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SOURCE LOCALIZATION AND SIGNAL RECONSTRUCTION IN A REVERBERANT FIELD USING THE FDTD METHOD

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ABSTRACT

Numerical methods applied to room acoustics are usually employed to predict the sound pressure at certain positions generated by a known source. In this paper the inverse problem is studied: given a number of microphones placed in a room, the sound pressure is known at these positions and this information may be used to perform a localization and signal reconstruction of the sound source. The source is assumed to be spatially sparse meaning it can be modeled as a point source. The finite difference time domain method is used to model the acoustics of a simple two dimensional square room and its matrix formulation is presented. A two step method is proposed. First a convex optimization problem is solved to localize the source while exploiting its spatial sparsity. Once its position is known the source signal can be reconstructed by solving an overdetermined system of linear equations.

Index Terms— Room acoustics, FDTD, source localization, source reconstruction, sparse approximation

1. INTRODUCTION

Source localization and signal reconstruction represent a challenge for researchers in many fields. One of the most popular approaches for source localization is beamforming of which robustness in a reverberant environment is still a topic of research [1]. Source signal reconstruction is also a challenging problem that has been studied in the context of dereverberation [2].

In [3] a method was introduced for the estimation of a static field using wireless sensor network measurements combined with the finite element method (FEM). The source was assumed to be a point source and its spatial sparsity was exploited. It was shown that the field could be accurately estimated by solving a convex optimization problem with ℓ^1-norm regularization. A similar approach involving the wave equation was proposed by other authors: here multi-channel microphone measurements combined with greedy algorithms were used to localize a sound source in a two-dimensional room with a frequency domain approach using the FEM [4] or with a time domain approach using finite differences [5].

In this paper a new method is presented that aims not only to localize the sound source but also to reconstruct the source signal once its position is determined. In contrast to [3, 4] the finite difference time domain (FDTD) method is used to model the sound field. This is a method which is less computationally expensive than the FEM and which is a subject of intensive study [6, 7]. Moreover, convex optimization is employed instead of the greedy algorithms used in [4, 5].

The paper is organized as follows: In Section 2 the FDTD method is briefly reviewed. In Section 3 the matrix formulation of the FDTD method derived. In Section 4 the two step method for source localization and signal reconstruction is described. Finally, in Session 5 simulation results are shown.

2. THE FINITE DIFFERENCE TIME DOMAIN METHOD

In this section the FDTD method is briefly introduced. The sound field in a two-dimensional room can be described by the following partial differential equation (PDE) [8]:

$$
\begin{aligned}
&\text{PDE} \quad \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = s \text{ on } \Omega \times \tau \\
&\text{BCs} \quad \frac{\partial p}{\partial t} = -c\xi \nabla p \cdot \mathbf{n} \text{ on } \partial\Omega \times \tau \\
&\text{ICs} \quad p(x, y, t_0) = \tilde{p}_0 \text{ and } \frac{\partial p}{\partial t}(x, y, t_0) = \tilde{u}_0 \text{ on } \Omega
\end{aligned}
\tag{1}
$$

Here, $\Omega \in \mathbb{R}^2$ represents the spatial domain, which defines the room geometry, τ represents the temporal domain and

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$p(x, y, t)$ and $s(x, y, t)$ represent the sound pressure and the source signal, respectively, which are both functions $p, s : \Omega \times \tau \to \mathbb{R}$, with $(x, y) \in \Omega$, $t \in \tau$ and $\Omega \times \tau \subset \mathbb{R}^3$. Notice that all the results presented in this paper can be easily extended to the three-dimensional case.

The boundary conditions (BCs) are used to model acoustical properties of the walls which introduce damping in the system. ξ defines the impedance of the walls and is assumed here to be frequency independent. The operator ∇ is defined as $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ and $\mathbf{n}$ is the normal vector with respect to the boundaries. The initial conditions (ICs) provide the initialization of the problem, both in the sound pressure and its time derivative, namely the initial particle velocity $\tilde{u}_0$. Notice that $\tilde{u}_0$ and $\tilde{p}_0$ are defined in $\Omega \subset \mathbb{R}^2$.

