Identification and Modeling of a Dynamical System

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Abstract

This paper first gives an introduction in the main aspects of existing time-domain methods for identifying linear continuous models of dynamical systems form sampled input-output data. The second part of the paper demonstrates these approaches via a real data example.
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1   Introduction

System modeling plays a fundamental role in modern engineering, as it is typically the first step in a design cycle. However, it is also one of the more complicated tasks in engineering, as it is more closely connected with reality (in contrast with the tasks of analysis and design, which are usually performed on a mathematical model). In some cases, one could build a so-called white-box model based on first principles (Newton's law, Kirchhoff's laws, laws of thermodynamics, reaction kinetics, etc.), but in many cases such models will be overly complex and possibly even impossible to obtain in reasonable time due to the complex nature of many systems and processes. A

much more common approach is therefore to start from measurements of the behavior of the system and the external influences (inputs to the system) and try to determine a mathematical relation between them without going into the details of what is actually happening inside the system. This approach is called system identification. Two types of models are common in the field of system identification:

- "Grey box" model: Sometimes the model obtained by invoking the first principles is incomplete because the value of some parameter is missing. For instance, a planet is subject to the gravitation law but its mass is unknown. In this case, it is necessary to collect experimental data and proceed to a tuning of the unknown parameters until the outputs predicted by the model match the observed data, The internal structure of the box is only partially known (there are grey zones).

- "Black box" model: When either the internal structure of the system is unknown or there are no first principles available, the only chance is to collect data and use them to guess the links between inputs and outputs. Black box modeling is useful to deal with very complex systems where the white box approach would be time consuming and expensive (an example: modeling the dynamics of an internal combustion engine in order to develop the idle-speed controller).

This paper is organized as follows. In section 2 we give an introduction to system identification. We discuss also methods for identifying linear models. Section 3 gives a application where these methods are applied. The results are given in section 4 and summarized in section 5.

2   Identifying Linear Multivariate Models

For this paper, linear, discrete-time, stochastic models are of interest. The books by Ljung [1] and Soderstrom and Stoica [2] are excellent references for the classical system identification theory. A summary of the relevant issues is presented within this section.

Identifying an empirical process model requires exciting the process to collect experimental data, and using these data for estimating a mathematical model. After the experimental data have been collected, the user makes three important selections: whether to use a parametric or nonparametric model, the model structure (i.e., the type of model and the model order), and the method for estimating parameters.

2.1 Introduction to System Identification

When attempting to identify a model of a dynamical system it is common practice to follow the procedure described in figure 1.
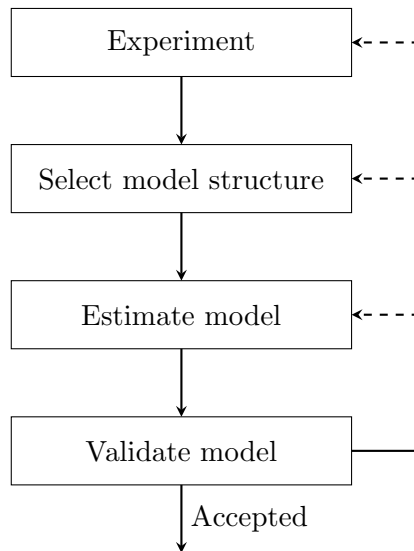
Figure 1: The basic system identification procedure.

Experiment   The purpose of the experiment is to collect a set of data that describes how the system behaves over its entire range of operation. The idea is to vary the input(s), u, and observe the impact on the output(s), y (see Figure 2).
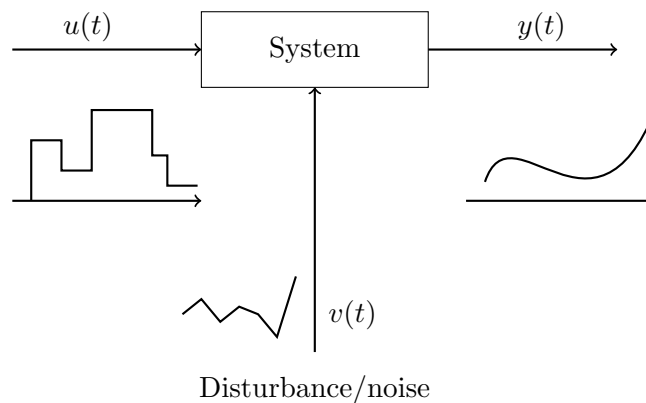
Figure 2: An input is applied to the system and it is observed how the output is affected.

