Optimal Time-Switching for Simultaneous Wireless Information and Power Transfer

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***Abstract*—The receiver is an important part of a simultaneous wireless information and power transfer (SWIPT) system. A single receiver that can simultaneously decode and harvest power from the same signal has yet to be developed, so practical schemes utilizing multiple co-located receivers have been utilized instead. Time-switching is one such scheme in which a receiver either performs information decoding or power harvesting, the durations of which are determined by the time-switching ratio. Existing methods to optimize the time-switching ratio either exploit problem convexity or reformulate it into a form that can utilize the Lambert W function. However, these techniques do not easily lend themselves for use with more complicated non-convex problem formulations. As an alternative, this paper examines the geometrical relationship between the optimal time-switching ratio and the weights used to solve the resource allocation problem when framed as a weighted sum. A simple relation is derived that will form the basis for an optimization algorithm in future works.**

***Index Terms*—time-switching, SWIPT, energy harvesting, rate-power region, resource allocation**

I. Introduction

The receiver is an important part of a simultaneous wireless information and power transfer (SWIPT) system. A single receiver that can simultaneously decode information and harvest power from the same signal has yet to be developed, so practical schemes utilizing multiple co-located receivers have been utilized instead [1]. Time-switching is one such scheme that was first proposed in [2]. In a time-switching scheme, a receiver either performs information decoding or power harvesting, where the duration for both is determined by the time-switching ratio.

Time-switching ratio optimization has been addressed in several SWIPT related works in which a variety of system formations have been considered. Most works examine optimizing one time-switching ratio for point-to-point relaying [3]–[7] or for a single user [8]. Multiple time-switching ratios are considered to a lesser degree. In [9], [10] each user is assigned a time-switching ratio, while each antenna is assigned a time-switching ratio in [11] for single user multiantenna SWIPT.

In optimizing the time-switching ratio, some works can exploit problem convexity and use standard convex optimization tools. In [9], [11], the time-switching ratio is shown to form a perspective function with the objective, which is concave. Energy efficiency optimization is considered in [10], which is convexified through non-linear precoding and the Dinkelbach method. The authors in [7] alternately optimize a single time-switching ratio and beamforming vectors with a line search and semidefinite programming, respectively. A suboptimal method is also presented that uses the Lambert W function to solve problems in the form of $z = xe^x$ for x. Other works that utilize the Lambert W function include [3]–[6].

Despite the variety of works addressing optimal time-switching, the available methods do not easily lend themselves for use with more complicated non-convex problem formulations. One example is [12], which solves a multiuser resource allocation problem for fixed time-switching ratios without resorting to removing user interference to convexify the problem. The coupling between users renders the system complex and non-convex and limits the applicability of existing methods, particularly those that optimize a single time-switching ratio. Reformulating the problem to use the Lambert W function would take considerable effort and may not be possible, while the non-convexity precludes using convex optimization techniques without further system-specific insight.

In place of finding the optimal time-switching ratio, [12] illustrated the impact of the ratio by presenting simulations for various fixed values. The resulting collection of rate-power curves implied that an optimal time-switching ratio produces a point on its envelope. Inspired by this, this paper examines the geometrical relationship between the optimal time-switching ratio and the envelope for a resource allocation problem framed as a weighted sum. A simple relation is derived between the ratio and the weights used to solve the resource allocation, which will form the basis for an optimization algorithm in future work.

Notation

Note that matrices are upright and sans-serif, vectors are italicized and bold, and scalars are italicized. The transpose operation is denoted with $(\cdot)^\top$, $\det$ denotes the determinant operation, $\bigcup$ denotes the union, and $\mathbb{E}[\cdot]$ is the expected value operator. "Big O" notation, denoted with $O\{\cdot\}$, defines the upper bound for the magnitude of an error term. Perpen-

Fonds de la Recherche Scientifique - FNRS and the Fonds Wetenschappelijk Onderzoek - Vlaanderen under EOS Project no 30452698 '(MUSE-WINET) MUlti-SErvice WIreless NETwork'

dicularity is denoted with $\perp$ and $\circ$ is the Hadamard product.

II. Context and Contributions

The work herein is inspired by [12]. To provide context and direction, a brief description of the resource allocation problem and the SWIPT system model used is provided. Some details are omitted for brevity, so for a complete description please refer to [12].

A. System Model

The SWIPT system is modeled as a broadcast channel (BC) that contains one access point using N_T antennas to deliver data and power wirelessly to N users, each with one antenna. Orthogonal frequency-division multiplexing (OFDM) is utilized, where the symbols are comprised of K orthogonal subcarriers. On each subcarrier k, the access point applies a precoder T^k to a symbol vector $\boldsymbol{x}^k$ before transmitting to the remote users.

