SUBSPACE IDENTIFICATION COMBINED WITH NEW MODE SELECTION TECHNIQUES FOR MODAL ANALYSIS OF AN AIRPLANE

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Abstract: Linear system identification is an important tool in experimental modal analysis. It allows for the extraction of resonance frequencies, damping ratios and mode shapes of a vibrating structure. In general, the model order is chosen quite high so as to catch all the important characteristics of the structure, even in the presence of large amounts of measurement noise. This often results in the appearance of non-physical, or so-called spurious modes. In this paper we will present a set of heuristic techniques to remove spurious modes from a previously identified model. The advantage of the techniques that will be presented is that they do not rely on statistical information, making them ideally suited for use in combination with subspace identification. The quality of the techniques will be assessed using simulated data and observations from in flight flutter tests.

Keywords: Parameter estimation, Subspace methods.

1. INTRODUCTION

System identification is a standard tool for the analysis of forcefully or ambient excited vibrating structures (Van der Auweraer, 2001). A linear model for the structure is built from available observations, based on which modal characteristics as resonance frequencies and modal shapes can be estimated. Typically, the vibrating structure is equipped with tens to hundreds of sensors and the average identification order needed to obtain a suitable model is reasonably high. Since measurements on the structure are often disturbed by large amounts of measurement noise, and unknown inputs do not always satisfy the white noise assumption, eg. during in flight tests with an aircraft subjected to turbulence, choosing a model order is often a difficult task, certainly when the amount of available measurements is limited. Not seldomly do models in which the order was chosen according to some order selection technique, therefore prove inadequate to describe all relevant characteristics of the structure. In modal analysis, one therefore typically uses an identification order that is guaranteed to be larger than necessary. Unfortunately however, as the order of the model is increased, so will the amount of identified modes. This will in many cases inevitably result in the appearance of so-called spurious modes which bear no immediate physical relevance. A common technique to remove spurious modes from a model is the stabilization diagram (Van der Auweraer, 2001), where models of increasing order are compared, and modes that are repeatedly found in these models with about the same characteristics are considered to be physical. A problem however is that the comparisson is highly user interactive, making the stabilization diagram unsuited for use in an online envionment. In this paper we will describe several automatic techniques to detect spurious modes and remove them from the model. Although the techniques are in general quite heuristic in nature, as is the stabilization diagram, we will show by means of a simulation and an example from the avionics industry that in many practical cases a quick discrimination between spurious and physical modes can effectively be made, without reverting to an analysis of the stabilization diagram. An advantage of the techniques that will be presented is that they are ideally suited to be applied to models obtained using subspace identification. Subspace identification is a popular concept that

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allows for a fast and robust identification of MIMO systems by using projections of subspaces spanned by the rows and columns of Hankel matrices containing input- and output measuremens (Verhaegen 1996; Van Overschee and De Moor, 1996; Bauer and Ljung, 2002), but does unfortunately not return stochastic information as confidence bounds around poles and zeros, which are used by some recently proposed mode selection techniques (Verboven *et al.*, 2002).

In section 2 we will briefly refresh the basic concepts of subspace identification, state space models and the theory of balanced model reduction from a practical point of view. Several mode selection techniques will then be presented in section 3. In section 4, we will assess the quality of the proposed techniques by means of a simulated example, and an analysis of data obtained from a test flight of an airplane. We will show that the methods presented in section 3 can effectively be applied to detect and remove spurious modes from a linear model, even in the presence of large amounts of measurement noise. Finally, in Section 5, some conclusions will be drawn.

Some common notations that will be used throughout this text are the following: $E\{\cdot\}$ will be used to denote the expected value of an expression. $A(i:j,k:l)$ denotes a submatrix of A, bounded by the ith and jth row and kth and lth column. If a colon (:) is used on its own (eg. A(:,k:l)) all available rows and/or columns are included in the submatrix.