The data set of corresponding inputs and outputs $\mathcal{D}_n = \{(u(t), y(t)), t = 1, \ldots, n\}$ is later used for inferring a model of the system.

Model structure selection. A model structure is a set of candidate models. That is, a set inside which one should search for a model. On a general level the problem of selecting a model structure is twofold:

1. Select a "'family"' of model structures considered appropriate for describing the system, e.g., linear model structures, Hammerstein models, wavelets, Support Vector Machines. This paper concentrates on the linear model structures.

2. Select a subset of the chosen family of model structures. In the family of linear structures this can for instance be an ARX(d,p,m) model structure where (d, p, m) signifies a time delay of m sampling periods and that the present output depends on d past outputs and p past inputs. Both input-output models and state space models will be considered, although most attention will be given to the former.

Estimate model. Estimate model. Once a set of candidate models has been chosen, the next step is to pick one particular model from this set. One will typically pick the model that performs best according to some type of criterion. This criterion can be formulated in many different ways but should ideally relate to the intended use of the model. The most common strategy is to pick the model that provides the best k-step, $k \in \mathbb{N}_0$, ahead predictions in terms of the smallest expected squared error between observed outputs and predictions.

Validation. When a model has been estimated it must be evaluated to investigate whether or not it meets the necessary requirements. The validation is closely connected to the intended use of the model.

Going backwards in the procedure. The paths going from the validation block and back to the previous stages indicate that the procedure is executed in an iterative manner. It is necessary to go back in the procedure to determine a number of different models, to try out various model structures, and in the worst case even redo the experiment.

2.2 Model Structure Selection

2.2.1 Input-output equations

Let us consider the case where the input $u(t)$ is an m-dimensional vector and the output $y(t)$ is a p-dimensional vector. Probably the most simple input-output relationship is obtained by describing it as a linear difference equation:

$$y(t) + A_1 y(t-1) + \cdots + A_{n_a} y(t-n_a) = \\ B_1 u(t-1) + \cdots + B_{n_b} u(t-n_b) + v(t) \tag{1}$$

where the A_i are $(p \times p)$ matrices, the B_i are $(p \times m)$ matrices and $v(t)$ is the white noise term. According to Ljung [1] a system can be written as

$$y(t) = G(q^{-1})u(t) + H(q^{-1})v(t) \tag{2}$$

where G and H are transfer functions in the time delay operator, q^{-1}. The operator works on a signal in the following way:

$$q^{-1}x(t) = x(t-1). \tag{3}$$

The model structure (1) corresponds to the generalized model structure (2) with

$$G(q^{-1}) = A^{-1}(q^{-1})B(q^{-1}), \qquad H(q^{-1}) = A^{-1}(q^{-1}). \tag{4}$$

where

$$A(q^{-1}) = I + A_1 q^{-1} + \cdots + A_{n_a} q^{-n_a} \tag{5}$$
$$B(q^{-1}) = B_1 q^{-1} + \cdots + B_{n_b} q^{-n_b} \tag{6}$$

These are matrix polynomials in q^{-1}. The input-output relationship (1) is an ARX model, where AR refers to the autoregressive part and X to the extra input (exogeneous variable).

