Flood Regulation by means of Model Predictive Control

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Abstract In this chapter flooding regulation of the river Demer is discussed. The Demer is a river located in Belgium. In the past the river was the victim of several serious flooding events. Therefore, the local water administration provided the river with flood reservoirs and hydraulical structures in order to be able to better manage the water flows in the Demer basin. Though this measures have significantly reduced the floods in the basin, the recent floods in 1998 and 2002 showed that this was not enough. In order to improve this situation a pilot project is started with as main goal to regulate the Demer with a model predictive controller. In this chapter the results of this project are discussed. First a simplified model of the Demer basin is derived based on the reservoir model. The model is calibrated and validated using historical data obtained from the local water administration. On the one hand the resulting model is accurate enough to capture the most important dynamics of the river; on the other hand the model is fast enough to be used in a real-time setting. Afterwards, the focus will be shifted to the model predictive controller. The use of the model predictive controller will be justified by comparing it to other control strategies used in practice for flood regulation. Then, the more technical details of the model predictive controller will be discussed in more detail. Finally the chapter will be concluded by historical simulations in which the model predictive controller is compared with the current control strategy used by the local water administration.

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1 Introduction

Flooding of rivers are worldwide the cause of great economic losses. The areas located in the surroundings of the Belgian rivers are no exception to this. This study focuses on the Demer river, a large river located in Belgium where severe floodings have occured in the past during periods of heavy rainfall. In order to prevent these flooding events the local water administration (VMM water authority) installed hydraulic structures (with movable gates) in order to influence the discharges and water levels in the water systems. Extra storage capacity for periods of heavy rainfall was provided by means of flood control reservoirs. Hydraulic structures to control the flow from and into the reservoirs were also installed. The hydraulical structures are controlled by an advanced version of a standard three position controller (see section 3). Though all these measures have reduced the amount and frequency of floodings in the Demer bassin, past events have shown that improvement is still necessary.

Table 1 Damage report of the latest floodings in the Demer bassin.

Period	**Estimated flood volume (km^3)**	**Cost in €**
dec 1993-jan 1994	23.5	47 000
jan 1994-feb 1995	22.9	11 000
sep 1998	32.6	16 169 000
feb 2002	15.7	still not known
dec 2002-jan 2003	18.0	still not known

In table 1 a summary of the latest flooding events in the Demer bassin is presented. From the table it can be seen that floodings have lead to big economical losses in the Demer bassin, especially the flooding from 1998. Simulations based on a complex finite element model (Infoworks) have shown that these flooding events could have been less severe and even completely avoided if the gates would have been controlled in a different way. Therefore, the local water administration has set its target to find a new control strategy in order to control the gates. The study presented in this chapter is an attempt to achieve a better control strategy by means of model predictive control.

The structure of this chapter is as follows. Section 2 presents the model used in this work. Section 3 presents the general idea behind control theory and gives a short overview about control techniques used in practice for the regulation of water systems. Also a nonlinear model predictive control (NMPC) scheme is presented. This NMPC scheme is applied on the model from section 2 and its results are presented in section 4. Finally in section 5 some guidelines and ideas are presented for future research.

2 Hydrodynamic water system modelling

Introduction

A Model Predictive Control (MPC) application has been tested for the river Demer case in Belgium (Fig. 1) in order to increase the efficiency of flood control by means of two flood control reservoirs installed upstream of the densily populated cities of Diest and Aarschot. Fig. 2 gives a schematic representation of the water system structure (river reaches and hydraulic control structures) in the region surrounding the two flood control reservoirs, which are called ”Schulensmeer” and ”Webbekom”.

The MPC controller being developed is based on two types of hydrodynamic river models. The first model has been developed during earlier studies for the VMM water authority and is of the full hydrodynamic type. The full hydrodynamic equations of de Saint-Venant (momentum and continuity equations; [1]) have been implemented in that model for the river Demer as well as for its main tributaries. This has been done for the entire Demer basin (this means also for the areas far up- and downstream of the area of interest for this study) (see green coloured river reaches in fig. 1). The full hydrodynamic model has been implemented in the InfoWorks-RS software of Wallingford Software in the UK ([4]). It is based on river bed cross-sectional data approximately every 50 meters along the modelled rivers, river bed roughness information and geometric data on all hydraulic structures (weirs, culverts, flow and water level control structures) and bridges along the course of all these rivers. The hydraulic control is based on the current fix regulations rules, which uniquely depend on up- and/or downstream water levels and discharges.

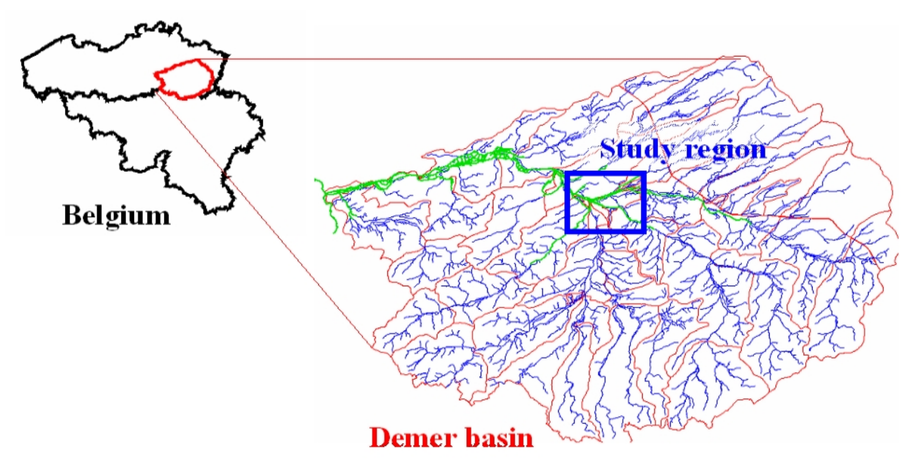


Fig. 1 Study region in the river Demer basin in Belgium.

