Subspace Methods for the Block Macaulay Matrix Framework

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1 Introduction

Many system identification problems, like the least-squares identification of linear time-invariant models [1] and the globally optimal identification of autoregressive moving-average models [3], can be rephrased as rectangular multi-parameter eigenvalue problems (RMEPs). Their coefficient matrices are generated using the observed data to which we want to fit the model. The solutions of these RMEPs correspond to the stationary points of the optimization problems and can be computed by solving a set of standard eigenvalue problems (SEPs) that follows from a multidimensional realization problem in the null space of the block Macaulay matrix (BMM) [4].

The BMM method, however, scales poorly with respect to the size of the generating RMEP, which is proportional to the number of observed data points in the related system identification problem. Current BMM methods compute all the solutions of the RMEP, which remains computationally intensive and is often unnecessary. Since we are often only interested in a subset of the solutions, we take inspiration from subspace methods to solve SEPs and square MEPs prevalent in the numerical linear algebra literature [2], and we develop for the first time subspace methods for the BMM framework.

2 Subspace methods for RMEPs

RMEPs are defined in a single matrix pencil format as

$$M(\lambda)\mathbf{z} = \left(\sum_{\{\omega\}} \mathbf{A}_\omega \lambda^\omega\right)\mathbf{z} = \mathbf{0}, \quad \|\mathbf{z}\| = \mathbf{1}, \qquad (1)$$

where $\mathbf{A}_\omega \in \mathbb{C}^{k\times l}$ are the coefficient matrices (with $k \geq l + n - 1$) and $\lambda^\omega = \prod_{i=1}^n \lambda_i^{\omega_i} = \lambda_1^{\omega_1}\cdots\lambda_n^{\omega_n}$ are the monomials. We call the n-tuples $\lambda = (\lambda_1,\ldots,\lambda_n) \in \mathbb{C}^n$ and vectors $\mathbf{z} \in \mathbb{C}^{l\times 1}\backslash\{0\}$ the eigentuples and eigenvectors of the RMEP, respectively.

Subspace methods to solve large SEPs and square MEPs consist of two alternating steps [2], which can also be developed for RMEPs in the BMM framework: (i) An *extraction* step, in which we project the large problem onto a smaller problem via approximation and projection spaces. In every extraction step, the basis of the approximation space helps to evaluate an approximation to the eigenvector of the original RMEP (1). The residual associated with the approximation is projected in a Petrov–Galerkin sense over the projection space, resulting in the new smaller problem. (ii) An *expansion* step, in which we use the right and left singular vectors associated with the eigentuple that makes the original pencil the most rank deficient, in order to expand the basis of the approximation and projection space, respectively. These two steps are applied in an alternating fashion until the convergence to an eigentuple of the original problem.

3 Presentation outline

In our presentation, we will start with a brief overview of RMEPs and the BMM method. Then, a detailed discussion regarding the two steps of the subspace methods for RMEPs will be provided. Finally, the convergence behavior and observed speed-up of the proposed subspace methods will be illustrated using numerical experiments.

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