The time-switching receiver architecture is incorporated through $\alpha^n \in [0,1]$, which determines the proportion of time user n decodes information and harvests power. For information transfer, the downstream data rate for user n on subcarrier k may be modeled using $\alpha^n \log_2(1+\gamma_{\mathrm{BC}}^{k,n}(\mathsf{T}^k, \boldsymbol{s}^k))$. Summing over all subcarriers results in the total data rate for user n. The downstream signal-to-interference-plus-noise ratio (SINR) for user n on subcarrier k is denoted with $\gamma_{\mathrm{BC}}^{k,n}(\mathsf{T}^k, \boldsymbol{s}^k)$, where $\boldsymbol{s}^k$ is the symbol power vector. The symbol power for user n, the n-th element of $\boldsymbol{s}_k$, is denoted by $s^{k,n}$ and defined as $s^{k,n} \triangleq \mathbb{E}\left[|x^{k,n}|^2\right]$, with $x^{k,n}$ the n-th element of $\boldsymbol{x}^k$ and $\mathbb{E}[\cdot]$ the expected value operator. For power transfer, the harvested power for user n on subcarrier k is denoted with $(1-\alpha^n)P^{k,n}(\mathsf{T}^k, \boldsymbol{s}^k)$. The total harvested power for user n is found by summing $(1-\alpha^n)P^{k,n}(\mathsf{T}^k, \boldsymbol{s}^k)$ over all subcarriers.

B. Resource Allocation Problem

The resource allocation problem is formulated as the maximization of a weighted sum of data rates over all users and subcarriers, where the total transmit power is constrained to P_{Total} and the minimum per-user harvested power is constrained to $C^n \; \forall \; n$. The problem is given as

$$\begin{aligned} \underset{\substack{\{\mathsf{T}^k, \boldsymbol{s}^k\}_{\forall k} \\ \{\alpha^n\}_{\forall\, n}}}{\text{maximize}} \; & \sum_{k=1}^{K}\sum_{n=1}^{N} \omega^n \alpha^n \log_2\left(1+\gamma_{\mathrm{BC}}^{k,n}(\mathsf{T}^k, \boldsymbol{s}^k)\right) \\ \text{s.t.} \; & \sum_{k=1}^{K}\sum_{n=1}^{N} s^{k,n}||\boldsymbol{t}^{k,n}||_2^2 \le P_{\text{Total}} \\ & \sum_{k=1}^{K}(1-\alpha^n)P^{k,n}(\mathsf{T}^k, \boldsymbol{s}^k) \ge C^n \; \forall \; n \\ & \alpha^n \in [0,1] \; \forall \; n, \end{aligned} \tag{1}$$

where $\boldsymbol{t}^{k,n}$ is the precoder vector for user n, i.e. the n-th column of T^k. The non-negative weights ω^n are used to weight the importance of individual user data rates as determined by the use-case. The optimal point is chosen by selecting a precoder matrix ($\mathsf{T}^k \; \forall \; k$), symbol power vector ($\boldsymbol{s}^k \; \forall \; k$), and time-switching ratio ($\alpha^n \; \forall \; n$) which maximizes the weighted sum.

C. Contributions

To solve (1), [12] optimized $\{\mathsf{T}^k, \boldsymbol{s}^k\} \; \forall \; k$ for fixed time-switching ratios $\alpha^n \; \forall \; n$, but did not optimize the time-switching ratio itself. Since the time-switching ratio is an important design parameter, this work endeavors to characterize its effect on the Pareto edge of the rate-power region (RPR) defined by $\{\mathsf{T}^k, \boldsymbol{s}^k\} \; \forall \; k$ and $\alpha^n \; \forall \; n$ in (1), where the optimal solution will reside. It will be shown that a simple mapping exists between a non-time-switched operating point and a single point on the Pareto edge. This mapping can be exploited in developing the optimization algorithm, which will be described elsewhere.

III. Rate-Power Regions

In the absence of time-switching, a resource allocation for SWIPT may be formulated as a maximization of the weighted sum of data rate and harvested power for all N users given by

$$\begin{aligned} \underset{\{\mathsf{T}^k, \boldsymbol{s}^k\}_{\forall k}}{\text{maximize}} \; & \sum_{n}^{N} w_y^n r_y^n + w_x^n r_x^n \\ \text{s.t.} \; & \sum_{n}^{N} p^n \le P_{\text{Total}}. \end{aligned} \tag{2}$$

For user n, r_y^n is the achieved data rate, w_y^n is the associated non-negative weight, r_x^n is the achieved harvested power, and w_x^n is the associated non-negative weight. The allocated transmit power $\{p^n\}_{\forall n}$ is a function of the resource allocation, which is represented with $\{r_x^n, r_y^n\}_{\forall n}$. Note that the x and y subscripts refer to the graphical axis in which the type of objective is assigned for graphing purposes (i.e. data rate is associated with the y-axis and harvested power with the x-axis).

A. Time-Switching Ratio Effect

The receiver for user n spends α^n proportion of time decoding information and $1-\alpha^n$ harvesting power. The achieved average data rate is then given by

$$\hat{r}_y^n = \alpha^n r_y^n, \tag{3}$$

where $\hat{r}_y^n$ is the resulting data rate for user n due to time-switching. Similarly, the achieved average harvested power is given by

$$\hat{r}_x^n = (1-\alpha^n) r_x^n, \tag{4}$$

where $\hat{r}_x^n$ is the resulting harvested power for user n due to time-switching. It is clear from (3) and (4) that there is an inverse relationship between the data rate and the harvested power for each user due to the time-switching ratio. The maximum data rate for user n occurs when $\alpha^n = 1$. Similarly, the maximum harvested power occurs for user n occurs when $\alpha^n = 0$.

B. Utopia RPR

The weights in (2) are used to adjust the relative importance of each objective for each user, where a larger weight may be interpreted as a larger incentive to allocate resources to that objective. By varying the weights, any point of a convex

Pareto edge of the resource allocation can be traced out [13].[1] The Pareto edge for (2) is the edge of the RPR.