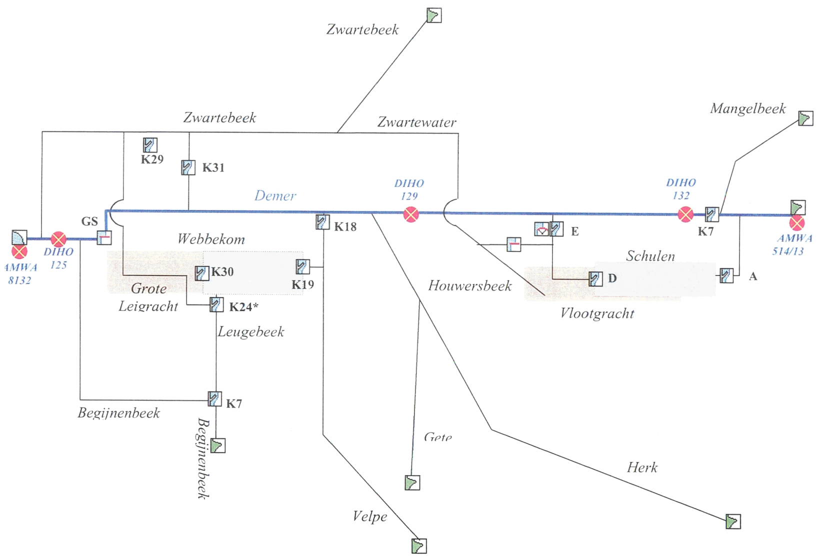


Fig. 2 Scheme of the river network in the study area, the flood control reservoirs "Webbekom" and "Schulensmeer" at the village Schulen, and the hydraulic regulating structures (OBM-Demer, 2003).

The InfoWorks-RS model has been recently extended with a real-time flood forecasting module, such that real-time flood predictions can be made (based on rainfall forecasts) with a time interval of 15 minutes and a prediction horizon of 2 days (depending on the time horizon of the rainfall forecasts). This real-time flood forecasting model has been implemented in the FloodWorks software (extension of InfoWorks; [5]) and is currently fully operational. The forecasting results are linked to a warning alert and alarm system, sending messaged to local fire brigades and crisis intervention authorities, but which can be consulted as well on-line by the public.

A next step in further advancing the flood control, also in view of the adaptations required due to the potential negative impacts of climate change, is the set up of a flood control system. This text aims to describe the development steps taken so far, where an MPC-based flood control prototype algorithm has been developed and tested.

The MPC-algorithm could not directly make use of the full hydrodynamic InfoWorks or FloodWorks model, because of the long calculation times of this model. One simulation of the entire model (to be done once in 15 minutes in the current real-time flood forecasting setting) takes only a little less than 15 minutes. The MPC-algorithm, however, requires multiple iterations during each prediction step.

With the aim to reduce the model calculation time, a simplified model has been developed. This second model is of a conceptual type. It simplifies the hydrodynamic river flow process by lumping the processes in space, and by limiting the study area to the region affected by the flood control. Lumping of the processes in space is done by modeling of the water levels, not every 50 meters as the full hydrodynamic model does, but only at the relevant locations. These are the locations up- and downstream of the hydraulic regulation structures, to be controlled by the MPC-controller, and the locations along the Demer where potential flooding is induced, to be limited by the controller. Depending on these locations, the river is subdivided in reaches, in which water continuity is modelled (in a spatially lumped way per reach) based on reservoir-type of models.

Conceptual model structure

Fig. 3 shows the scheme of the model components for the study area around the two flood control reservoirs ”Schulensmeer” and ”Webbekom” in the river Demer basin. This area receives rainfall-runoff inflow via the tributary rivers Mangelbeek, Herk, Gete, Velpe, Zwartebeek, Zwartewater and Begijnenbeek. By means of the hydraulic regulating structures A and K7, the local water engineers can anticipate on future flood risks. Through closing gate K7 and opening gate A, the Schulensmeer reservoir is being filled, the downstream Demer flow reduced, and consequently the flood risk of the cities Diest and Aarschot downstream of the study area of the reservoirs reduced. After the flood period, the Schulensmeer reservoir (which consists of different reservoir compartments) can be emptied through the hydraulic regulating structures D and E. The second reservoir ”Webbekom” is regulated in a similar way by means of the hydraulic structures K18, K19, K7 at the Leugebeek river, K24* and K30. Fig. 2 gives an overview of the structure of the conceptual model developed for the study area. The river reaches are in this scheme represented by means of lines with positive flow in the direction of the arrows, the hydraulic regulating structures by means of the full rectangles, the fixed spills or overflows by open rectangles, and the model units where water storage (in the reservoir compartments or along river reaches) and water levels are simulated by nodes. The symbol ”q” denotes discharges, ”h” water levels, ”v” storage volumes, and ”k” controllable gate crest levels. The water levels and volumes are the model variables describing the state of the water system in the MPC controller. The gate crest levels are in the inputs in the MPC controller, the upstream (rainfall-runoff) discharges the disturbances of the MPC controller.

In order to test the MPC controller in the first phase of the project, a ”reduced area” around the Schulensmeer reservoir has been considered. A separate reduced conceptual model has been developed for this area. The scheme of this conceptual model for the reduced area is shown in Fig. 4. Next to the reduction in area, also some simplifications to the physical reality have been implemented, in order to enable disconnection of the Schulensmeer area from the more downstream areas. Fig. 5 shows photos of the region upstream of the Schulensmeer reservoir and shows

