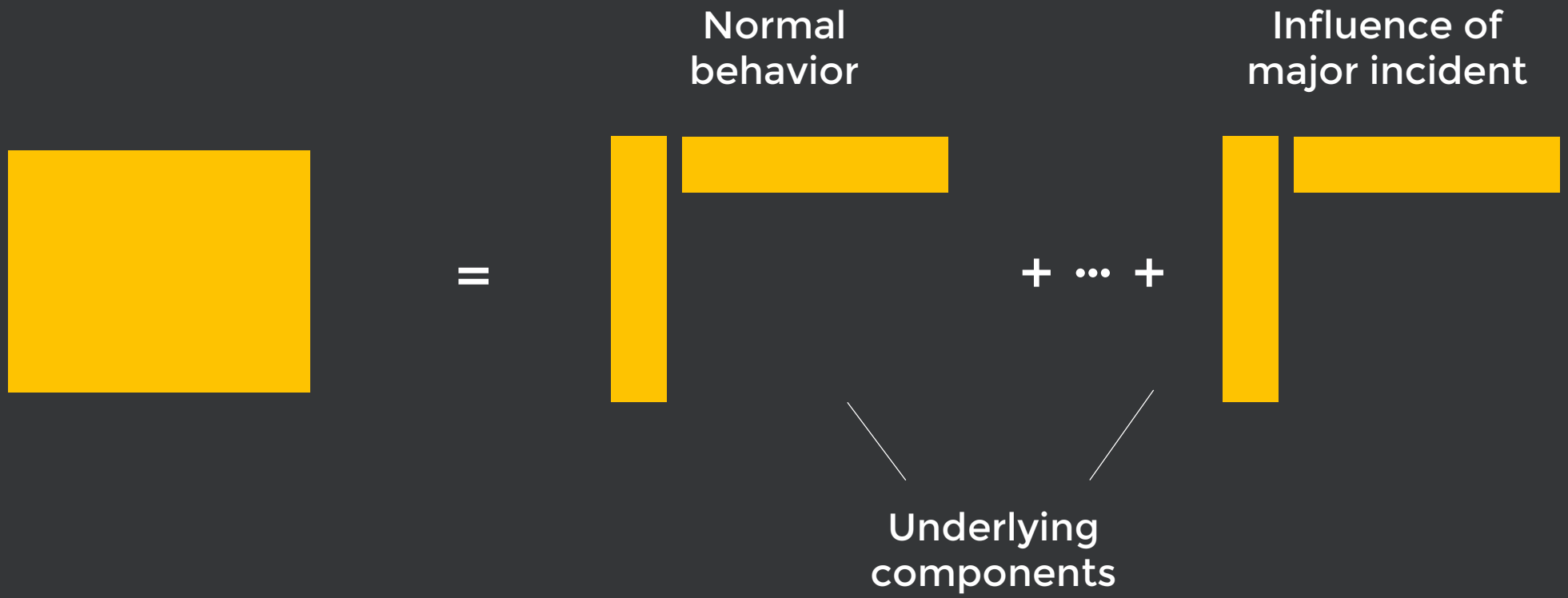

R0
E40
R1
Church Str.
8h
12h
16h
20h
80 20 40 4
40 10 20 2
120 30 60 6
20 5 10 1
=
8h
12h
16h
20h
4
2
6
1
R0
E40
R1
Church Str.
20 5 10 1