If the users simultaneously decode information and harvest power then $\{r_x^n, r_y^n\}_{\forall n}$ is the best operating point produced by the particular resource allocation. However this point is unobtainable, so a point on the Pareto edge for (2) may be considered a Utopia point from a time-switching perspective. The collection of Utopia points defines the Pareto edge of the Utopia RPR. Figure 1 depicts the Utopia RPR and illustrates the tradeoff between the data rate and the harvested power for $N = 1$.

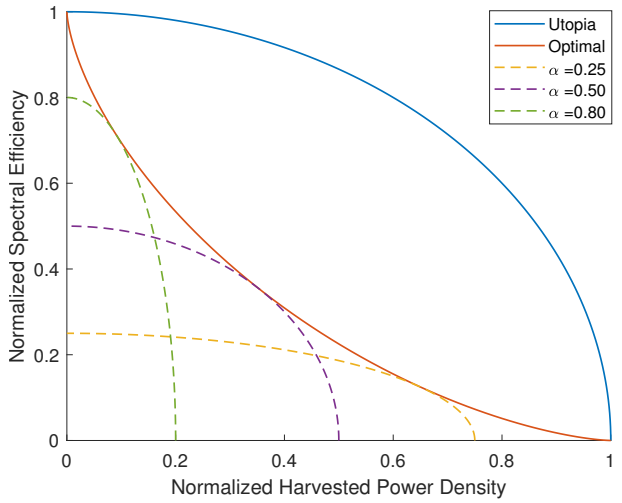


Fig. 1. The normalized RPRs for $N = 1$ are illustrated here. The Pareto edge of the Optimal RPR, labeled "Optimal", is the envelope of the fixed-α RPRs. The fixed-α RPRs, labeled with the α parameter, are time-switched versions of the Utopia RPR, which is labeled "Utopia".

C. Fixed-α RPR

The influence of the time-switching ratio $\alpha^n \in [0, 1]\ \forall\ n$ on the Utopia RPR may be examined by including fixed $\{\alpha^n\}\forall n$ in optimization (2) as follows:

$$\begin{aligned} \underset{\{\mathsf{T}^k, s^k\}_{\forall k}}{\text{maximize}} \ & \sum_n^N \hat{w}_y^n \hat{r_y}^n + \hat{w}_x^n \hat{r_x}^n \\ \text{s.t.} \ & \sum_n^N p^n \leq P_{\text{Total}}, \end{aligned} \tag{5}$$

where $\hat{w}_y^n$ and $\hat{w}_x^n$ are now the associated non-negative weights for $\hat{r_y}^n$ and $\hat{r_x}^n$, respectively. Plotting the RPR for fixed time-switching values for a single user results in the curves labeled with "α" in Figure 1. Each curve is referred to as a fixed-$\boldsymbol{\alpha}$ RPR, denoted with $\mathbf{RPR}_{\boldsymbol{\alpha}}$, and represents the Pareto edge of possible operating points for a given time-switching ratio.

It is observed that a resource allocation on the Pareto edge of the Utopia RPR, with r_x^n and r_y^n and weights w_x^n and w_y^n for user n, defines a resource allocation on the Pareto edge of $\mathbf{RPR}_{\boldsymbol{\alpha}}$ with $(1-\alpha^n)r_x^n$ and $\alpha^n r_y^n$ for user n, corresponding to (5) with weights $\hat{w}_x^n = \frac{w_x^n}{1-\alpha^n}$ and $\hat{w}_y^n = \frac{w_y^n}{\alpha^n}$. With this, each $\mathbf{RPR}_{\boldsymbol{\alpha}}$ is straightforwardly derived from the Utopia RPR. It is also observed that for a given resource allocation in the Utopia RPR, the time-switching ratio varies the operating point along a manifold, given by (3) and (4), and ranges from $(0, r_y^n)$ to $(r_x^n, 0)$ for all n. The manifold is referred to as the rate-power manifold and all operating points along this manifold have the same underlying resource allocation.

[1]The Pareto edge of (1) is concave due to the time-sharing property of multicarrier systems [12].

D. Optimal RPR

A set that contains all possible combinations of time-switching parameters may be defined with

$$\mathcal{D} := \{\boldsymbol{\alpha} \in \mathbb{R}^N \mid \alpha^n \in [0, 1],\ \forall\ n \in [1, N]\}. \tag{6}$$

For a fixed set of time-switching parameters $\boldsymbol{\alpha}$ such that $\boldsymbol{\alpha} \in \mathcal{D}$, the Utopia RPR can be scaled leading to the fixed-$\boldsymbol{\alpha}$ RPR containing the scaled points. The union of all fixed-$\boldsymbol{\alpha}$ RPRs for all possible $\boldsymbol{\alpha} \in \mathcal{D}$ is the Optimal RPR and is referred to here as

$$\mathbf{RPR}^* = \bigcup_{\boldsymbol{\alpha} \in \mathcal{D}} \mathbf{RPR}_{\boldsymbol{\alpha}}. \tag{7}$$

Each fixed-$\boldsymbol{\alpha}$ RPR contains a point for which the particular α is optimal and produces the maximum data rate for a particular harvested power, as seen in Figure 1. The Pareto edge of $\mathbf{RPR}^*$ contains these optimal points and is the envelope formed by the collection of fixed-$\boldsymbol{\alpha}$ RPRs, as shown in Figure 1 with the curve labeled "Optimal".