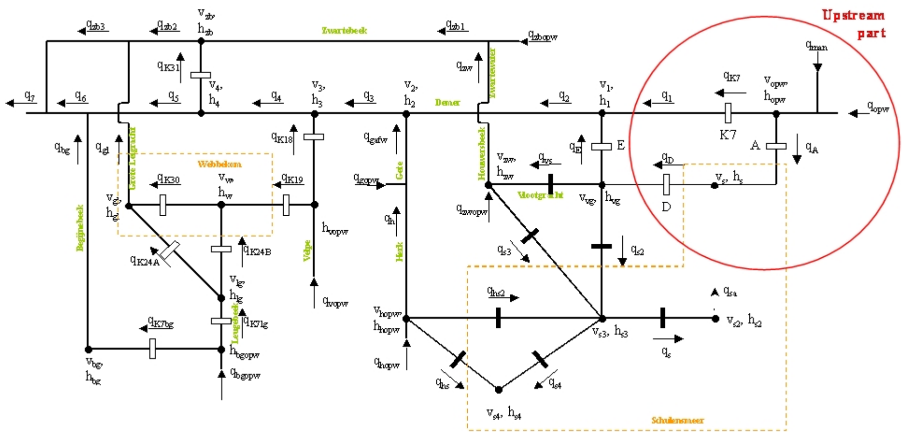


Fig. 3 Scheme of the conceptual model for the study area of Figure 2 (dots for the calculation nodes = river or reservoir storage elements; lines for the river reaches; open rectangles for the hydraulic regulating structures; closed rectangles for the fixed spills or weirs).

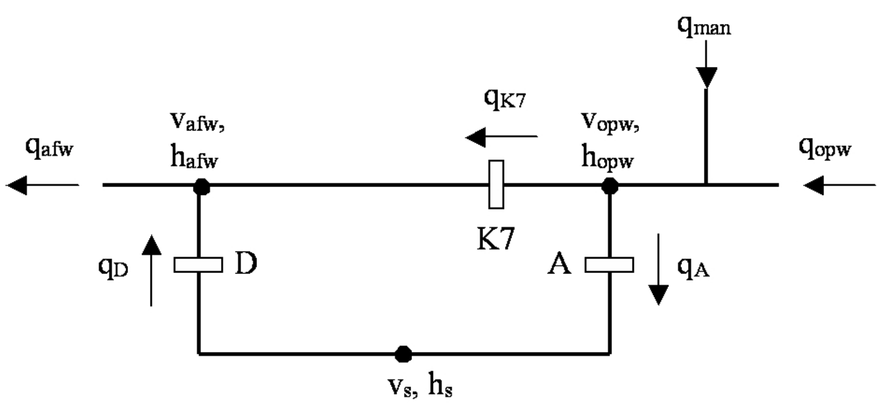


Fig. 4 Schematic representation of the ”Schulensmeer” reduced area, together with the conceptual model variables.

some upstream model components and model variables.

Conceptual model building and calibration process

The conceptual model-structure has been identified and its parameters calibrated based on simulation results with the full hydrodynamic InfoWorks model. In order to do so, a combination of different modelling concepts has been applied. The flood control reservoirs have been modelled in the simplest way as storage nodes (vs, vs2, vs3, vs4, and vw), maintaining water continuity. Also the large water storage areas, which in the conceptual model are modelled in a combined way with their drainage canals, are schematized as storage nodes. This is the case for the nodes ”vvg” (draining the Schulensmeer reservoir) and ”vgl” (draining the Webbekom reservoir). For each of these storage nodes, a continuous relationship has been implemented between the storage volume and the mean water level (which is assumed static at each time step). These relationships have been calibrated based on the Digital Elevation Model underlying the IWRS model. Example of such calibration result is shown in Fig. 6 for the Webbekom flood control reservoir. The water levels are shown as levels above the ”TAW” standard reference level for Belgium.

The static level assumption is valid for the flood control reservoirs (thus for the water levels hs, hs2, hs3, hs4 and hw). This is, however, not the case for the water levels ”hvg” and ”hgl” of the drainage areas. These water levels indeed vary a lot along the course of the drainage canals. The water level considered in the conceptual model then is taken at one specific location along the drainage canal. Water levels at other locations along the drainage canal are only included in the conceptual model when they are really needed (e.g. because they control hydraulic structures). They are assumed related to the primary levels considered (”hvg” and ”hgl” in this case). In case no unique relationship exists with these primary levels, the drainage canal (together with the flood zones on its banks) is modelled as a long regular river reach, as described next.

Long regular river reaches have been modelled in two different ways: (a) schematized by means of a serial connection of reservoirs, and (b) by means of the water surface profile concept (considering the water level differences from down- to upstream along the river). In approach (a), water levels are modelled in a step-wise way from up- to downstream, while approach (b) starts from the most downstream locations along the reach and considers water level changed to upstream. Approach (a) guarantees water continuity, but has the disadvantage that more downstream reservoirs in the serial connection do not necessarily have lower water levels. Outflow corrections are then needed to avoid this type of anomalies. Approach (b) does not pose this problem, because water level differences are always taken positive from

Fig. 5 Photo of the river Demer and the ”Schulensmeer” flood control reservoir in the background, together with the locations of the main water level and discharge variables.

down- to upstream.

In approach (b) the water level differences are depending on the discharge in the reach and the downstream water level. Primarily the following relation is tested:

$$h = h_{afw} + \ a\ (q^2 / \, (h_{afw} \text{ - } h_{afw,0})^2)^b \tag{1}$$

where $h - h_{afw}$ is the water level difference to be modelled depending on the discharge q and the downstream water level h_{afw}, and where h_{afw},0 is the bed level at the downstream location. This equation is based on the assumption of the equation of Manning ([1]):

$$q = \frac{1.49}{n} \, A \, R^{\frac{2}{3}} \, S^{\frac{1}{2}} \tag{2}$$

where the discharge q depends on cross-section area A, the hydraulic radius R, the friction slope S and the Manning coefficient n. Under the uniform flow approximation, the friction slope S equals the river bed slope, which under the normal depth assumption furthermore equals the water surface slope $h - h_{afw}$. For wide river sections, the hydraulic radius becomes approximately equal to the river bed width and thus independent on the water level. For rectangular sections, the cross-section area becomes linearly proportional to the water depth. Under these assumptions, the water level difference becomes proportional to the ratio of the squared discharge and the squared water depth:

$$q^2 / \, (h_{afw} \text{ - } h_{afw,0})^2 \tag{3}$$

which leads to equation (1). Fig. 7 gives an example of the calibration of equation (1) to conceptually model the water level differences between h2 en h3 along the Demer. Fig. 8 shows another example of the same approach but for the Grote