ARMAX model structure The basic disadvantage with (1) is the lack of adequate freedom in describing the properties of the disturbance term. One could add flexibility to that by describing the equation error as a moving average of white noise. This gives the model

$$y(t) + A_1 y(t-1) + \cdots + A_{n_a} y(t-n_a) = B_1 u(t-1) + \cdots \\ + B_{n_b} y(t-n_b) + v(t) + C_1 v(t-1) + \cdots + C_{n_c} v(t-n_c) \tag{7}$$

With

$$C(q^{-1}) = I + C_1 q^{-1} + \cdots + C_{n_c} q^{-n_c}, \tag{8}$$

the ARMAX model structure (7) corresponds to the generalized model structure (2) with

$$G(q^{-1}) = A^{-1}(q^{-1})B(q^{-1}), \qquad H(q^{-1}) = A^{-1}(q^{-1})C(q^{-1}). \tag{9}$$

Output Error model structure If one supposes that the relation between input and undisturbed output z can be written as a linear difference equation, and that the disturbances consist of white measurement noise, then we obtain the following description:

$$z(t) + F_1 z(t-1) + \cdots + F_{n_f} z(t - n_f) =$$
$$B_1 u(t-1) + \cdots + B_{n_b} z(t - n_b) \qquad (10)$$
$$y(t) = z(t) + v(t)$$

With

$$F(q^{-1}) = I + F_1 q^{-1} + \cdots + F_{n_f} q^{-n_f}, \qquad (11)$$

the model can be written as

$$y(t) = F^{-1}(q^{-1}) B(q^{-1}) u(t) + v(t). \qquad (12)$$

Comparing with the generalized model structure (2), $H(q^{-1}) = I$, which gives the natural predictor

$$\widehat{y}(t) = F^{-1}(q^{-1}) B(q^{-1}) u(t) = z(t). \qquad (13)$$

2.2.2 State space form

Another important class is state space representations, which have been popularized by the seminal work of Kalman (see, e.g., Kalman, 1963). They are of the form

$$x(t+1) = Ax(t) + Bu(t) + He(t) \qquad (14)$$
$$y(t) = Cx(t) + v(t) \qquad (15)$$

where $x \in \mathbb{R}^{n_x}$ is a vector containing the states of the system, $e \in \mathbb{R}^{n_x}$ is the vector of the state noise variables, $v \in \mathbb{R}^{n_y}$ is the vector of measurement errors and A, B, C and D are matrices of compatible dimensions. The state and the measurement noise vectors are assumed to be uncorrelated. A simple relationship between the state space form and the general input-output form exists [1].

3 Practical application: identification of a distillation column

As an example of a dynamic system, some parts of a distillation column are identified. This is a nonlinear system, but due to the need for linear models for many control techniques, a linear model will be identified. Such a model will only be valid around the working point where the column is identified.

3.1 Column

The column (figure 3 is about 6 m high and consists of 3 sections of about 1.5m with packing responsible for the separation). The content of the column is a binary mixture of methanol (CH_3OH, boiling point: 64.7 $°C$ (337.8K), molar mass: 32.04 g/mol, density: 0.7918 g/cm^3, liquid) and isopropanol (C_3H_8O, boiling point: 82.3 $°C$ (355K), molar mass: 0.785 g/mol, density: 0.785 g/cm^3, liquid).

The distillation column is a multiple input multiple output (MIMO) system with 4 manipulated variables (inputs) and 10 controlled variables (outputs). The manipulated variables are: reboiler power (Qr), feed (Fv), feed temperature (Qv) and distillate flow (Fd). The controlled variables (outputs) are distillate flow (Fd), feed flow (Fv) and nine temperatures: temperature at the top of the column (Tt); temperature in the center of every packing section, respectively Ts1, Ts2 and Ts3; temperature between section 1 and 2 (Tv2); temperature between the second and the third packing section (Tv1). This is where the feed enters the column; temperature in the reboiler of the column (Tb) and the temperature of the feed before heating (Tv0).

