Globally Optimal ARMA Model Identification is an Eigenvalue Problem

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Abstract

The identification of ARMA models emerges in various application domains [1], e.g., in modeling industrial processes and financial time series, anomaly detection, and time series prediction. Moreover, the ARMA model structure is an important building block for more sophisticated models [3], e.g., ARMAX and ARIMA models. Although numerous techniques to identify ARMA models already exist, none of these methods guarantees to find the globally optimal model parameters. In our poster, we tackle this hiatus and propose a new approach to find the globally optimal least-squares parameters of ARMA models.

The identification of ARMA models corresponds to a multivariate polynomial optimization problem. Although typically solved via nonlinear numerical optimization techniques, we approach this optimization problem from a linear algebra point of view and translate, via the method of Lagrange, the optimization problem into a (potentially large) system of multivariate polynomial equations [2]. This system corresponds to a set of difference equations. Consequently, the null space of the Macaulay matrix (constructed from the coefficients of the multivariate polynomials), which is spanned by Vandermonde-like vectors constructed from the roots, is a multidimensional observability matrix. The roots of the system, of which at least one yields the globally optimal parameters of the ARMA model, follow from the construction of an autonomous multidimensional (nD) linear state space model (realization theory) and an eigenvalue calculation.

In essence, this approach reformulates the globally optimal identification of ARMA models as an eigenvalue problem. A lot of interesting research still lies ahead, but preliminary insights give cause for great optimism.

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