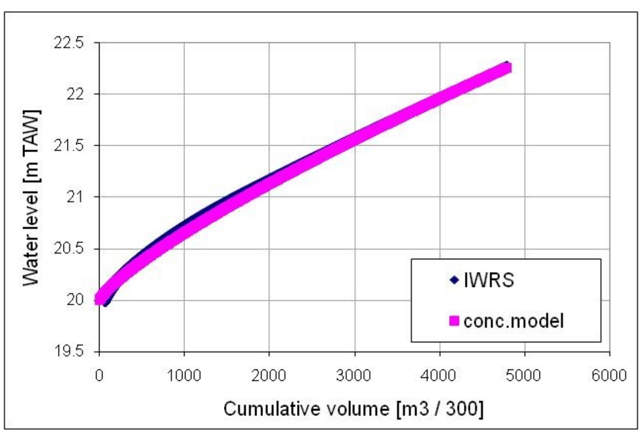


Fig. 6 Calibration result for the storage volume - mean water level relation of the ”Webbekom” flood control reservoir.

Leigracht river (water level differences between hgl en hzb).

Approach (a), on the other hand, makes use of reservoir models, where the outflow discharge (flow to downstream) depends on the reservoir storage volume and (potentially also) on the upstream inflow discharges. The relationships between outflow discharge and storage volume and (potentially) inflow discharge, are to be calibrated to simulation results with the full hydrodynamic model. These relationships are linear in the most parsimonious case, but often of non-linear nature. Any nonlinear relation can, however, be approached by a set of piece-wise linear relations. In this study, the reservoir-model relations are identified in the following step-wise approach. First the relationship between water storage in the river reach and the outflow discharge is studied based on the IWRS results. When this relation is unique, a linear, non-linear, piece-wise linear or non-linear function is calibrated to that relation. Fig. 8 shows for the same Demer reach as in Fig. 6 the relation between the water level h3 (depending on the storage volume v3) and the downstream discharge. A unique relation is found, indicating that the reservoir concept is applicable, thus that both approach (a) and approach (b) could be implemented for this case. In Fig. 8, the relation is not linear but a piecewise relation could be calibrated: a linear relation up to water level 20.2 m TAW, and a power relation for the higher water levels.

When the relation is not unique, but shows ”hysteresis effects” (see example in Fig. 10), a more complex calibration method has to be followed. As shown in [3] based on hydrodynamic sewer system applications, hysteresis effects are to be explained by different storage - outflow relationships in the decreasing and increasing flanks of the flow hydrographs. These differences are to be explained by the differences in ”dynamic storage” in the system, and thus by the inflow discharges. In this case, the reservoir model is advanced separating the total storage in the river reach in static and dynamic storage parts. The static storage is identified as the lowest

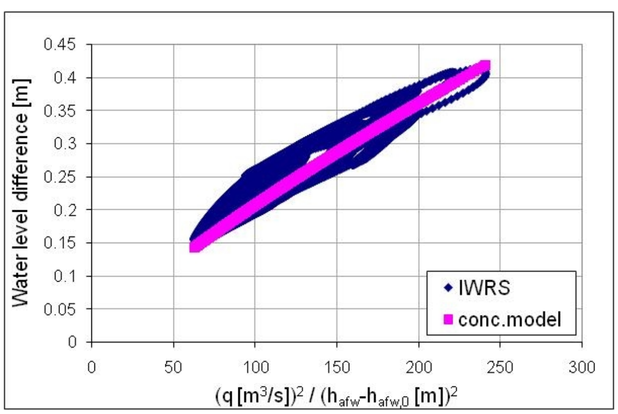


Fig. 7 Calibration result applying approach (b) for the Demer river reach between the conceptual storage nodes h2 and h3.

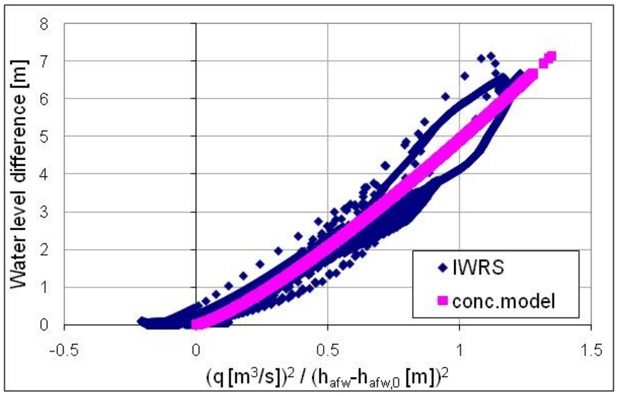


Fig. 8 Calibration result applying approach (b) for the Grote Leigracht river between the conceptual storage nodes hgl and hzb.

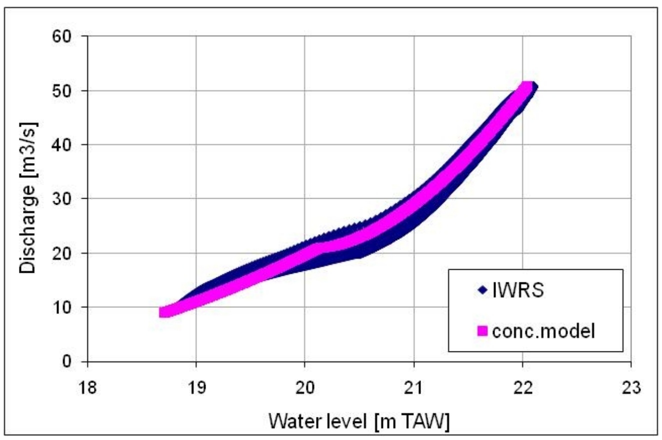


Fig. 9 Calibration result applying approach (a) for the Demer storage node describing h3. A unique relation is shown between the water level h3, which depends on storage volume v3, and the outflow discharge (q3) of this storage.