There is no on line measurement of the concentration of the distillate flow. The concentration can be measured by its refractive index on a refractometer. Because the concentration of the column can be derived from the temperatures, only a model with the measured inputs and outputs is made. Gibbs phase rule for binary vapor-liquid systems shows that if pressure, temperature and relative volatility is fixed, phase compositions are defined. In this column, only temperature can be changed, so if the temperature is known, the concentration is known [3]. To know the concentration of the column, an identification process similar as the one to make the model of the column with the measured inputs and outputs, can be done with the temperatures and the concentration, but will not be done in this work.

3.2 Excitation signals

The actuators and sensors are connected to a fieldpoint (National Instruments, controller interface: cFP-2100, I/O modules: cFP-AIO-610 and cFP-AI-110). A Labview program is written to control the actuators and register the variables.

As an excitation signal a PRBS (Pseudorandom binary sequence) signal is generated with constant intervals with a length of 400 seconds. From this signal, the first 10000 samples are used and applied to the system with a sample time of one second.
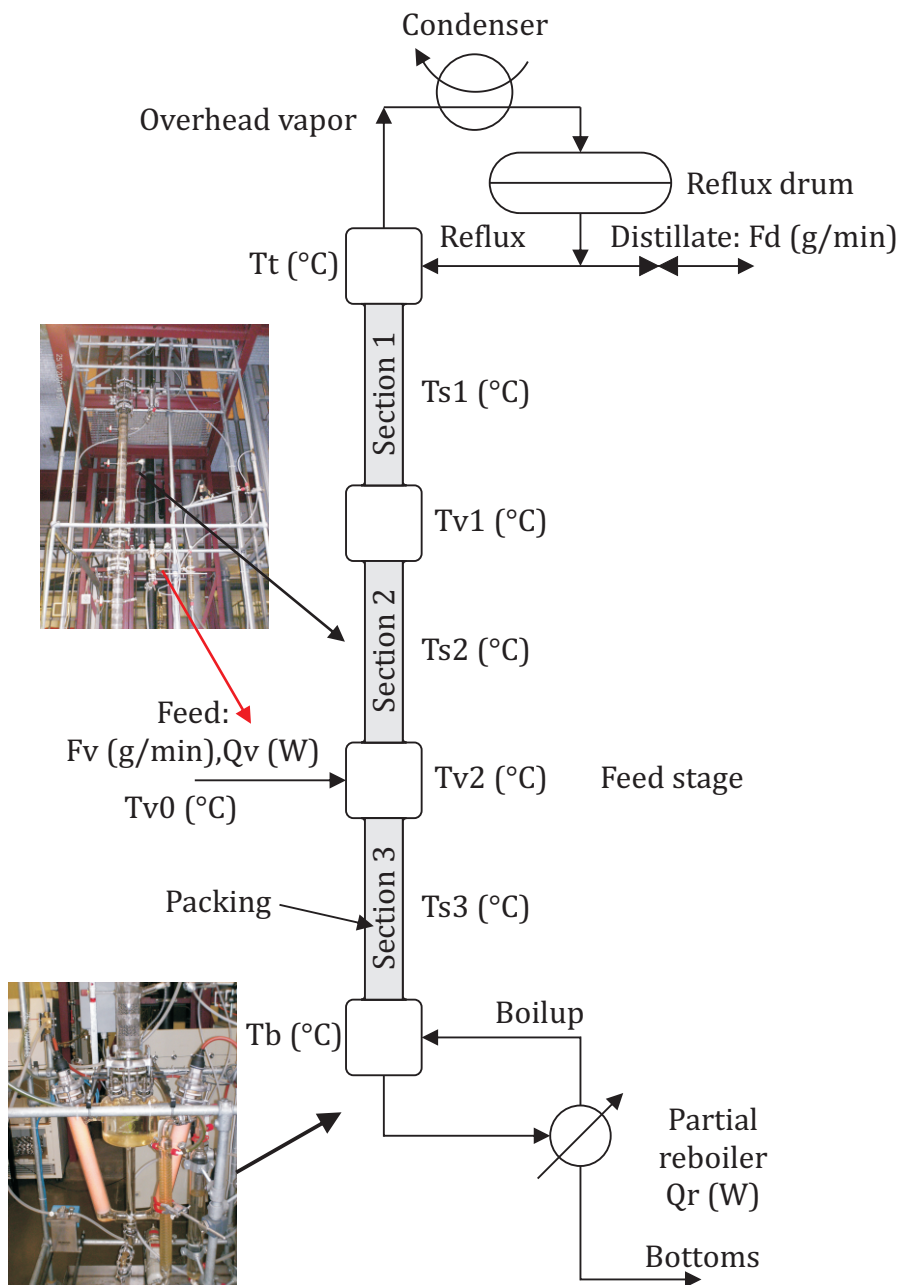
Figure 3: Schematical overview of the distillation kolom.