Finding the solution p of the problem (1) for a given s is often a difficult task. Numerical approaches have to be used and these usually consist in discretizing the domain $\Omega \times \tau$ in order to reduce the problem to a linear system of equations. In the FDTD method this discretization is performed in a simple fashion: p and s are sampled uniformly in space and time,

$$p(x, y, t) = p(lX, mY, nT) = p^n_{l,m}, \tag{2}$$

where X and Y represent the spatial steps with respect to the x and y axis respectively and T the time step. The same notation is used for the discretized source term $s^n_{l,m}$ and X is set to be equal to Y.

The simplest FDTD scheme is obtained by approximating the space and time second order derivatives as follows, e.g. for the time derivative:

$$\frac{\partial^2 p}{\partial t^2} \approx \frac{p^{n+1}_{l,m} - 2p^n_{l,m} + p^{n-1}_{l,m}}{T^2}. \tag{3}$$

By substituting these approximations in the wave equation (1) it is possible to write the *standard leapfrog* (SLF) scheme update equation

$$\begin{aligned} p^{n+1}_{l,m} = \lambda_c^2 (p^n_{l+1,m} + p^n_{l-1,m} + p^n_{l,m+1} + p^n_{l,m-1}) + \\ 2(1 - 2\lambda_c^2) p^n_{l,m} - p^{n-1}_{l,m} + s^n_{l,m}, \end{aligned} \tag{4}$$

where $\lambda_c = cT/X$ is the Courant number.

The update equation (4) reveals the iterative nature of the FDTD method. The sound pressure at time index $n+1$ is evaluated by using samples from the previous two time indices. In particular, multiple spatial samples of the sound pressure at time index n are employed; in the SLF scheme only the axially neighboring samples are used.

The Courant number is an important parameter in terms of stability and numerical errors [6]. For the SLF scheme in a two-dimensional case the Courant number is chosen to be $1/\sqrt{2}$. Lower values increase the numerical errors while higher values make the resulting solution unstable. This implies that the resolution in time and space are inherently connected by the maximum of the achievable Courant number.

Numerical errors in the FDTD method result in dispersion, i.e., waves travel with different speed depending on their frequency and direction in space. The most common approach to reduce numerical errors is to use a different scheme to discretize the derivatives of the wave equation. The general update equation is

$$\begin{aligned} p^{n+1}_{l,m} = d_1 \left(p^n_{l+1,m} + p^n_{l-1,m} + p^n_{l,m+1} + p^n_{l,m-1} \right) + \\ d_2 (p^n_{l+1,m+1} + p^n_{l+1,m-1} + p^n_{l-1,m+1} + p^n_{l-1,m-1}) + \\ d_3 p^n_{l,m} - p^{n-1}_{l,m} + s^n_{l,m} \end{aligned} \tag{5}$$

where the parameters d_1, d_2 and d_3 define the scheme (e.g., for SLF $d_1 = 1/2$ and $d_2 = d_3 = 0$). A collection of the currently known stable schemes can be found in [6].

The update equation (5) must be modified at the boundaries, as it would require samples at grid points that lie outside the domain. These points are known as *ghost points* and can be used to model the BCs in (1). The modified update equation that has to be used for the boundary grid points is then given (e.g. for a right wall) by

$$\begin{aligned} (1 + \lambda_c/\xi_x) p^{n+1}_{l,m} = d_1 (2p^n_{l-1,m} + p^n_{l,m+1} + p^n_{l,m-1}) \\ 2d_2 (p^n_{l-1,m+1} + p^n_{l-1,m-1}) + d_3 p^n_{l,m} \\ - (1 - \lambda_c/\xi_x) p^{n-1}_{l,m}. \end{aligned} \tag{6}$$

Note that when a corner is present more grid points need to be eliminated. The update equation (5) can be further modified to include frequency-dependent boundaries using a digital filter at each boundary point [6].

Finally, initial conditions must be specified as well. The first-order derivative is approximated with a finite difference,

$$\frac{p^{+1}_{l,m} - p^{-1}_{l,m}}{2T} = \tilde{u}^0_{l,m}, \tag{7}$$

and $p^0_{l,m} = \tilde{p}^0_{l,m}$.