Normal behavior




Normal
behavior
Influence of
major incident
=
+ ••• +


Normal
behavior
Influence of
major incident
=
+ ••• +
Underlying
components


Normal
behavior
Influence of
major incident
=
+ ••• +

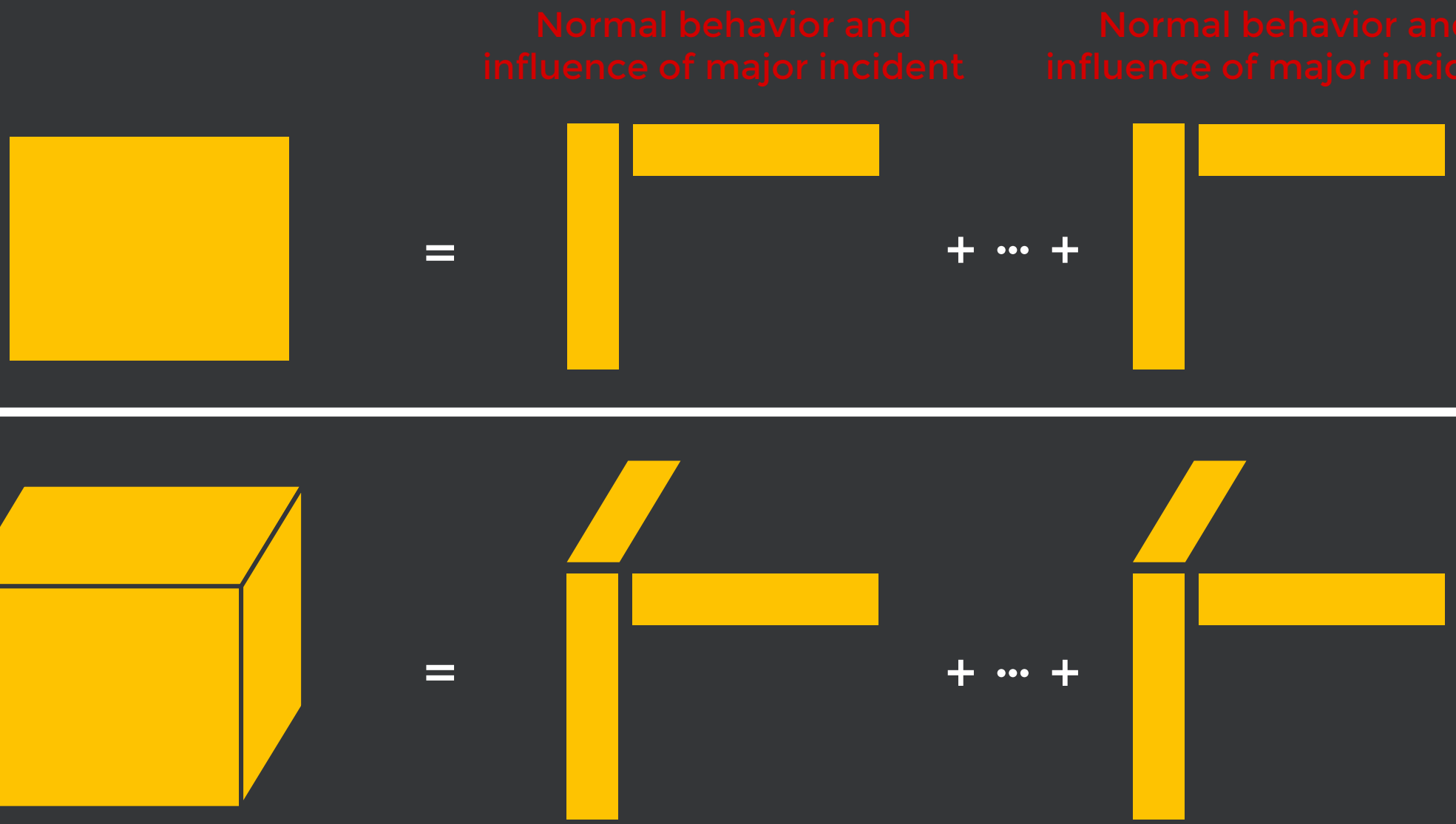



Normal
behavior
Influence of
major incident
=
+ ••• +


=
Normal
behavior


Normal behavior
Influence of major incident
=
+ ... +


=
+ ... +
Normal behavior
Influence of major incident

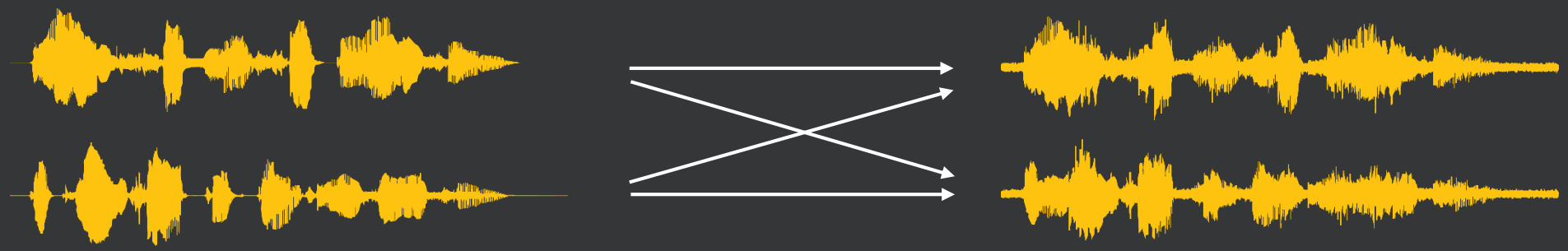

Normal behavior and influence of major incident
Normal behavior and influence of major incident
=
+ ... +
=
+ ... +
Normal behavior
Influence of major incident