3.3 Experiment

Before the identification experiment was performed, the column was stirred to reach steady state. The steady state values were: reboiler power: 4500 W, feed flow: 150 g/min, feed temperature: 40 °C and distillate flow: 70 g/min. After 2 hours, the test signals where applied for 9620 seconds. The binary signal for the reboiler power was switched between 4200 and 4800 Watt. The feed (Fv) was manipulated from 125 to 175 g/min. The feed temperature (Qv) was manipulated between 145 and 175 Watt and the distillate flow (Fd) was manipulated form from 45 to 95 g/min during the experiment.

The registered signals are resampled to a sampling period of 5 seconds for use in the identification process.

4 Results

The two systems that will be identified are MISO (Multiple Input, Single Output) systems with 6 inputs and one output. The output is the top temperature (Tt) or the reboiler temperature (Tr) of the column . The inputs are in both cases: feed flow (Fv, g/min), feed heating (Qv, W), the temperature of the feed before heating (Tv0, °C), the temperature of the feed (Tv2, °C), distillate flow (Fd, g/min) and the reboiler power (Qr, W). With this system, different linear system models are fitted. The first 8000 data points are used for estimation. The next 1920 points are used for validation. The results of the model estimations are showed in figure 4 and Table 1.

To compare the different models a fit factor is defined [4]:

$$Fit = 100. \left(1 - \frac{\left| Y - \hat{Y} \right|}{\left| Y - \bar{Y} \right|} \right) \qquad (16)$$

Where $\hat{Y}$ is the predicted output, $\bar{Y}$ is the mean value of Y and Y is the measured output.

As suggested in paragraph 2.2, ARX models are the simplest models and are first tried. The next model types tried, are ARMAX and output error models. The last type tried for this work is the state space class. As can be seen in table 1, the amelioration of the fit is very less when taking more complex models. Output error models are not even able to produce a satisfying fit. As a rule of thumb, the simplest model structure is taken for the final model. Thats why ARX models are taken in both cases.

Table 1: The quality of some models for a 1 step ahead prediction and a 12 step (one minute) ahead prediction of the model. The fit factor is defined by 16. ARX(n_a,n_b,n_k) is an ARX model with a time delay of n_k samples and and a present output depending on n_b past inputs and n_a past outputs. AMX(n_a,n_b,n_c,n_k) is an ARMAX model with n_a,n_b and n_c the orders of the matrices A, B and C in 7 and n_k a time delay of n_k samples. n4s1 and n4s3 are both state space models with respectively order 1 and 3.

(a) Results for the reboiler temperature.

model	reboiler temperature	
	1 step ahead	12 step ahead
arx(3,5,6)	**86.28**	**76.46**
arx(3,2,10)	86.2	72.91
amx(3,2,5,6)	86.16	74.17
amx(3,1,1,0)	86.09	70.22
amx(3,5,5,6)	86.01	74.9
amx(3,1,1,9)	85.98	67.77
n4s1	85.11	64.28

(b) Results for the top temperature.