2. SUBSPACE IDENTIFICATION AND THE STATE SPACE FORMULATION

The aim of subspace identification is to identify models of the form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k, \\ y_k &= Cx_k + Du_k + v_k, \end{aligned} \tag{1}$$

with

$$E\left\{\begin{bmatrix} w_p \\ v_p \end{bmatrix}\begin{bmatrix} w_q^T & v_q^T \end{bmatrix}\right\} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}\delta_{pq} \geq 0, \tag{2}$$

where $E\{\cdot\}$ denotes the expected value operator and δ_{pq} the Kronecker delta. It is assumed that:

$$E\left\{\begin{bmatrix} w_p \\ v_p \end{bmatrix} x_k^T\right\} = 0, \forall p \geq k\ . \tag{3}$$

The elements of the vectors $y_k \in \mathbb{R}^l$ and $u_k \in \mathbb{R}^m$ are given observations of the outputs and inputs of the system at the discrete time index *k*. The vector $x_k \in \mathbb{R}^n$ is the unknown state vector at time *k*. The unobserved process and measurement noise $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^l$ are assumed to be white, zero mean, gaussian with covariance matrices as given in (2). The system matrices *A,B,C,D* and the covariance matrices *Q, S,* and *R* have appropriate dimensions.

In subspace identification, the model (1) is obtained using projections of rows and columns of so-called block Hankel matrices containing the inputs and outputs of the system. These projections can typically be calculated using basic tools as QR- and SVD-decompositions, eliminating the need for a costly optimization of a non convex cost function as in many predictor-error methods, and making the method inherently robust.

The obtained state-space representation (1) is not unique. Applying a basis transformation $x \rightarrow Tx$ and a corresponding transformation of the state space matrices, $(A,B,C,D) \rightarrow (TAT^{-1}, TB, CT^{-1}, D)$, the model (1) can be written in a multitude of forms, which all describe the same input-output behavior. A common representation in modal analysis is the so-called modal representation.

$$\begin{aligned} x_{k+1}^m &= \Lambda x_k^m + B^m u_k + w_k, \\ y_k &= C^m x_k^m + Du_k + v_k, \end{aligned} \tag{4}$$

where the system matrix Λ is diagonal and mainly consists of pairs of complex conjugated eigenvalues $\lambda, \bar{\lambda}$ being the poles of the system. For this to be possible the original system matrix *A* needs to be diagonalizable which is in practical applications usually the case. The modal characteristics of the structure under study can then easily be obtained from (6) as follows:

$$\begin{aligned} f_i &= \arg\left(\lambda_i \tfrac{T_s}{2\pi}\right), \\ d_i &= \frac{\ln\left(|\lambda_i|\right)}{\sqrt{\ln\left(|\lambda_i|\right)^2 + \arg\left(\lambda_i\right)^2}}, \\ v_i &= C^m(;,i), \end{aligned} \tag{5}$$

with f_i, d_i and v_i the resonance frequency, damping and mode shapes corresponding to the ith pole $\Lambda(i,i) = \lambda_i$. T_s is the sampling rate.

Another commonly used representation is the so-called Balanced representation (Obinata and Anderson, 2001). The idea of the Balanced representation is to decompose the controllabity and observability grammians of the model into principal components in order to evaluate the contributions of each mode to the overall input/output behavior of the model. The controllability Grammian *P* and observability Grammian *Q* can easily be obtained as solutions to the following Lyapunov equations:

$$\begin{aligned} APA^T + P - BB^T &= 0 \\ A^T QA + Q - C^T C &= 0 \end{aligned} \tag{6}$$

The key property of a balanced realization is that a state transformation $x^b = Tx$ and a corresponding similarity transformation $(A^b, B^b, C^b, D) = (TAT^{-1}, TB, CT^{-1}, D)$ is selected such that the controllability and observability grammians are both equal to a diagonal matrix Σ.

$$\begin{aligned} A^b \Sigma A^{b^T} + \Sigma - B^b B^{b^T} &= 0 \\ A^{b^T} \Sigma A^b + \Sigma - C^{b^T} C^b &= 0 \end{aligned} \qquad (7)$$

The larger a diagonal entry of the grammians, the bigger the contribution of the corresponding entry of the state vector to the overall input/output behavior of the model. The diagonal entries are therefore usually sorted on the diagonal in descending order. The so called concept of Balanced model reduction is then nothing else than the removal of the last entries of the state. More concretely, if the balanced system matrices are partitioned as follows:

$$A^b = \begin{bmatrix} A_{11}^b & A_{12}^b \\ A_{21}^b & A_{22}^b \end{bmatrix} \qquad (8)$$

$$B^b = \begin{bmatrix} B_1^b \\ B_2^b \end{bmatrix}, \quad C^b = \begin{bmatrix} C_1^b & C_2^b \end{bmatrix} \qquad (9)$$

The reduced model would be $(A_{11}^b, B_1^b, C_1^b, D)$.