storage for a given outflow discharge (thus during the decreasing flanks of the flow hydrographs), while the dynamic storage is the difference between the total storage in the reach and the static storage identified. By plotting this dynamic storage at each time step versus the inflow discharge in the reach, and calibrating the identified relation by means of a linear, non-linear, piece-wise linear or non-linear model, a first reservoir submodel becomes ready: the submodel to predict at each time step the dynamic storage based on the inflow discharge. When this dynamic storage is subtracted from the total storage (at each time step to be calculated considering water continuity) static storage predictions are obtained. Analyzing the relationship between this static storage and the outflow discharge, and fitting a function to this

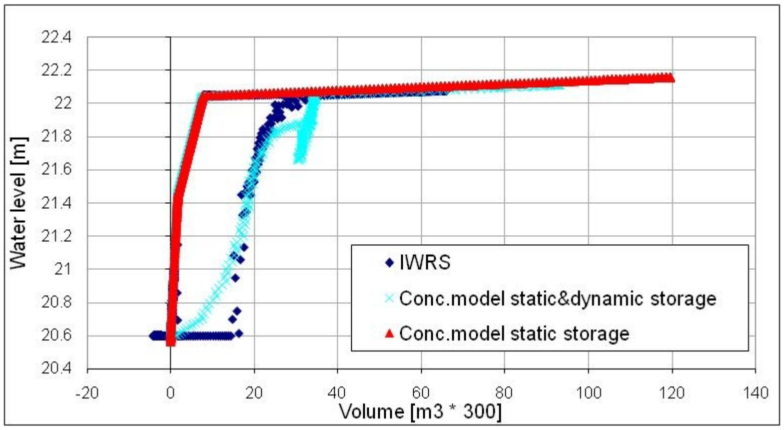


Fig. 10 Calibration result applying approach (a) for the Leugebeek describing hlg. The hysteresis effect in the storage-volume relation has been modeled separating the total storage in static and dynamic storages.

relation, completes the reservoir model-structure.

The ”hysteresis” fenomenon was seen for reaches of the ”Leugebeek” river (node ”lg”) (see Fig. 10). This is a river reach where part of the reach has water levels that are upstream controlled (steep hydraulic slope) and another part downstream controlled (mild hydraulic slope).

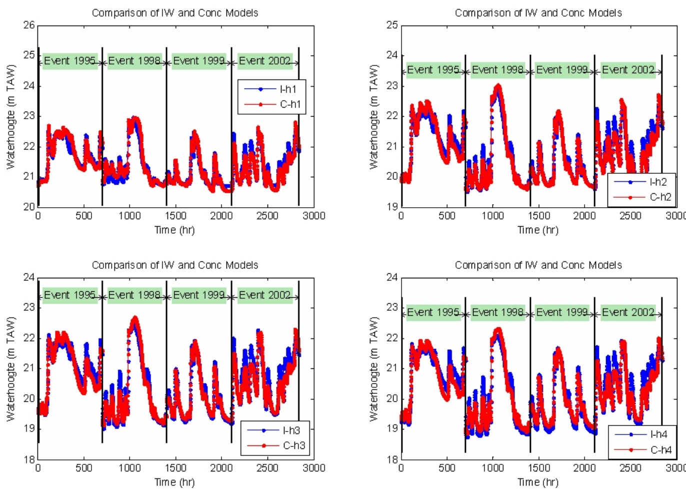
Both approaches (a) and (b) model the water levels in a ”lumped” way, which means that only 1 water level (at one single location along the reach) is considered. When water levels at more locations are required along the reach (e.g. because of their importance in controlling hydraulic structures, or in controlling water levels in upstream river reaches, through back-water effects), relations are sought between the levels at these locations and the primary modelled water level. Because of its stability and mathematical elegance, preference has been given to approach (b). The hydraulic structures in the conceptual model are implemented in a way identical to the IWRS-model; the same hydraulic model equations are considered.

Model equations are in the conceptual model solved based on finite differences between successive time steps. No iteration per time step is aimed, in order to have the model calculation times as limited as possible. A time step of 5 minutes is considered, which is coarse in comparison with the spatial resolution of the model (average distance between the calculation nodes). For this reason, the first version of the conceptual model was strongly affected by instabilities. Flow delay terms therefore have been added to the model, making use of the linear reservoir model equation:

$$q_{out}(t) = \exp(-\frac{1}{k})\, q_{out}(t-1) + \; (1-\exp(-\frac{1}{k})\, q_{in} \tag{4}$$

where q_{in} is the original flow (before delay) and q_{out} the flow after application of the delay equation.

Fig. 11 Comparison of the InfoWorks-RS and conceptual model results for the discharges and water levels at selected nodes; for the historical floods of 1995, 1998, 1999-2000 and 2002.(a)



Conceptual model validation

The identification of the conceptual model-structure and the calibration of its parameters, based on the methodology described above, has been based on the simulation results with the full hydrodynamic model for two historical high flow or flood events: the flood events of September 1998 and January 2002. Validation was done based on the full set of flood events during the pas 15 years; being the flood events of 1995, 1998, 1999 – 2000 and 2002. Fig. 11-13 show some validation results, comparing the water level and discharge simulation results of the conceptual model with the one of the InfoWorks-RS (IW) models. Both models have a time step of 5 minutes. Model output results shown in Fig. 11-13 are aggregated at the hourly time step.

Next to the comparison with the full hydrodynamic model results for historical flood events, it also has been checked whether the conceptual model still performs well for conditions (hydraulic structure regulations or gate levels) which did go beyond the range of conditions during the historical flood events. Also the model has been checked for discontinuities and stability.