E. Optimal RPR and Rate-Power Manifolds

Choosing an operating point from a fixed-$\boldsymbol{\alpha}$ RPR and varying the time-switching ratio will produce a rate-power manifold. This is illustrated in Figure 2 for three operating points, where the manifolds are labeled A, B, and C. The manifolds are derived from resource allocations that produce a point on the Pareto edge of the Utopia RPR, as illustrated by the black dashed lines in Figure 2. Notice that a pair of manifolds intersect close to the Pareto edge of $\mathbf{RPR}^*$ if the underlying resource allocations are sufficiently similar (e.g. A and B). Manifolds derived from dissimilar resource allocations intersect away from the Pareto edge of $\mathbf{RPR}^*$ (e.g. A and C). As the underlying resource allocations become equivalent, the intersection converges with the Pareto edge of $\mathbf{RPR}^*$. Since a point on $\mathbf{RPR}^*$ has an optimal time-switching ratio, this observation implies that the time-switching ratio on the manifold of interest (e.g. A), where the adjacent manifold (e.g. B) intersects, converges to the optimal value as the resource allocations become equivalent. A fixed-$\boldsymbol{\alpha}$ RPR using the optimal time-switching ratio will also intersect the Optimal RPR at the same point.

A similar observation is made by [14] concerning the envelope created by the straight lines used in the art form known as string art. The authors observed that adjacent straight lines intersect close to the envelope. By parameterizing the x- and y-intercepts for a family of lines, it was shown in [14] that an analytical expression can be derived for the envelope based on these intersections. Although an analytical expression cannot be found for the envelope in the problem considered here, the associated optimal α and operating point

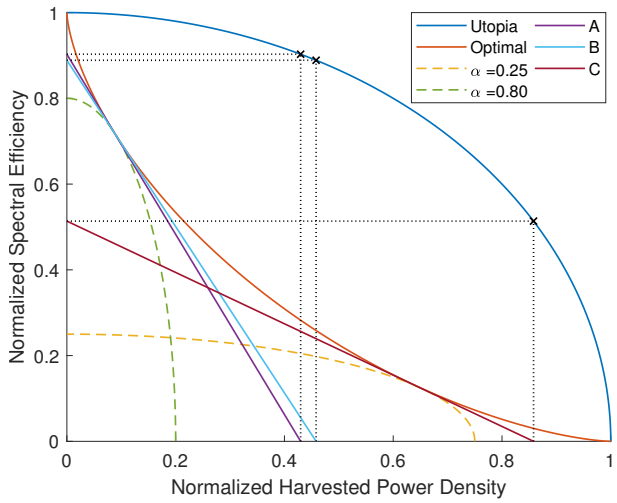


Fig. 2. For two rate-power manifolds, the point of intersection approaches the Pareto edge of the Optimal RPR as the underlying resource allocations become equivalent (e.g. manifolds A and B). Dissimilar resource allocations intersect away from the Pareto edge of the Optimal RPR (e.g. manifolds A and C). The black-dotted lines indicate which point on the Utopia RPR each manifold is derived from.

may still be determined by finding where the intersection occurs.

IV. Optimal Operating Points: Single User

In Section III-E, it was observed that the intersection of two rate-power manifolds moved closer to and converged with the Pareto edge of **RPR*** as the underlying resource allocations became equivalent. Hence, the point on the Pareto edge of **RPR***, associated with a particular operating point on the Pareto edge of **RPR**$_{\boldsymbol{\alpha}}$, may be determined by finding the intersection of the associated rate-power manifold with another rate-power manifold for a sufficiently similar resource allocation.

Consider the solution $\hat{\boldsymbol{r}}$ to a fixed-$\boldsymbol{\alpha}$ RPR weighted sum data rate sum power maximization given by (5) for a single user (e.g. $N = 1$) with weights $\hat{\boldsymbol{w}}$, where $\hat{\boldsymbol{w}} = [\hat{w}_x, \hat{w}_y]^\top$, $\hat{\boldsymbol{r}} = [\hat{r}_x, \hat{r}_y]^\top$, and $\hat{\boldsymbol{w}}$ is a vector normal to the surface of the Pareto edge at the point $\hat{\boldsymbol{r}}$. For convenience in defining the permutation matrix later, the time-switching ratio is chosen to be $\alpha = 0.5$ and the RPR, namely **RPR**$_{\mathbf{0.5}}$, is normalized such that the maximum value on either axis is 1.

A. Rate-Power Manifold

If the rate-power region's Pareto edge is a smooth surface, then the points $\hat{\boldsymbol{r}}'$ in the neighborhood of $\hat{\boldsymbol{r}}$ satisfy:

$$\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}' \approx \hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}} \tag{8}$$

where $\hat{\boldsymbol{r}}' \approx \hat{\boldsymbol{r}} + \mathbf{a}$ and $\mathbf{a} \perp \hat{\boldsymbol{w}}$. The value of $\mathbf{a}$ falls some amount ϵ along the manifold normal to $\hat{\boldsymbol{w}}$, which in 2-dimensional space is equivalent to

$$\boldsymbol{a} = \epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}}, \tag{9}$$

and yields

$$\hat{\boldsymbol{r}}' \approx \hat{\boldsymbol{r}} + \epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}}, \tag{10}$$

where $\mathsf{R} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is a 90 degree rotation. The difference between $\hat{\boldsymbol{r}} + \epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}}$ and $\hat{\boldsymbol{r}}'$ may be bounded with

$$\hat{\boldsymbol{r}}' = \hat{\boldsymbol{r}} + \epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}} + O\{\epsilon^2\} \text{ as } \epsilon \to 0, \tag{11}$$

where $O\{\epsilon^2\}$ is "Big O" notation meaning that the absolute-value of the error for $\hat{\boldsymbol{r}}'$ is at most some constant times $|\epsilon^2|$.