3. MATRIX FORMULATION OF FDTD

In Section 2 it has been shown that the wave equation can be converted into a set of linear equations with FDTD. Usually this linear system is solved iteratively. However, since this set of equations will be used as a constraint in an optimization problem, it is more convenient to write it in the following manner:

$$\mathbf{B}\mathbf{p} = \mathbf{s} \tag{8}$$

where $\mathbf{B} \in \mathbb{R}^{N_X^2(N_T+2) \times N_X^2(N_T+2)}$ is a matrix containing geometry and boundaries information and $\mathbf{p}$ and $\mathbf{s} \in \mathbb{R}^{N_X^2(N_T+2)}$ are a vector containing respectively sound pressure and source signal samples at all positions and time samples. Here, N_X indicates the number of spatial samples along one axis (in the following it is assumed this is the same for the x and y axis) and $N_T + 2$ is the number of time samples plus 2 initial conditions.

The vector $\mathbf{p}$ is defined as follows:

$$\mathbf{p} = [\mathbf{p}_{-1}^T, \mathbf{p}_0^T, \dots \mathbf{p}_{N_T}^T]^T, \tag{9}$$

where $\mathbf{p}_n$ is the vector containing all the sound pressure samples at time indices n. The vector $\mathbf{s}$ has the same structure but includes also the initial conditions

$$\mathbf{s} = [\tilde{\mathbf{u}}_0^T, \tilde{\mathbf{p}}_0^T, \mathbf{s}_1^T, \dots \mathbf{s}_{N_T}^T]^T. \tag{10}$$

The update equation (5) can be written as:

$$p_{l,m}^{n+1} - \mathbf{p}_n^T \mathbf{a}_{l,m} + p_{l,m}^{n-1} = s_{l,m}^n \tag{11}$$

where $\mathbf{a}_{l,m} \in \mathbb{R}^{N_X^2}$ is a vector that selects the samples at position (l, m) and its neighbor points. Writing $\mathbf{p}_n$ as

$$\mathbf{p}_n = [\dots \overbrace{p_{l,m-1}^n, p_{l+1,m-1}^n \dots}^{N_X} p_{l,m}^n, p_{l+1,m}^n \dots]^T, \tag{12}$$

it can be seen that $p_{l,m-1}^n$ is N_X samples away from $p_{l,m}^n$, and so if $\mathbf{p}_n[i] = p_{l,m}^n$ then $\mathbf{p}_n[i + N_X] = p_{l,m+1}^n$ and $\mathbf{p}_n[i - N_X] = p_{l,m-1}^n$. Hence $\mathbf{a}_{l,m}$ can be defined as:

$$\mathbf{a}_{l,m} = [\dots \overbrace{d_2, d_1, d_2 \dots}^{N_X} d_2, \overbrace{d_3}^{(l,m)}, d_2 \overbrace{\dots d_2, d_1, d_2}^{N_X} \dots]^T. \tag{13}$$

where dots represent zero values.

Now the matrix $\mathbf{A} \in \mathbb{R}^{N_X^2 \times N_X^2}$ can be defined as:

$$\mathbf{A} = [\mathbf{a}_{0,0}, \mathbf{a}_{1,0}, \dots \mathbf{a}_{N_X-1,N_X}, \mathbf{a}_{N_X,N_X}]. \tag{14}$$

This matrix consists of 9 diagonals. Equation (5) may be rewritten as

$$\mathbf{I}\mathbf{p}_{n+1} - \mathbf{A}\mathbf{p}_n + \mathbf{I}\mathbf{p}_{n-1} = \mathbf{s}_n \tag{15}$$

where $\mathbf{I}$ is the identity matrix.

The initial conditions are defined in the first $2N_X^2$ samples of the vector $\mathbf{s}$, see (10). For the initial particle velocity, the first N_X^2 samples must satisfy (7) which in vector notation can be written as $(\mathbf{I}\mathbf{p}_{+1} - \mathbf{I}\mathbf{p}_{-1})/2T = \tilde{\mathbf{u}}_0$ and similarly for the initial sound pressure $\mathbf{I}\mathbf{p}_0 = \tilde{\mathbf{p}}_0$.