3. METHODS FOR MODE DISCRIMINATION

3.1. Introduction

In this section we will describe some techniques to remove spurious modes from a model of the form (1). We will thereby make extensive use of the modal and balanced representations of a system, as introduced in section 2.

3.2. H_2 and H_∞ modal truncation

A first naive approach would be to write the model in its modal form, as given in (4), remove a certain mode, and assess the "damage" done to the model in H_2 and H_∞ norm. Hence, if the full order model is called H_{full}, and H_{reduced} is the lower order model formed by removing the complex conjugated poles λ_i and $\overline{\lambda}_i$ from (4), the following expressions are evaluated:

$$\left\| H_{\text{full}} - H_{\text{reduced}} \right\|_2, \quad \left\| H_{\text{full}} - H_{\text{reduced}} \right\|_\infty, \qquad (10)$$

where

$$H_{\text{full}}(z) - H_{\text{reduced}}(z) = H_i(z) = \begin{bmatrix} C(:,i) & \overline{C(:,i)} \end{bmatrix} \left(zI_2 - \begin{bmatrix} \lambda_i & 0 \\ 0 & \overline{\lambda}_i \end{bmatrix} \right)^{-1} \begin{bmatrix} B(i,:) \\ \overline{B(i,:)} \end{bmatrix}. \qquad (11)$$

The expressions in (10) can be calculated as:

$$\begin{aligned} \left\| H_{\text{full}} - H_{\text{reduced}} \right\|_2 &= Tr\left(C(:,i) P_i C(:,i)^T \right), \\ \left\| H_{\text{full}} - H_{\text{reduced}} \right\|_\infty &= \max_\omega \overline{\sigma}\left(H_i(e^{j\omega}) \right), \end{aligned} \qquad (12)$$

where P_i is the controllability matrix of the second order model (11), and can be obtained by solving a Lyapunov equation as in (6). Numerical procedures are widely available for the calculation of the infinity norm (Boyd *et al.*, 10). After repeatedly calculating (12), once for each mode, the distance measures obtained are divided by their maximal value, this is, the maximal distance that can be obtained by removing 1 mode. The result is a number between 0 and 1 for each mode, and each criterium (H_2 and H_∞) which will be used as a significance parameter describing the importance of the mode in section 4.

In general it seems reasonable to assume that the more important a mode is, the bigger will be the influence of its removal from the model, and hence its significance parameter. Furthermore it is shown in (Jonckheere, *1984*) that for nearly undamped structues, the grammians of the modal form are almost diagonal, meaning that the modal and the balanced form are "close" to each other in some sense. Hence, for such structures, modal truncation makes perfect sense since it leads to similar results as balanced model reduction. In many practical cases, however, the structure under study is not nearly undamped, and modal truncation may lead to an inadequate rejection of spurious modes due to phenomena as mode coupling, which make it very hard to assess the importance of a mode by examining a single second order subsystem. This will also be shown in the examples in section 4. In order to draw better conclusions for complicated structures, we will therefore look into the connection between the balanced and the modal representation of the identified model.

3.3. Connection between the balanced and the modal form

Since the balanced and the modal form are both representations of the same model, there is always a similarity transformation T linking one form to the other:

$$\begin{aligned} \Lambda &= TA^b T^{-1}, \\ B^m &= TB^b, \\ C^m &= C^b T^{-1}, \\ D^m &= D^b. \end{aligned} \qquad (13)$$

From $\Lambda = TA^b T^{-1}$ it follows that the diagonal elements of Λ are a linear combination of the entries of A^b, where we know that the entries of A^b that are most relevant for the input/output behavior of the model in the sense of Moore are situated in its upper left part. A formal way to exploit this fact in a mode selection context is to replace A^b with a significance matrix S of the same dimensions, where the elements of S give a measure of the importance of the corresponding entries in A^b, eg.

$$S = \begin{bmatrix} n & n-1 & n-2 & \cdots \\ n-1 & n-1 & n-2 & \cdots \\ n-2 & n-2 & n-2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \qquad (14)$$

and inspect the diagonal elements of $|T| \cdot S \cdot |T^{-1}|$, with $|T|$ the elementwise absolute value of T, to obtain a measure for the significance of the corresponding pole in Λ. Again, the significance parameters are rescaled so as to lie between 0 and 1.