For any point $\hat{\boldsymbol{r}}$ on the rate-power region, the associated manifold of points $\boldsymbol{k}$ of achievable rate-power combinations (achieved by varying α) satisfy

$$(\boldsymbol{k} - \hat{\boldsymbol{r}})^\top \Pi \, \hat{\boldsymbol{r}} = 0, \tag{12}$$

where

$$\Pi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{13}$$

is the permutation matrix. Note that the simple nature of the permutation matrix is due to normalization and the choice of $\alpha = 0.5$. For $\alpha \neq 0.5$, a different permutation matrix would be needed. A normal vector at $\hat{\boldsymbol{r}}$ is described by $\Pi \, \hat{\boldsymbol{r}}$, $\boldsymbol{k}$ describes a point on the manifold, and $(\boldsymbol{k} - \hat{\boldsymbol{r}})$ is on the manifold. Any vector on the manifold will have a 0 dot product with the normal vector.

B. Optimal Operating Point

The optimal operating point is determined by finding the intersection of the two rate-power manifolds as the difference between the resource allocations go to zero. There are two points on the fixed-$\boldsymbol{\alpha}$ RPR:

$$\hat{\boldsymbol{r}} \tag{14}$$

$$\hat{\boldsymbol{r}}' = \hat{\boldsymbol{r}} + \epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}} + O\{\epsilon^2\} \tag{15}$$

with the associated rate-power manifold definitions:

$$(\boldsymbol{k} - \hat{\boldsymbol{r}})^\top \Pi \, \hat{\boldsymbol{r}} = 0 \tag{16}$$

$$(\boldsymbol{k} - \hat{\boldsymbol{r}}')^\top \Pi \, \hat{\boldsymbol{r}}' = 0. \tag{17}$$

Substituting (15) into (17) and simplifying yields

$$(\boldsymbol{k} - 2\,\hat{\boldsymbol{r}} - \epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}} - O\{\epsilon^2\})^\top \Pi (\epsilon \, \mathsf{R} \, \hat{\boldsymbol{w}} + O\{\epsilon^2\}) = 0. \tag{18}$$

Dividing by ϵ and taking the limit as $\epsilon \to 0$, reduces the equation to

$$(\boldsymbol{k} - 2\,\hat{\boldsymbol{r}})^\top \Pi \, \mathsf{R} \, \hat{\boldsymbol{w}} = 0. \tag{19}$$

Since $\Pi = \Pi^{-1}$, equating (16) and (19) yields

$$\boldsymbol{k} = \Pi [\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^{-\top} [2 \, \mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^\top \Pi \, \hat{\boldsymbol{r}}. \tag{20}$$

This can be simplified further by first noticing that

$$[\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^{-\top} [2 \, \mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^\top = 2\mathsf{I} - [\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^{-\top} [0, \hat{\boldsymbol{r}}]^\top. \tag{21}$$

Given

$$\mathsf{A}^{-1} = \frac{1}{\det \mathsf{A}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}, \tag{22}$$

it can be shown that

$$-\mathsf{R} \, \mathsf{A} \, \mathsf{R} = \mathsf{A}^{-\top} \det \mathsf{A}, \tag{23}$$

where $\det \mathsf{A}$ refers to the determinant of A. If $\mathsf{A}^{-\top} = [\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^{-\top}$ then

$$2\mathsf{I} - [\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]^{-\top} [0, \hat{\boldsymbol{r}}]^\top = 2\mathsf{I} + \frac{\mathsf{R} [\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}] \, \mathsf{R} [0, \hat{\boldsymbol{r}}]^\top}{\det [\mathsf{R} \, \hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]}. \tag{24}$$

$\mathsf{R}\,\mathsf{R} = -\mathsf{I}$ and $\det[\mathsf{R}\,\hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}] = \hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}$, so

$$2\mathsf{I} + \frac{\mathsf{R}[\mathsf{R}\,\hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]\,\mathsf{R}[0, \hat{\boldsymbol{r}}]^\top}{\det[\mathsf{R}\,\hat{\boldsymbol{w}}, \hat{\boldsymbol{r}}]} = 2\mathsf{I} - \frac{[\hat{\boldsymbol{w}}, \mathsf{R}\,\hat{\boldsymbol{r}}][\hat{\boldsymbol{r}}, 0]^\top}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}. \quad (25)$$

Since $[\hat{\boldsymbol{w}}, \mathsf{R}\,\hat{\boldsymbol{r}}][\hat{\boldsymbol{r}}, 0]^T = \hat{\boldsymbol{w}}\,\hat{\boldsymbol{r}}^\top$, the simplification results in

$$\boldsymbol{k} = \Pi\left(2\mathsf{I} - \frac{\hat{\boldsymbol{w}}\,\hat{\boldsymbol{r}}^\top}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}\right)\Pi\,\hat{\boldsymbol{r}}\,. \quad (26)$$