As for the boundary conditions, the above equations have to be modified according to (6) and similar equations for other walls and corners. Due to these modifications, the $\mathbf{A}$ matrix will have zero elements in the diagonals since the ghost points must be removed. Moreover, the boundary samples $\mathbf{p}_{n+1}$ and $\mathbf{p}_{n-1}$ are multiplied by a constant representing the impedance and hence the identity matrices in (15) have to be slightly modified. The FDTD matrix can finally be written as

$$\mathbf{B} = \begin{pmatrix} \mathbf{I}/2T & & -\mathbf{I}/2T & \\ & \mathbf{I} & & \\ \mathbf{I}_{+1} & -\mathbf{A} & \mathbf{I}_{-1} & \\ & \ddots & \ddots & \ddots \\ & \mathbf{I}_{+1} & -\mathbf{A} & \mathbf{I}_{-1} \end{pmatrix}, \tag{16}$$

where $\mathbf{I}_{+1}$ is an identity matrix multiplied by the coefficients $(1 + \lambda_c/\xi)$ in each row that belongs to a boundary sample (notice that the coefficient is different for corners) and $\mathbf{I}_{-1}$ is similarly defined with coefficients $(1 - \lambda_c/\xi)$. For frequency-dependent boundaries there would be more $\mathbf{I}_{-i}$ matrices on the left of $\mathbf{I}_{-1}$, representing the *memory* of the boundary digital filters. Note that $\mathbf{B}$ can become a huge matrix for high spatial/temporal sampling rates. However, it has a sparse multi-diagonal structure which allows an efficient storage and computation.

4. SOURCE LOCALIZATION AND RECONSTRUCTION

4.1. Reconstruction of an impulse

In the following it will be assumed that the sound field can be simulated correctly at each position of the room, i.e. that FDTD returns an accurate physical model. Numerical methods are usually employed to predict the sound pressure from a known source signal $\mathbf{s}$. Here the inverse problem is considered: the sound pressure is known at a subset of the FDTD grid points where microphones are present, hence $\mathbf{p}$ is partially known and $\mathbf{s}$ must be estimated.

A convex optimization problem can be formulated as follows [3],

$$\begin{aligned} \min_{\mathbf{p},\mathbf{s}} \quad & \sum_{i=1}^{K} \|\mathbf{D}_i\mathbf{p} - \hat{\mathbf{p}}_i\|_2 + \lambda\|\mathbf{C}\mathbf{s}\|_1 \\ \text{s.t.} \quad & \mathbf{B}\mathbf{p} = \mathbf{s} \end{aligned} \tag{17}$$

where K is the number of microphones, $\hat{\mathbf{p}}_i$ is a vector containing the signal measured in the microphone i and $\mathbf{D}_i$ is a selection matrix that selects the samples at the microphone position in the vector $\mathbf{p}$. The equality constraints correspond to the FDTD method. The cost function consists of two terms: The squared error norm forces the sound pressure represented by $\mathbf{p}$ to be close to the sound pressure of the measurements in the microphones. The term $\lambda\|\mathbf{C}\mathbf{s}\|_1$ is a *regularisation* term inducing sparsity in the source signal vector. This term is crucial for the problem to be solvable since the FDTD system of linear equations is heavily underdetermined. λ is a regularization parameter that can increase the importance of the sparsity-inducing term at the cost of a decreased accuracy between measured and simulated sound pressure values. $\mathbf{C}$ is a selection matrix that removes the first $2N_X^2$ samples corresponding to the initial conditions, where the source signal vector is no longer sparse but can be dense.

By imposing sparsity in such a manner, the source signal $s_{l,m}^n$ is enforced to be non-zero for only few indices l, m and n. Hence the regularization encourages solutions with *impulsive* sources. Simulations show that when the source signal is indeed an impulse in both time and space, reconstruction is possible even with only one microphone. In the next subsections the more interesting case of reconstructing and localizing a more general source signal will be studied.

4.2. Source localization

When a general point source signal is used instead of an impulse, problem (17) does not allow source signal reconstruction to be performed but it can achieve a correct source localization.