model	top temperature	
	1 step ahead	12 step ahead
arx(10,3,8)	**98.74**	**72.35**
arx(5,1,8)	98.42	71.45
arx(5,2,10)	98.42	71.67
arx(3,1,8)	98.42	71.16
amx(3,1,1,8)	98.38	70.79
amx(5,2,1,9)	98.37	69.71
arx(3,1,2,8)	98.36	70.17
n4s3	98.32	71.36
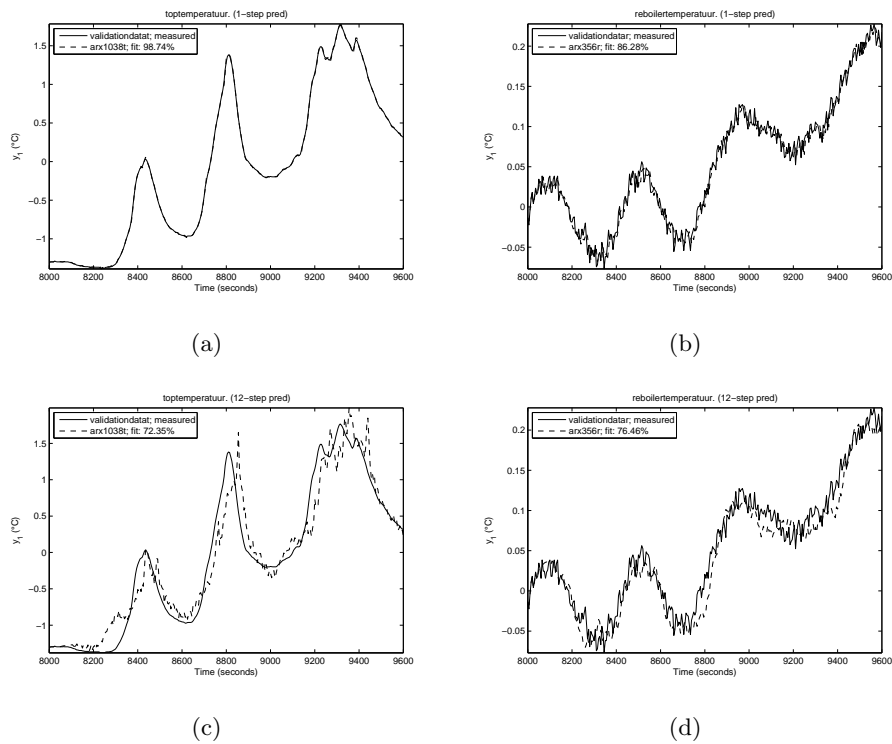
Figure 4: Overview validation plot of the best fitting model. On the left the ARX model for the top temperature (Tt) is shown. The model used for these plots is ARX(10,3,8). On the right the validation of the reboiler temperature (Tb) is shown. The mode used for these plots is ARX(3,5,6). (a) and (b) are a 1 step ahead prediction of the model. (c) and (d) are a 12 step ahead prediction of the model.

The models will be used to predict the behavior of the temperature of the column. Therefor the k-step ahead predictions are of more importance than the simulated output.

The final model chosen for the reboiler temperature is ARX model with $n_a = 3$, $n_b = 5$ and a time delay $n_k = 6$. This ARX model is converted to a state space model with 35 states. After model reduction 10 states are left. The final model chosen for the top temperature is also an ARX model with $n_a = 10$, $n_b = 3$ and a time delay $n_k = 8$. This ARX model is converted to a state space model with 52 states. After model reduction, also is this case 10 states are left.

5 Discussion

This article illustrates linear system identification techniques applied on a nonlinear dynamic system. Two ARX models are estimated. These models can be used for prediction of the top and reboiler temperature of the column. Short time prediction will be very good. Long time prediction will result in a bigger deviation.

In the future other input signals will be applied to the column, for instance Gaussian noise and sum-of-sinusoid signals. These signals have a richer frequency spectrum and will provide more information which results in better models. Nonlinear models can be fitted too, but will not be used because this work must result in a linear model predictive controller for this distillation column.

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