C. Optimal Time-Switching Ratio

Equation (26) finds the point $\boldsymbol{k}$ on the Pareto edge of the Optimal RPR that utilizes the same resource allocation as $\hat{\boldsymbol{r}}$ from the fixed-$\boldsymbol{\alpha}$ RPR. Simplifying (26) further, since

$$\Pi\mathsf{I}\Pi = \mathsf{I} \quad (27)$$

and

$$\Pi\,\hat{\boldsymbol{w}}\,\hat{\boldsymbol{r}}^\top\,\Pi = \begin{bmatrix} \hat{w}_y\hat{r}_y & \hat{w}_y\hat{r}_x \\ \hat{w}_x\hat{r}_y & \hat{w}_x\hat{r}_x \end{bmatrix}, \quad (28)$$

it is easy to see that

$$\begin{aligned} \boldsymbol{k} &= \frac{1}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}(2\,\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}\,\mathsf{I} - \Pi\,\hat{\boldsymbol{w}}\,\hat{\boldsymbol{r}}^\top\,\Pi)\,\hat{\boldsymbol{r}} \\ &= \frac{1}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}\begin{bmatrix} 2\hat{w}_x\hat{r}_x + \hat{w}_y\hat{r}_y & -\hat{w}_y\hat{r}_x \\ -\hat{w}_x\hat{r}_y & 2\hat{w}_y\hat{r}_y + \hat{w}_x\hat{r}_x \end{bmatrix}\hat{\boldsymbol{r}} \\ &= \frac{1}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}\begin{bmatrix} 2\hat{w}_x\hat{r}_x^2 \\ 2\hat{w}_y\hat{r}_y^2 \end{bmatrix} = 2\frac{\hat{\boldsymbol{r}}\circ\hat{\boldsymbol{r}}\circ\hat{\boldsymbol{w}}}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}, \end{aligned} \quad (29)$$

where $\circ$ is the Hadamard product.

Equation (29) reveals a simple relationship between the weights for the fixed-α resource allocation and the optimal time-switching ratio. Recalling that the influence of $\alpha = 0.5$ was included through the permutation matrix Π, the factor of 2 in (29) can be attributed to α since

$$\frac{1}{\alpha} = \frac{1}{1-\alpha} = 2. \quad (30)$$

Combining α with the point on the fixed-$\boldsymbol{\alpha}$ RPR, $\hat{\boldsymbol{r}}$, produces a point on the Utopia RPR,

$$\boldsymbol{r} = \left[\frac{\hat{r}_x}{1-\alpha}, \frac{\hat{r}_y}{\alpha}\right]^\top. \quad (31)$$

The remaining portion of (29) is then seen to be the optimal time-switching ratio given by

$$1 - \alpha^* = \frac{\hat{w}_x\hat{r}_x}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}} \quad (32)$$

and

$$\alpha^* = \frac{\hat{w}_y\hat{r}_y}{\hat{\boldsymbol{w}}^\top \hat{\boldsymbol{r}}}. \quad (33)$$

Furthermore, the weights $\hat{\boldsymbol{w}}$ incorporate the impact of α during the optimization, such that

$$\boldsymbol{w} = [w_x, w_y]^\top = [(1-\alpha)\hat{w}_x, \alpha\hat{w}_y]^\top. \quad (34)$$

Substituting (31) and (34) into (32) and (33) yields

$$1 - \alpha^* = \frac{w_x r_x}{\boldsymbol{w}^\top \boldsymbol{r}} \quad (35)$$

and

$$\alpha^* = \frac{w_y r_y}{\boldsymbol{w}^\top \boldsymbol{r}}, \quad (36)$$

which shows that α^* can be directly calculated based on the Utopia point and the associated weights without considering an intermediate α.

V. Optimal Operating Points: Multiple Users

In this section, the results from Section IV are generalized and extended to the multiuser case. Consider the maximization of a weighted sum of data rates and harvested powers for N users described in (2). Using all possible values for α to time-switch a particular point from the Pareto edge of the Utopia RPR produces an N-dimensional manifold described by

$$\{(1-\alpha^n)r_x^n, \alpha^n r_y^n \mid \alpha^n \in [0,1]\}_{\forall\, n}. \quad (37)$$

From (2), it also follows that each point within the manifold lies on the Pareto edge of a fixed-$\boldsymbol{\alpha}$ RPR, namely $\mathbf{RPR}_{\boldsymbol{\alpha}}$, and maximizes a weighted data rate and power sum criterion with weights $\frac{w_x^n}{1-\alpha^n}$ and $\frac{w_y^n}{\alpha^n}$ for user n. These weights define the 2N-1 dimensional hyperplane

$$\sum_n^N \left(\frac{w_x^n}{1-\alpha^n}\right)x^n + \left(\frac{w_y^n}{\alpha^n}\right)y^n = C \quad (38)$$

with constant C and is tangent to $\mathbf{RPR}_{\boldsymbol{\alpha}}$ at the considered point.