Fig. 12 Comparison of the InfoWorks-RS and conceptual model results for the discharges and water levels at selected nodes; for the historical floods of 1995, 1998, 1999-2000 and 2002.(b)

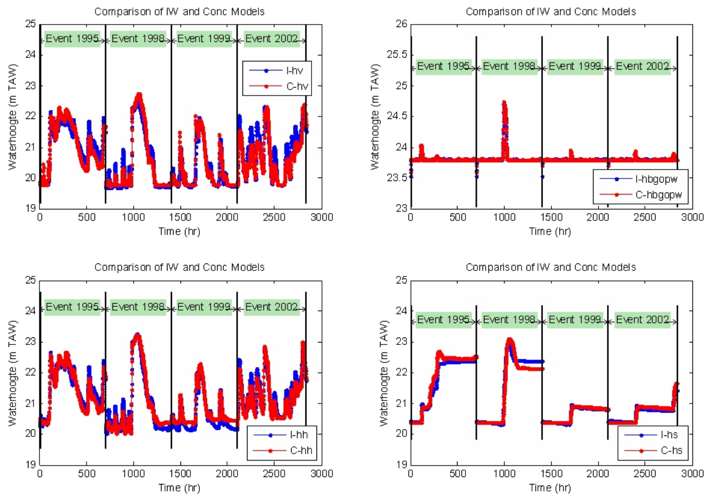
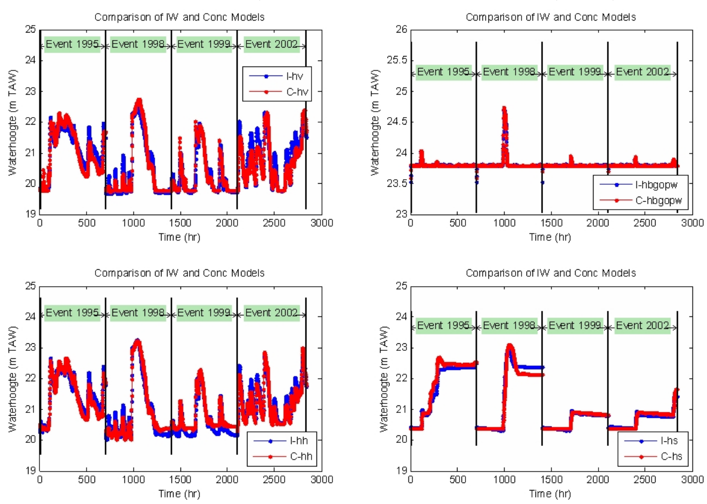


Fig. 13 Comparison of the InfoWorks-RS and conceptual model results for the discharges and water levels at selected nodes; for the historical floods of 1995, 1998, 1999-2000 and 2002.(c)

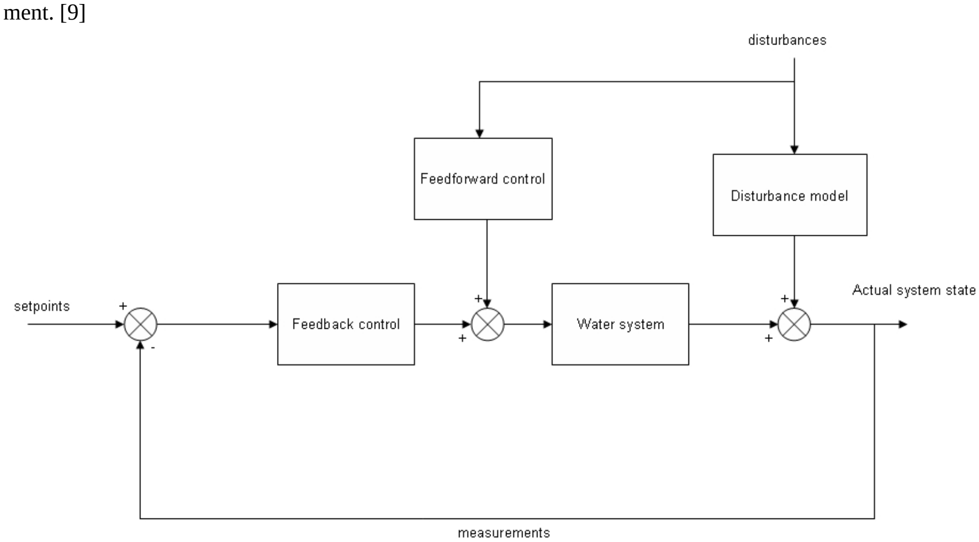


3 Flood Regulation

Classical Feedback and Feedforward Control

In Fig. 14 a diagram of a classical feedback and feedforward controller for water system management is presented. The setpoint represents the desired water level. The objective of the controller is to steer the water levels as close as possible to the desired setpoints. In order to achieve this at each sampling time the different water levels are measured. This measured water level is then compared to the setpoint and the deviation is passed to the feedback controller. The feedback controller will then determine a control action that tries to remove this deviation. In water system management the feedback controller is typically a proportional integral controller (PI) [6]. The feedforward controller uses measurements of the disturbance (in this case rainfall) and an inverse model in order to counteract the effect the measured disturbance will have on the water level. In the ideal case this would lead to perfect setpoint regulation. In practice, however, there are uncertainties on the models and the measured disturbances which make perfect setpoint regulation with a feedforward controller impossible. In order to cancel out these uncertainties a feedforward controller is always combined with a feedback controller [16].

Fig. 14 A diagram of a classical feedback and feedforward controller for water system management. [9]



Three-position controller

Three-position control corresponds to a control mode that is based on some very simplistic rules and typically has the purpose to steer a water level to a desired setpoint. A three-position controller achieves this by reacting on each deviation from the setpoint by adjusting the gate position for a predetermined amount of time. A standard three position controller consists of the following rules:

- The water level lies between desired upper and lower limit - no corrective action. The setpoint lies between the upper limit and lower limit.

- The water level exceeds the upper limit - the gate is lowered in order to lower the water level.