Normal behavior and influence of major incident
Normal behavior and influence of major incident
=
+ ... +

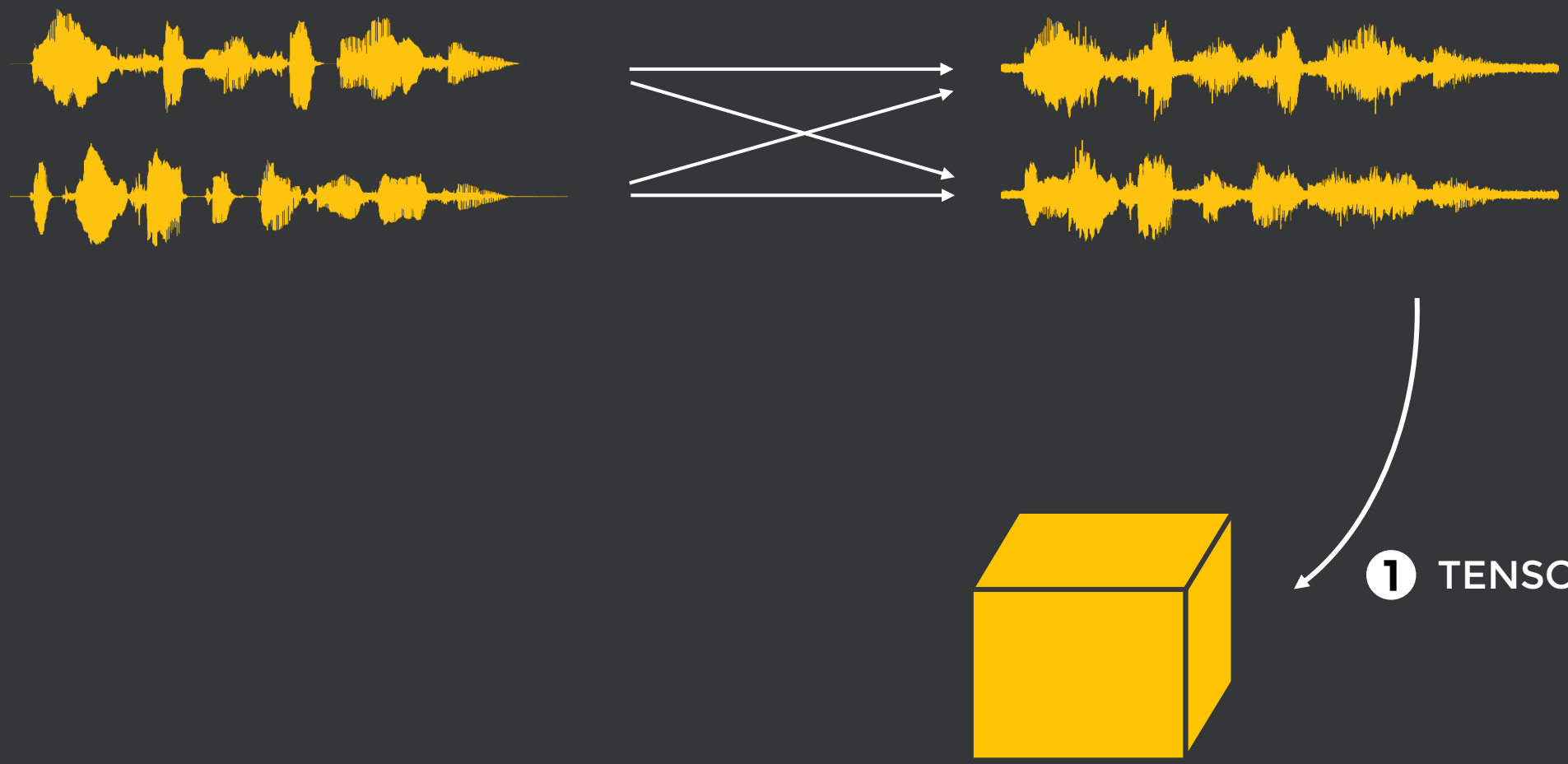

=
+ ... +
Normal behavior
Influence of major incident

TENSOR DECOMPOSITIONS ALLOW THE UNIQUE RECOVERY OF UNDERLYING COMPONENTS