Proposition 1. *For a given set of weights $\{w_x^n, w_y^n\}_{\forall\, n}$ and the corresponding optimal resource allocation $\{r_x^n, r_y^n\}_{\forall\, n}$ in* (2)*, consider a particular time-switching ratio set $\hat{\boldsymbol{\alpha}}$ containing elements $\hat{\alpha}^n$ such that*

$$\hat{\alpha}^n = \frac{w_y^n r_y^n}{w_x^n r_x^n + w_y^n r_y^n} \quad (39)$$

and

$$1 - \hat{\alpha}^n = \frac{w_x^n r_x^n}{w_x^n r_x^n + w_y^n r_y^n} \quad (40)$$

for all n. The corresponding point on the $\mathbf{RPR}_{\hat{\boldsymbol{\alpha}}}$ is

$$\{(1-\hat{\alpha}^n)r_x^n, \hat{\alpha}^n r_y^n \mid \forall\, n \in [1, N]\} \quad (41)$$

and the corresponding hyperplane is

$$\sum_n^N \left(\frac{w_x^n}{1-\hat{\alpha}^n}\right)x^n + \left(\frac{w_y^n}{\hat{\alpha}^n}\right)y^n = C. \quad (42)$$

The N-dimensional manifold in (37) *lies completely within the hyperplane defined in* (42)*.*

Proof: Substituting

$$x^n = (1-\alpha^n)r_x^n,\ y^n = \alpha^n r_y^n \quad (43)$$

and the definition for $\hat{\boldsymbol{\alpha}}$ into the left-hand side of (42) provides

$$\begin{aligned} &\sum_n^N \frac{w_x^n}{1-\hat{\alpha}^n}(1-\alpha^n)r_x^n + \frac{w_y^n}{\hat{\alpha}^n}\alpha^n r_y^n \\ &= \sum_n^N \frac{w_x^n}{\frac{w_x^n r_x^n}{w_x^n r_x^n + w_y^n r_y^n}}(1-\alpha^n)r_x^n + \frac{w_y^n}{\frac{w_y^n r_y^n}{w_x^n r_x^n + w_y^n r_y^n}}\alpha^n r_y^n \\ &= \sum_n^N (w_x^n r_x^n + w_y^n r_y^n) = C, \end{aligned} \quad (44)$$

which is indeed the original weighted sum for all α and proves that the manifold lies within the hyperplane. □

Proposition 1 shows that the Utopia RPR can be mapped to the Optimal RPR if the specific value of α^n given by (39) and (40) are used. It is easy to see this is true for the $N = 1$ case by drawing it and noting that the manifold and the hyperplane are the same line. For $N > 1$, it is now shown in Proposition 2 that using this particular time-switching ratio for all users n results in a Pareto optimal point on the **RPR*** for the given resource allocation.

Proposition 2. *For a given set of weights* $\{w_x^n, w_y^n\}_{\forall\, n}$ *and the corresponding optimal resource allocation* $\{r_x^n, r_y^n\}_{\forall\, n}$ *in* (2)*, the only point in the N-dimensional manifold* (37) *that is Pareto optimal on* ***RPR**** *has*

$$\alpha^n = \hat{\alpha}^n = \frac{w_y^n r_y^n}{w_x^n r_x^n + w_y^n r_y^n} \tag{45}$$

for all n*.*

Proof: Consider 2^N points in the neighborhood of point A,

$$(r_x^n + \epsilon w_y^n) \;\text{ and }\; (r_y^n - \epsilon w_x^n) \tag{46}$$

or

$$(r_x^n - \epsilon w_y^n) \;\text{ and }\; (r_y^n + \epsilon w_x^n) \tag{47}$$

for user n and $\epsilon > 0$. For sufficiently small ϵ, these points achieve the same weighted data rate and power sum as point A and hence lie on the Pareto edge of the Utopia RPR. The point on the manifold $\{(1-\alpha^n)r_x^n, \alpha^n r_y^n\}_{\forall\, n}$ is not Pareto-optimal if $\tilde{\alpha}^n$ exists for all n such that either

$$(1-\tilde{\alpha}^n)(r_x^n + \epsilon w_y^n) > (1-\alpha^n) r_x^n \tag{48a}$$
$$\tilde{\alpha}^n (r_y^n - \epsilon w_x^n) > \alpha^n r_y^n \tag{48b}$$

or

$$(1-\tilde{\alpha}^n)(r_x^n - \epsilon w_y^n) > (1-\alpha^n) r_x^n \tag{49a}$$
$$\tilde{\alpha}^n (r_y^n + \epsilon w_x^n) > \alpha^n r_y^n \tag{49b}$$

are statisfied.

An uppper bound (UB) and lower bound (LB) for $\tilde{\alpha}^n$ derived from (48a) and (48b) are given by

$$\frac{\alpha^n r_y^n}{r_y^n - \epsilon w_x^n} < \tilde{\alpha}^n < 1 - \frac{(1-\alpha^n) r_x^n}{r_x^n + \epsilon w_y^n}. \tag{50}$$

No solution can be found if LB $>$ UB, which is equivalent to

$$\begin{aligned} \frac{\alpha^n r_y^n}{r_y^n - \epsilon w_x^n} &> 1 - \frac{(1-\alpha^n) r_x^n}{r_x^n + \epsilon w_y^n} \\ \alpha^n &> \frac{w_x^n (r_y^n - \epsilon w_y^n)}{w_x^n r_x^n + w_y^n r_y^n} \\ \alpha^n &> \left(1 - \epsilon \frac{w_y^n}{r_y^n}\right) \hat{\alpha}^n. \end{aligned} \tag{51}$$