In fact, if the estimated source signal, without the ICs part, is squared and summed over time for all l,m, its maximum value over l, m is found to reveal a good estimate of the position $[l_s, m_s]$ of the source

$$[l_s, m_s] = \underset{l,m}{\text{argmax}} \sum_{n=1}^{N_T} (s_{l,m}^n)^2. \tag{18}$$

4.3. Reconstruction with known position

It is possible to exploit the estimated source position to perform the source signal reconstruction. The source signal vector $\mathbf{s}$ may be written as

$$\mathbf{s} = \mathbf{F}\mathbf{s}_T \tag{19}$$

where $\mathbf{s}_T \in \mathbb{R}^{N_T}$ is a vector containing the time signal at the estimated source position and $\mathbf{F} \in \mathbb{R}^{N_X^2(N_T+2)\times N_T}$ is an expansion matrix. Let k be the index of the estimated source position corresponding to $[l_s, m_s]$ in the vectors $\mathbf{s}_i$ for $i = 1, \dots, N_T$. The upper $2N_X^2$ rows of $\mathbf{F}$ will have only zeros, since these correspond to the initial conditions $\tilde{\mathbf{u}}_0$ and $\tilde{\mathbf{p}}_0$ that are not present in $\mathbf{s}_T$. The next N_X^2 rows correspond to $\mathbf{s}_1$ and have only one non-zero element in position $[k + 2N_X^2, 1]$. Then the next N_X^2 rows have a 1 in position $[k + 3N_X^2, 2]$ and so on until the second index reaches N_T.

The sound pressure vector can then be written as:

$$\mathbf{p} = \mathbf{Z}\mathbf{s}_T \tag{20}$$

where $\mathbf{Z} = \mathbf{B}^{-1}\mathbf{F} \in \mathbb{R}^{N_X^2(N_T+2)\times N_T}$.

The vector $\mathbf{s}_T$ can thus be obtained by simply solving the system:

$$\mathbf{D}\mathbf{Z}\mathbf{s}_T = \mathbf{D}\mathbf{p} \tag{21}$$

which is always overdetermined. Here $\mathbf{D}$ represent the selection matrix that selects all the rows where microphones measurements are present.

4.4. Reconstruction without initial conditions

It is also possible to create an expansion matrix that reconstructs the ICs. The matrix $\mathbf{F}$ then has to be modified in the following manner:

$$\mathbf{F}_{\text{IC}} = \begin{pmatrix} \mathbf{I}_{2N_X^2\times 2N_X^2} & | & \\ & | & \mathbf{F} \\ \mathbf{0}_{N_T N_X^2\times 2N_X^2} & | & \end{pmatrix}. \tag{22}$$

$\mathbf{F}_{\text{IC}}$ then becomes a $(N_T+2)N_X^2 \times (2N_X^2 + N_T)$ matrix and the signal can be reconstructed by solving (21), which is an overdetermined set of equation only if $KN_T \geq 2N_X^2 + N_T$ since now $\mathbf{DZ} \in \mathbb{R}^{KN_T\times(2N_X^2+N_T)}$. This problem manages to partially reconstruct the initial conditions. As simulation results in Section 5 will show, the source signal can be reconstructed with high accuracy, even though the first few samples should be discarded since these are usually corrupted by the inaccurate reconstruction of the initial conditions.

Being able to have a method that does not need the initial conditions means that the signal can be reconstructed using shorter successive time windows. This reduces the numerical cost since the matrices that have to be computed become smaller.

5. SIMULATION RESULTS

An FDTD simulation is performed in a 1 m^2 2D room using a spatial resolution of $N_X = 10$ and $N_T = 500$ time samples, meaning a sampling frequency of $f_S = 4.4$ kHz, with the SLF scheme. The impedance is chosen to be $\xi = 200$ and initial conditions are set to zero. The system is excited using a physically constrained source [7] of filtered white noise, from 20 to 600 Hz. The K microphone signals are then simulated by adding Gaussian white noise (GWN) to the FDTD field values at the microphone positions. The source localization problem (17) is then solved with CVX [9]. Here the parameter λ is set to 10^{-4} meaning sparsity is not strongly enforced.