- The water level is lower than the lower limit - the gate is raised in order to raise the water level.

Note that this controller is an example of a feedback controller. The local water administration responsible for the water management of the Demer bassin uses three-position controllers for the control of the gates. However, their controllers are more advanced than the standard three-position controller. During normal operation the purpose is to steer the water levels to their setpoints. This is done by standard three-position controllers. During periods of heavy rainfall the focus shifts in prevention of flooding. Therefore, the standard rules are replaced by new rules. These new rules determine the gate position based on the measurement of water levels in the water system. These rules are formulated by the local water administration based on many years of experience in regulating the Demer bassin and can therefore be considered to be expert knowledge. These type of controllers have the advantage that the movement of the gate is limited which is positive with respect to wear and tear of the gates . Another advantage is its straightforward implementation.

Model Predictive Control

Management of modern water systems require more advanced control methods than the classical feedback and feedforward. The first reason is that the classical methods are not capable of dealing with constraints on the water levels, neither can they cope with physical limitations on the gates. The second reason is that these methods are not capable to deal with future rain predictions. The third reason is that these methods are only suitable for linear systems.

A control strategy that is better suited for the management of water systems is model predictive control (MPC) ([13],[14]). MPC is a control strategy that can deal with constraints on the system as well as with future rain predictions. Besides that MPC can deal with linear as well as with nonlinear models. In literature several studies can be found in which automatic control techniques are used to control a water system. There are also several studies available in which MPC is used to control water systems. These works however have as main goal to control the different water levels to some desired target value and not to prevent flooding. In these applications it is usual sufficient to linearize the system around the desired steady state in order to obtain good results. However, when trying to avoid/reduce flooding linearization around the steady state is not sufficient. During periods of heavy rainfall the complete nonlinear dynamics of the system are excitated so it is very important that the MPC can deal with nonlinear model behaviour. In the sequel the nonlinear

MPC scheme applied in this research will be discussed in detail.

Components of MPC

MPC is a model based controller. These types of controllers emerge from the chemical industry where the fabricated products are produced very closely to the limits of the quality specifications. Therefore, it is important that the applied control strategy is capable to work as close as possible to the limits of the constraints on the process. The same observations hold for flooding regulation. In order to avoid flooding it is important that the controller pushes the water levels as close as possible to the constraints if necessary. If the controller isn't capable of doing this, this can lead to flooding of some water levels that could have been avoided. MPC is a control strategy that is capable of doing this. MPC contains the following components:

- internal model: The internal model describes the physical behaviour of the controlled water system and is used by MPC in order to predict the future water levels within a certain time window, also called prediction horizon. The more accurate the model, the more accurate the predictions by which the performance of the controller increases. However, a trade-off must be made between accuracy of the model and its complexity. A very accurate model can come with the expense of an unacceptable high calculation time which makes it impossible to use it in a MPC framework.

- objective function: This function captures the objective of the controller. The objective function penalizes deviations from desired reference values. More important water levels can be given more importance by increasing the corresponding penalization. Typically, movements of the gates are also penalized in order to avoid excessive wear of the gates.

- constraints: MPC is a control strategy that differentiates itself from other control strategies by its capability to deal with constraints. In water systems two types of constraints can be distinguished. The first one concerns physical limitations of the gates. There are constraints on the maximum and minimal gate height as well as on the maximum gate movement. These constraints are hard constraints and should always be satisfied. The second type of constraints concerns limitations on the water levels. In order to avoid flooding it is important each water level stays under its corresponding flood level, the level at which flooding occurs. These constraints are soft constraints in the sense that they may be violated if the rainfall is too excessive to avoid flooding. Also note that the internal model can also be seen as a constraint on the system.

- optimization: MPC performs an optimization in which it tries to optimize the objective function taking the constraints into account. This optimization leads to an input sequence that optimizes the objective function within the given prediction horizon. In this chapter the optimization will be a nonlinear program because of the nonlinear system model.

- receding horizon: The first input of the optimal input sequence is applied to the system. Then, in the next sampling time the time window is shifted with 1 time step and based on the new state of the system a new optimization procedure is performed. This strategy provides robustness against uncertainties coming from modelling errors, rain prediction errors and measurement noise.

- measurements and state estimation: In order to be able to make future predictions with the internal model it is necessary to know the current state of the system. This can be obtained by measurements. However, in practice it is not possible to measure the full state of the system. Therefore, based on the measurement of a subset of all the states and the internal model the full state of the system is estimated by a state estimator.

Mathematical formulation

In this work the internal model is assumed to be a discrete nonlinear dynamic state space model defined by

$$\begin{aligned} x_{k+1} &= f\left(x_k, u_k, d_k\right) \\ y_k &= Cx_k \end{aligned} \tag{5}$$

with $x_k \in \mathbb{R}^{n_x}$ and $u_k \in \mathbb{R}^{n_u}$ vectors representing respectively the state, the input and the disturbances (rainfall) of the system at time step k and $y_k \in \mathbb{R}^{n_y}$ a vector containing the outputs of the system at time step k. The state x_k consists of the water levels, the discharges and the node volumes of the water system. The output y_k consists of the water levels of the system as the goal of MPC is to steer the water levels. The input u_k consists of the level of the different gates.