3.4. Continuous extension to balanced truncation

Closely related to the former technique is the concept of a continuous extension to balanced truncation. Instead of truncating the model and completely removing the last entries of the state, one might opt to change the balanced system matrix,

$$A^b = \begin{bmatrix} a_{11}^b & a_{12}^b & \cdots & a_{1n}^b \\ a_{21}^b & a_{22}^b & \cdots & a_{2n}^b \\ \vdots & \vdots & & \vdots \\ a_{n1}^b & a_{n2}^b & \cdots & a_{nn}^b \end{bmatrix} \qquad (15)$$

and introduce a parameter ε to continuously remove the last entries of the state, eg. as follows:

$$\tilde{A}^b(\varepsilon) = \begin{bmatrix} a_{11}^b & \varepsilon a_{12}^b & \cdots & \varepsilon^{n-1} a_{1n}^b \\ \varepsilon a_{21}^b & \varepsilon a_{22}^b & \cdots & \varepsilon^{n-1} a_{2n}^b \\ \vdots & \vdots & & \vdots \\ \varepsilon^{n-1} a_{n1}^b & \varepsilon^{n-1} a_{n2}^b & \cdots & \varepsilon^{n-1} a_{nn}^b \end{bmatrix} \qquad (16)$$

While ε is continuously decreased, starting from one, the influence on the system poles can be assessed, eg. by using the euclidean distance measure in the complex plane. Again it is assumed that modes that are mainly related to the least important elements of A^b will be influenced more and can hence be classified as spurious.

3.5. Pole/zero cancellations

Pole/zero cancellations, a zero of a rational entry in the transfer function matrix that is almost or completely equal to a system pole, rendering a mode nearly uncontrollable or unobservable with respects to some or all of the inputs and outputs, are not uncommon in models identified from vibrating structures, especially when a high modeling order is used, and are often an indication that the cancelled pole is spurious. It is however seldom a good idea to revert to measures as the distance between a pole and some nearby transfer function zeros as the basis of mode selection techniques, as lowly damped, weakly excited modes may well be accompanied by a nearby zero, even if the mode is quite important for the physical characteristics of the structure as a whole. In (Verboven *et al*. 2002), it was therefore proposed to take extra statistical information as the variance of a pole into account when examining pole/zero cancellations. One of the techniques described in this work is to construct a confidence region around every pole and count the number of transfer function zeros within this region. Such confidence regions, however, are not available if the model is obtained using subspace identification. It is however reasonably acceptable that if we started moving a pole in our model, the influence on the model as a whole would be inversely proportional to the unknown variance on the pole position. As a mode selection rule, similar to the one presented in (Verboven *et al*. 2002) we therefore propose to move each pole to its $m \times l$ closest transfer function zeros and assess the influence on the model as a whole in each of the $m \times l$ cases, eg. by calculating the sum of the 2- or infinity-norms of the differences between the adjusted and the original models. As usual, a significance parameter is obtained by dividing the distances so obtained by their maximal value resulting in a value between 0 and 1.

3.6. Cross correlations with SISO models

An obvious critertium, proposed in (Verboven et al. 2002), is to check to what extent poles of the full MIMO model (1) can be retrieved from smaller models obtained from individual or groups of input and output sequences. For our examples in section 4, we constructed l MISO models from the available observations, one for each output, and compared the poles so obtained with the ones from the full MIMO model using the standard euclidean distance measure in the complex plane. To speed up the procedure, the individual models can for instance be obtained using a fast ARX modeling procedure.

4. EVALUATION

In order to evaluate the techniques outlined in section 3, we applied them to two datasets. The first was generated by filtering white noise through a known twelfth order model and adding 40% of measurement noise to the simulated output so obtained. A second example involves observation data from an in flight flutter test of an aircraft. This dataset was also analyzed by the aircraft’s manufacturer to allow for a comparisson with our selection methods.

