Globally Optimal Least-Squares ARMA Model Identification is an Eigenvalue Problem

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1 Introduction

Autoregressive moving-average (ARMA) models regress an observed output sequence on its own lagged values and on a linear combination of unobserved, latent input samples [1]. They emerge in a wide variety of domains, e.g., in modeling industrial processes, financial time series, or smart utility grid applications (electricity, water, etc.). Moreover, the ARMA model structure is an important building block for more sophisticated models, e.g., ARMA models with exogenous inputs (ARMAX) and autoregressive integrated moving-average models (ARIMA). Although numerous identification techniques already exist, most of them rely on nonlinear numerical optimization and do not guarantee to find the global optimum. We tackle and resolve this hiatus and identify ARMA models exactly, i.e., find the globally optimal least-squares ARMA model parameters, by solving an eigenvalue problem.

2 Research methodology

An ARMA model combines a regression of the output sample $y_k \in \mathbb{R}$ on its own lagged values y_{k-i} with a linear combination of unobserved, latent inputs $e_{k-j} \in \mathbb{R}$ [1]:

$$\sum_{i=0}^{n_a} \alpha_i y_{k-i} = \sum_{j=0}^{n_c} \gamma_j e_{k-j}, \tag{1}$$

where n_a and n_c are the orders of the autoregressive and moving-average part, respectively. Without loss of generality, we fix $\alpha_0 = \gamma_0 = 1$. The identification of ARMA models corresponds to a multivariate polynomial optimization problem and searches, for given data $y \in \mathbb{R}^N$, the unknown model parameters α_i and γ_j, $\forall i = 1, \ldots, n_a$ and $\forall j = 1, \ldots, n_c$. Hence, it minimizes the squared 2-norm of the unknown latent input vector $e \in \mathbb{R}^{N-n_a+n_c}$, subject to the ARMA model structure:

$$\begin{aligned} &\min_{a,c,e} \|e\|_2^2 \\ &\text{s.t. } T_a y = T_c e \end{aligned} \tag{2}$$

The model matrices T_a and T_c are banded Toeplitz matrices of appropriate dimensions in the parameters α_i and γ_j. Although typically solved via nonlinear numerical optimization techniques, we approach this optimization problem from a linear algebra point of view and find the globally optimal model parameters by solving an eigenvalue problem.

After rewriting the cost function, we obtain, via the first order optimality conditions, a system of multivariate polynomial equations, in which most variables appear linearly. This system corresponds to a multiparameter eigenvalue problem (MEP), which we solve using the block Macaulay matrix (an extension of the ordinary Macaulay matrix for MEPs). Its null space is a multidimensional observability matrix with a multi-shift-invariant structure (see Dreesen et al. [2]). We apply multidimensional realization theory to exploit this structure and to set up an eigenvalue problem in that null space, of which the eigenvalues correspond to the roots of the system, hence, giving us the globally optimal least-squares model parameters.

3 Presentation outline

In our presentation, we will explain how we can find the system of multivariate polynomial equations and set up, using the new block Macaulay matrix, the eigenvalue problem, of which one of the eigenvalues corresponds to the globally optimal least-square model parameters. Furthermore, the presentation will elaborate on this block Macaulay matrix and the multi-shift-invariant structure of its null space.

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References

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