Figure 1 shows the results of Monte Carlo simulations. For each point in the graph 100 simulations were run using random positions of the microphones and source. The figures on top show the probability of correct localization (i.e., finding the correct grid point) while the ones on the bottom show the mean squared error (MSE) between the original and reconstructed signal. Both the mean and median of the MC simulations for the MSE are shown in order to emphasize the fact that few outliers significantly decrease the average of the MSE. The plots on the left show simulation results for different numbers of microphones. Here, GWN was added to each microphone measurement with 40 dB signal to noise ration (SNR). Localization of the source works efficiently with only three microphones but at least 5 are needed to perform an accurate signal reconstruction. The graphs on the middle show results for different values of SNR, using 5 microphones. Again, the performance of the localization is somewhat better than the signal reconstruction performance: at 20 dB SNR the source is localized correctly with a 98% probability while 40 dB SNR is needed for good signal reconstruction. Finally, the plots on the right show the behavior when ICs are non-zero. The initial particle velocity and initial sound pressure are set to $\tilde{u}_{l,m}^0 = \sin(2\pi 2l/N_X)$ and $\tilde{p}_{l,m}^0 = -10^{-3}\sin(2\pi m/N_X)$. The field that these ICs generate is then added to the one generated by the sound source and multiplied by a constant. In fact here simulations are re-

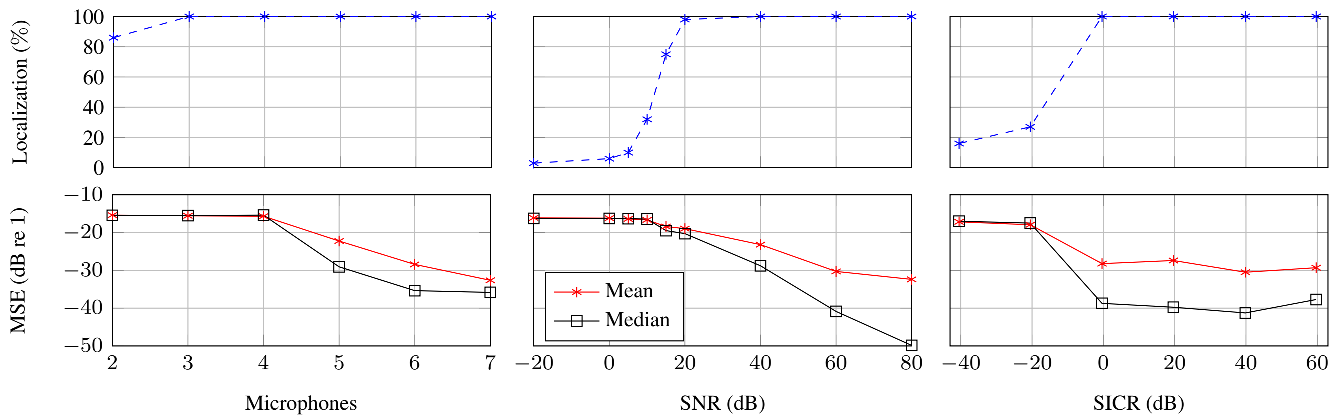

Fig. 1: Localization of source in probability and reconstruction error for different numbers of microphones, different values of SNR and SICR.

peated for different values of signal to initial conditions ratio (SICR), i.e. the ratio between the sound pressure power due to the sound source and the sound pressure power due to the ICs recorded by the microphones. In these simulations GWN was added in each microphone measurement corresponding to a SNR of 60 dB. The ICs can be thought as the source of *noise coming from the past*. This type of noise can be removed efficiently when the ICs can be estimated correctly. The resulting localization and signal reconstruction performance is much more robust when most of the noise is coming from the previous sound field created by the ICs. Good accuracy can be achieved even at 0 dB SICR, meaning that the sound field due to the sound source and the one due to the ICs have the same magnitude.

6. CONCLUSIONS

A new two step method for localization and signal reconstruction using the FDTD method has been proposed. First the sound source is localized by solving a convex optimization problem that exploits spatial sparsity and then signal reconstruction is achieved by solving an overdetermined system of linear equations. Simulations have shown that the method can be robust against noise in particular when this is coming from a past sound field that can be partially estimated.

The main issue for the applicability of the proposed method in a real world scenario is that the geometry and boundary conditions of the room are assumed to be known. Matching room acoustic properties with simulations can be a difficult task especially due to the modeling of the boundary conditions. Moreover, the computational cost of the FDTD method and the dispersion error it suffers can be problematic. These errors can be removed or reduced by either using very high resolutions, which would inevitably lead to a huge increase of the computational cost, or by limiting the method to the low frequency range.

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