Similarly, (49a) and (49b) lead to $\alpha^n < \left(1 + \epsilon \frac{w_y^n}{r_y^n}\right) \hat{\alpha}^n$. Taking the two together results in

$$\left(1 - \epsilon \frac{w_y^n}{r_y^n}\right) \hat{\alpha}^n < \alpha^n < \left(1 + \epsilon \frac{w_y^n}{r_y^n}\right) \hat{\alpha}^n. \tag{52}$$

As $\epsilon \to 0$, the only point that satisfies (52) is $\alpha^n = \hat{\alpha}^n$ (i.e. $\{(1-\hat{\alpha}^n) r_x^n, \hat{\alpha}^n r_y^n\}_{\forall\, n}$) and is thus the only point that can be Pareto optimal. □

VI. Conclusion

This paper examined the relationship between the Utopia RPR and the Optimal RPR as a function of the time-switching ratio. In Section III, it was observed that the rate-power planes for similar resource allocations intersect close to the Optimal RPR and converge with the Optimal RPR as the difference between them approaches zero. This observation was used in Section IV for the single-user case to derive a point on the Optimal RPR from a point on the fixed-$\boldsymbol{\alpha}$ RPR. The derivation also revealed a simple relationship between the weights and resource allocation from the Utopia RPR and the optimal time-switching ratio. In Section V, the derivations were shown to apply to the multiuser case. As a result, this mapping can be exploited in developing the optimization algorithm, which will be described elsewhere.

References

[1] T. D. Ponnimbaduge Perera, D. N. K. Jayakody, S. K. Sharma, S. Chatzinotas, and J. Li, "Simultaneous Wireless Information and Power Transfer (SWIPT): Recent Advances and Future Challenges," *IEEE Communications Surveys & Tutorials*, vol. 20, no. 1, pp. 264–302, 2018.

[2] R. Zhang and C. K. Ho, "MIMO Broadcasting for Simultaneous Wireless Information and Power Transfer," *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 1989–2001, May 2013.

[3] T. M. Hoang, B. C. Nguyen, N. N. Thang, M. Tran, and P. T. Tran, "Performance and optimal analysis of time-switching energy harvesting protocol for MIMO full-duplex decode-and-forward wireless relay networks with various transmitter and receiver diversity techniques," *Journal of the Franklin Institute*, vol. 357, no. 17, pp. 13 205–13 230, Nov. 2020.

[4] M. Mohammadi, B. K. Chalise, H. A. Suraweera, C. Zhong, G. Zheng, and I. Krikidis, "Throughput Analysis and Optimization of Wireless-Powered Multiple Antenna Full-Duplex Relay Systems," *IEEE Transactions on Communications*, vol. 64, no. 4, pp. 1769–1785, Apr. 2016.

[5] C. Zhong, H. A. Suraweera, G. Zheng, I. Krikidis, and Z. Zhang, "Wireless Information and Power Transfer With Full Duplex Relaying," *IEEE Transactions on Communications*, vol. 62, no. 10, pp. 3447–3461, Oct. 2014.

[6] C. Zhong, X. Chen, Z. Zhang, and G. Karagiannidis, "Wireless Powered Communications: Performance Analysis and Optimization," *IEEE Transactions on Communications*, vol. 63, no. 12, pp. 5178–5190, Dec. 2015.

[7] B. K. Chalise, H. A. Suraweera, and G. Zheng, "Throughput maximization for full-duplex energy harvesting MIMO communications," in *2016 IEEE 17th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jul. 2016, pp. 1–5.

[8] L. Ma, Y. Wang, and Y. Xu, "Sum rate optimization for SWIPT system based on zero-forcing beamforming and time switching," in *2017 13th International Wireless Communications and Mobile Computing Conference (IWCMC)*, Jun. 2017, pp. 351–356.

[9] X. Zhou, R. Zhang, and C. K. Ho, "Wireless Information and Power Transfer in Multiuser OFDM Systems," *IEEE Transactions on Wireless Communications*, vol. 13, no. 4, pp. 2282–2294, Apr. 2014.

[10] J. Tang, D. K. C. So, N. Zhao, A. Shojaeifard, and K.-K. Wong, "Energy Efficiency Optimization With SWIPT in MIMO Broadcast Channels for Internet of Things," *IEEE Internet of Things Journal*, vol. 5, no. 4, pp. 2605–2619, Aug. 2018.

[11] S. Kang, H. Lee, S. Hwang, and I. Lee, "Time Switching Protocol for Multi-Antenna SWIPT Systems," in *2020 IEEE Wireless Communications and Networking Conference (WCNC)*, May 2020, pp. 1–6.

[12] R. P. Ketcham, J. Verdyck, and M. Moonen, "Joint Beamforming and Power Allocation for Multiuser MISO Broadcast Channel SWIPT Employing OFDM," *IEEE Access*, vol. 9, pp. 165 154–165 172, 2021.

[13] W. Jakob and C. Blume, "Pareto Optimization or Cascaded Weighted Sum: A Comparison of Concepts," *Algorithms*, vol. 7, no. 1, pp. 166–185, Mar. 2014.

[14] G. Quenell, "Envelopes and String Art," *Mathematics Magazine*, vol. 82, no. 3, pp. 174–185, 2009.