4.1. A simulation

Six dominant modes, obtained from a ground vibration test of an airplane were used to create a twelfth order model with 38 outputs, one known input and three unknown noise sources. White, zero mean, stationary, gaussian noise with unit variance was applied to the known, as well as the three unknown inputs in order to create 64 seconds of output data, sampled at 256 Hz. 40% of measurement noise was added and the output data together with the known input where thereafter used for identification using a robust N4SID subspace algorithm, described in (Van Overschee *et al.*, 1996), where the modelling order was set to 30. Frequencies, dampings, and significance parameters as returned by the different heuristic mode selection techniques are given in table 1, where the six true modes are located on top. The heuristic mode selection techniques are ordered from left to right in the same order as described in this paper, and the last column (sum) is nothing else than the sum of all the significance parameters for a certain mode. It is important to note here that a significance parameter smaller than 0.5 does not necessarily mean that the mode is unimportant. The significance parameters are mostly rescaled distances which means that their absolute value has little meaning. In a modal analysis context a proper way to remove spurious modes would be to sort the modes in descending order of significance and look for a sudden decrease in significance or, if not available, set a treshhold for the maximal number of modes you are willing to consider in your further analysis. From table 1 it is clear that the six true modes are correctly identified as the six most important ones by all heuristic techniques except for H_2, which can be explained by the effect of mode coupling of the modes around 5 Hz, where individual modes reinforce each-other so as to create a clear resonance, although the individual modal subsystems have a low H_2 norm.

4.2. In Flight flutter testing of an aircraft

67 seconds of measurement data, sampled at 256 Hz, obtained during in flight flutter tests of a fly-by-wire airplane equipped with 12 accelerometers and excited by white noise where identified using the same subspace algorithm as in the previous example, with the modelling order set to 40. As in the previous example, frequencies, dampings, and results for the different heuristic techniques are given in table 2. The most important modes, as given by the airplane's manufacterer are printed in bold, and a stabilization diagram is added for comparison in figure 1. Note that all modes are classified correctly, except for the mode at 3.897 Hz, printed in italic, which is stabilized in figure 1, but was not accepted as such by our algorithms. Further analysis revealed that the mode in question was extremely poorly excited during the test-flight, which clarifies its classification as insignificant. For a correct classification of such weak modes, a combination of the proposed techniques with an automatic analysis of the

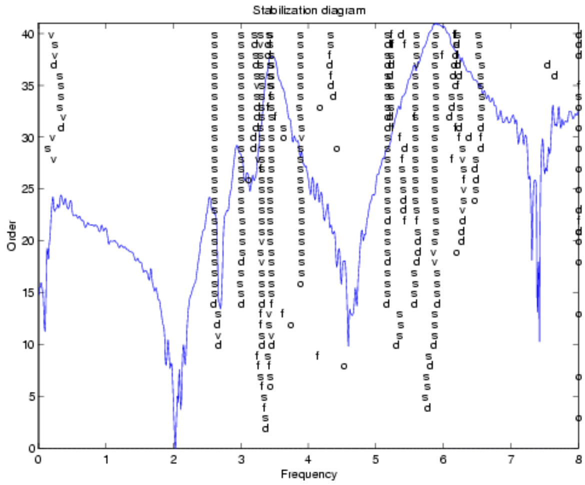

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Fig. 1. Stabilization diagram from in-flight flutter test measurements. 'f' is used for stable frequencies (1%), 'd' for stable damping ratios (5%) and 'v' for stable vectors (2%). If all conditions are satisfied, the pols is labeled as stable 's'.

stabilization diagram, as presented in (Vechhio *et al.*, 2003) might be useful

5. CONCLUSIONS

In this paper, several techniques were proposed for the removal of spurious mode from a previously identified model. A special property of the presented techniques is that they do not rely on statistical information, making them ideally suited for use with models obtained using subspace identification. Although the techniques presented are quite heuristic in nature, it was shown by a simulation and an example from the avionics inductry that in many cases, a distinction between true and spurious modes can effectively be made, even in the presence of large amounts of measurement noise. Problems may however occur if modes are very poorly excited.

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Table 1 Frequencies, dampings, and significance parameters for a simulation using a 12th order model. Modes are ordered according the the sum of their significance paramaters. The true modes are printed in bold.