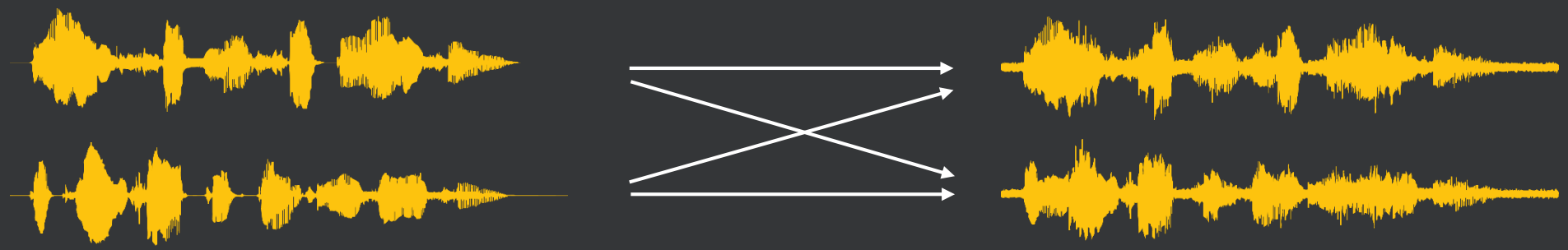
TENSOR-BASED BLIND SIGNAL SEPARATION



TENSOR-BASED BLIND SIGNAL SEPARATION



TENSOR-BASED BLIND SIGNAL SEPARATION

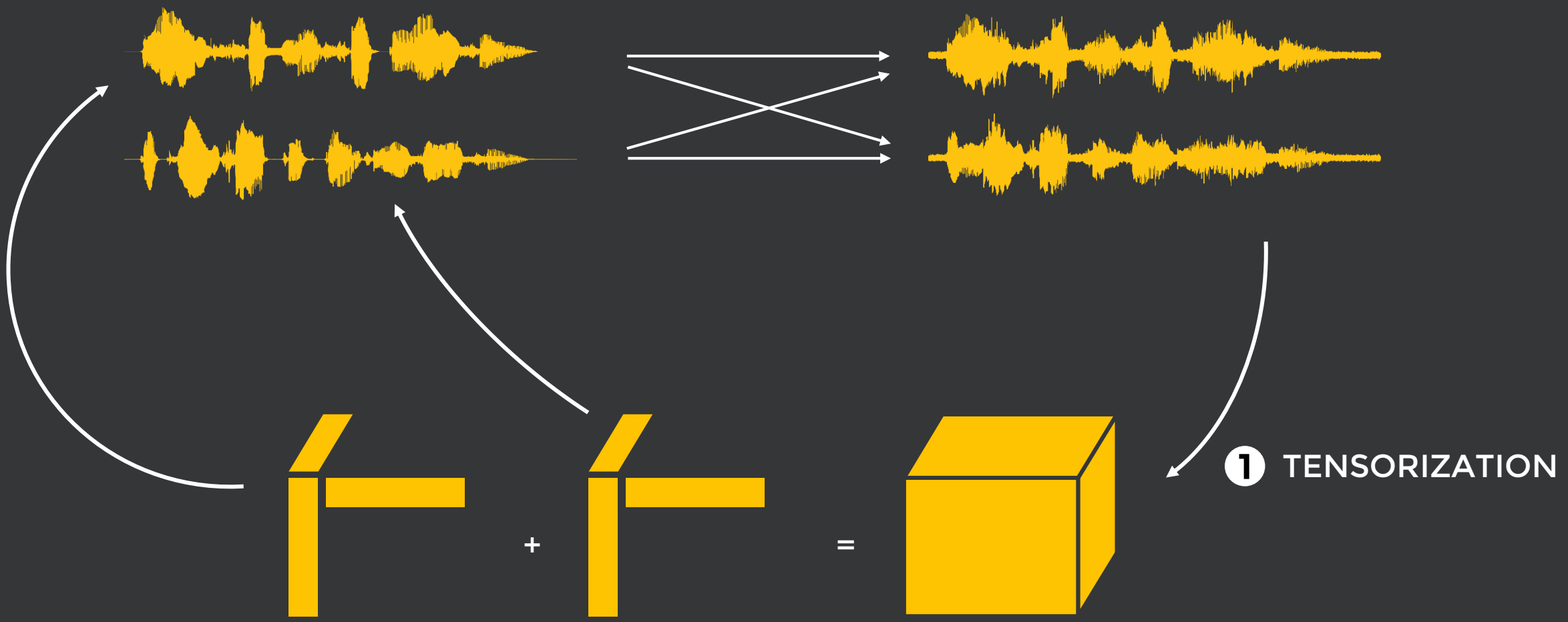




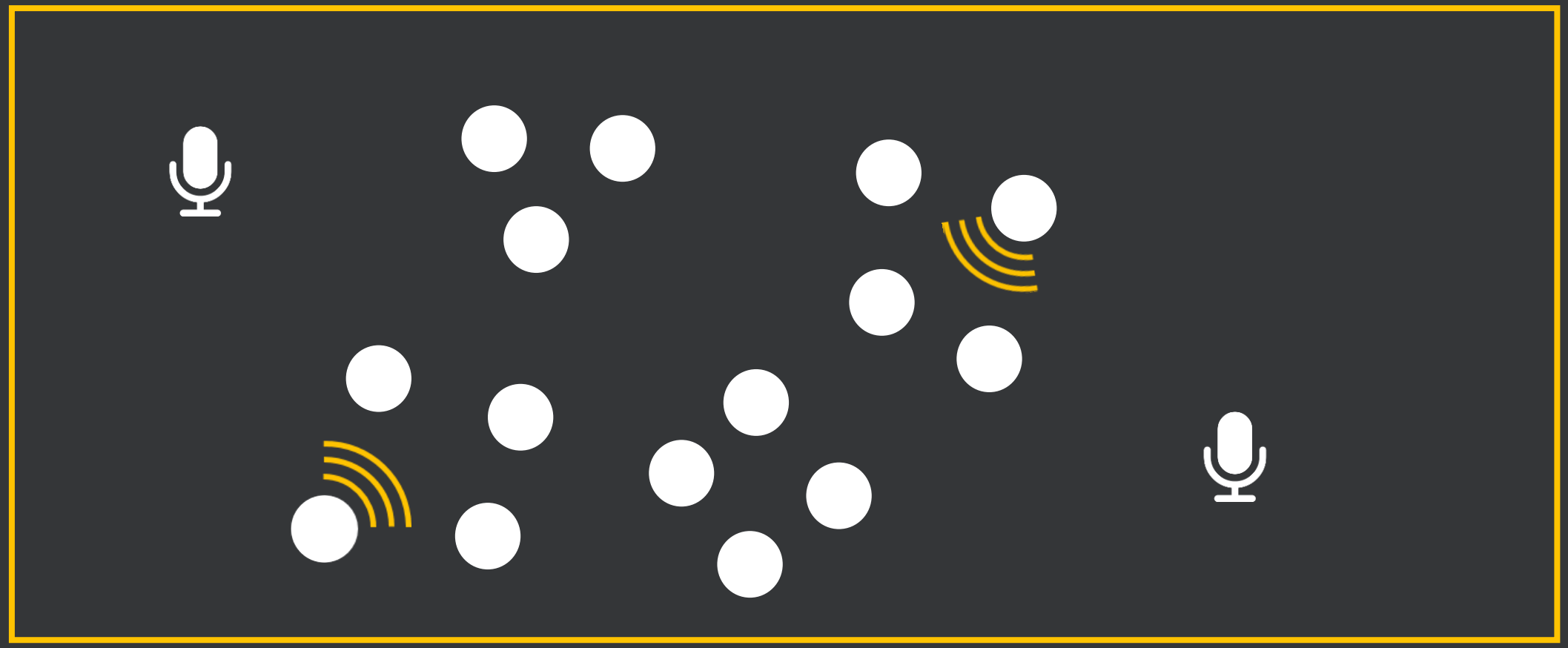
1 TENSORIZATION

2 UNIQUE TENSOR DECOMPOSITION

TENSOR-BASED BLIND SIGNAL SEPARATION



COCKTAIL PARTY PROBLEM



Recovered signals:



TENSORIZATION ONLY USEFUL FOR SIGNAL SEPARATION?

AS A PARADIGM, FOR MUCH MORE!

Graph or data clustering
Training neural networks
Function approximations
Decoupling of systems
Filter banks
Wavelet analysis
⋮


Questions?