Frequency	Damp	H2	Hinf	Balanced	Continuous	Pole/zero	Cross-corr.	Sum
3.169Hz	**0.69%**	**0.3565**	**0.1303**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**4.4867**
3.743Hz	**0.82%**	**0.2871**	**0.3605**	**0.9288**	**0.9288**	**0.7273**	**0.6563**	**3.8887**
4.627Hz	**1.30%**	**0.1401**	**0.1043**	**0.7485**	**0.7485**	**0.0853**	**0.1802**	**2.0069**
5.099Hz	**1.95%**	**0.2760**	**0.1951**	**0.7557**	**0.7557**	**0.1395**	**0.0778**	**2.1998**
5.885Hz	**1.27%**	**1.0000**	**1.0000**	**0.8474**	**0.8474**	**0.2756**	**0.2607**	**4.2311**
8.392Hz	**1.54%**	**0.1237**	**0.1107**	**0.6425**	**0.6425**	**0.0515**	**0.0673**	**1.6381**
6.161Hz	6.23%	0.2324	0.0322	0.5486	0.5486	0.0132	0.0076	1.3826
5.610Hz	7.48%	0.1752	0.0247	0.4871	0.4871	0.0059	0.0084	1.1885
0.541Hz	55.73%	0.0224	0.0042	0.3780	0.3779	0.0016	0.0048	0.7888
1.923Hz	13.29%	0.0139	0.0025	0.3368	0.3368	0.0006	0.0056	0.6961
9.030Hz	2.38%	0.0070	0.0019	0.2630	0.2630	0.0002	0.0061	0.5412
3.175Hz	9.30%	0.0195	0.0037	0.2277	0.2277	0.0001	0.0072	0.4859
3.490Hz	25.20%	0.0602	0.0099	0.1845	0.1844	0.0004	0.0026	0.4419
7.976Hz	11.05%	0.0121	0.0021	0.1516	0.1516	0.0002	0.0069	0.3245
3.414Hz	8.56%	0.0119	0.0035	0.1318	0.1318	0.0000	0.0071	0.2862

Table 2 Frequencies, dampings, and significance parameters for data obtained from an in-flight flutter test. Modes are ordered according the the sum of their significance paramaters. The true modes are printed in bold. The status of the mode at 3.897 Hz, printed in italic, is unsure. It is well stabilized in the stabilization diagram, but not recognized as such by the selection procedures. Further analysis revealed that the mode in question is very poorly excited.

Frequency	Damping	H2	Hinf	Balanced	Continuous	Pole/zero	Cross-corr.	Sum
3.296Hz	**4.92%**	**1.000**	**1.000**	**0.948**	**0.244**	**1.000**	**0.473**	**4.665**
3.439Hz	**2.75%**	**0.692**	**0.338**	**1.000**	**0.590**	**0.678**	**1.000**	**4.298**
5.901Hz	**4.09%**	**0.748**	**0.891**	**0.914**	**1.000**	**0.499**	**0.241**	**4.293**
2.609Hz	**2.53%**	**0.017**	**0.023**	**0.859**	**0.187**	**0.163**	**0.847**	**2.096**
5.590Hz	**2.77%**	**0.183**	**0.104**	**0.836**	**0.059**	**0.144**	**0.289**	**1.615**
5.179Hz	**2.73%**	**0.112**	**0.072**	**0.790**	**0.096**	**0.066**	**0.165**	**1.302**
3.006Hz	**3.05%**	**0.032**	**0.020**	**0.715**	**0.124**	**0.072**	**0.212**	**1.175**
6.597Hz	5.44%	0.035	0.018	0.548	0.157	0.043	0.076	0.877
3.418Hz	4.22%	0.114	0.066	0.436	0.060	0.042	0.078	0.796
5.259Hz	4.98%	0.141	0.064	0.429	0.072	0.045	0.036	0.786
6.008Hz	2.99%	0.059	0.055	0.479	0.074	0.018	0.090	0.775
3.220Hz	3.93%	0.067	0.035	0.489	0.078	0.015	0.013	0.697
6.416Hz	1.24%	0.008	0.023	0.437	0.075	0.003	0.134	0.679
6.118Hz	6.27%	0.093	0.055	0.189	0.215	0.029	0.039	0.619
4.813Hz	4.60%	0.013	0.006	0.368	0.069	0.028	0.121	0.605
3.897Hz	***4.11%***	***0.005***	***0.013***	***0.124***	***0.052***	***0.000***	***0.226***	***0.421***
4.514Hz	1.72%	0.002	0.001	0.245	0.057	0.001	0.082	0.388