MPC tries to steer the future outputs to the reference value y_r while trying to keep the inputs as close as possible to the reference input u_r without violating constraints on the inputs and the states. MPC achieves this by optimizing on-line at each time step an optimal control problem. In the case of the nonlinear system (5) the online optimal control problem is a nonlinear programming problem defined as:

$$\min_{\substack{x_{k+1},\ldots,x_{k+N_p} \\ y_{k+1},\ldots,y_{k+N_p} \\ u_k,\ldots,u_{k+Nc-1}}} \sum_{i=1}^{N_p} \left(y_{k+i}-y_r\right)^T Q \left(y_{k+i}-y_r\right) + \sum_{i=0}^{N_c-1} \left(u_{k+i}-u_r\right)^T R \left(u_{k+i}-u_r\right) \tag{6}$$

subject to the following constraints for $i = 1,\ldots,N$, $j = 0,\ldots,N_c-1$:

$$x_{k+1} = f(x_k, u_k, d_k) \tag{7}$$
$$y_k = Cx_k \tag{8}$$
$$\underline{y} \le y_{k+i} \le \overline{y} \tag{9}$$
$$\underline{u} \le u_{k+j} \le \overline{u} \tag{10}$$
$$\left\| u_{k+j+1} - u_{k+j} \right\|_\infty \le \Delta_{max} \tag{11}$$

with N_p the prediction horizon and N_c the control horizon. The prediction horizon is usually bigger than the control horizon. This optimal control problem is a non-linear program due to the nonlinear equality constraint (7) representing the internal model of the water system. In order to solve this nonlinear program the nonlinear constraint (7) is replaced by a linear-time varying system that is obtained by doing a linearization around the states obtained by applying the optimal input sequence

$$\{u(k|k-1), u(k+1|k-1), \ldots, u(k+N_c-2|k-1), u(k+N_c-2|k-1)\} \tag{12}$$

obtained from the optimal control problem solved at the previous time step $k-1$. Note that for time step $k+N_c-1$ no future input can be obtained from the previous time step which is the reason this input is chosen equal to that of time step $k+N_c-2$. This input sequence can be seen as a first guess for the solution of the optimal control problem at the current time step k. For notaional convenience in the sequel this input sequence will be referred to as $\left\{u_k^0, u_{k+1}^0, \ldots, u_{k+N_c-2}^0, u_{k+N_c-1}^0\right\}$. At time step k a simulation of the nonlinear model (5) is performed by applying this input sequence and taking the future rain predictions $\left\{d_k, d_{k+1}, \ldots, d_{k+N_p-1}\right\}$ into account. This results into a sequence of states defined by $\left\{x_k^0, x_{k+1}^0, x_{k+2}^0, \ldots, x_{k+N_p}^0\right\}$. Linearizations around these states give raise to the following linear time-varying system:

$$\left(x_{k+1} - x_{k+1}^0\right) = \left(\frac{\partial f}{\partial x}\right)_k \left(x_k - x_k^0\right) + \left(\frac{\partial f}{\partial u}\right)_k \left(u_k - u_k^0\right). \tag{13}$$

This can be re-written as following linear time-varying system

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k + z_k \\ y_k &= C x_k \end{aligned} \tag{14}$$

with

$$A_k(i,j) = \frac{\partial f_i(k)}{\partial x_j(k)} \tag{15}$$
$$B_k(i,l) = \frac{\partial f_i(k)}{\partial u_l(k)} \tag{16}$$
$$z_k = x_{k+1}^0 - A_k x_k^0 - B_k u_k^0. \tag{17}$$

The partial derivatives in (15) and (16) are calculated numerically by means of finite differences. The linear time-varying system (14) is an approximation of the nonlinear model and is only valid within a certain region around the linearized states. In optimization this region is called a trust region. Taking this into account within the trust region the following optimization program gives an approximate optimal control sequence for the nonlinear model (5)

Optimization 1

$$\min_{\substack{x_{k+1},\dots,x_{k+N_p}\\ y_{k+1},\dots,y_{k+N_p}\\ u_k,\dots,u_{k+Nc-1}}} \sum_{i=1}^{N_p} (y_{k+i}-y_r)^T Q (y_{k+i}-y_r) + \sum_{i=0}^{N_c-1} (u_{k+i}-u_r)^T R (u_{k+i}-u_r) \tag{18}$$

$$x_{k+1} = A_k x_k + B_k u_k + z_k \tag{19}$$

$$y_k = C x_k \tag{20}$$

$$\underline{y} \leq y_{k+i} \leq \overline{y} \tag{21}$$

$$\underline{u} \leq u_{k+j} \leq \overline{u} \tag{22}$$

$$\left\| u_{k+j+1} - u_{k+j} \right\|_\infty \leq \Delta_{max} \tag{23}$$

$$\left\| u_{k+j} - u^0_{k+j} \right\|_\infty \leq \Delta_T \tag{24}$$

with (24) the constraints defining the trust region for which the linear time-varying approximation is valid. Note that the optimization program is now a quadratic program. After solving optimization (1) the obtained optimal control sequence can be used to perform a new simulation leading to a new sequence of future states around which a new linearization can be made and the optimization can be redone. This sequence can be repeated untill convergence is obtained. The following algorithm summarizes this optimization procedure

Algorithm 1

1. *At time step k initialize the inputs* $\left\{u^0_k, u^0_{k+1}, \dots, u^0_{N_c-1}\right\}$ *with (12).*
2. *Perform a simulation with the input sequence* $\left\{u^0_k, u^0_{k+1}, \dots, u^0_{N_c-1}\right\}$ *and obtain the future states* $\left\{x^0_k, \dots, x^0_{k+N_p}\right\}$.
3. *Perform a linearization around these future states and solve optimization (1).*
4. *If the difference of the new optimal input sequence* $\left\{u^*_k, u^*_{k+1}, \dots, u^*_{N_c-1}\right\}$ *with* $\left\{u^0_k, u^0_{k+1}, \dots, u^0_{N_c+1}\right\}$ *is smaller than a prespecified tolerance then stop the algorithm, else replace* $\left\{u^0_k, u^0_{k+1}, \dots, u^0_{N_c-1}\right\}$ *with* $\left\{u^*_k, u^*_{k+1}, \dots, u^*_{N_c-1}\right\}$ *and go to step 2.*

So after the solution of a sequence of quadratic programs algorithm 1 returns a local optimal solution to the initial nonlinear optimal control problem. Then the first input u_k^* of this input sequence is applied to the real system. At the next sampling time measurements on the system are performed and the current state of the system is estimated by means of a state estimator. However, in the sequel it will be assumed that the state of the real system and the internal model are exactly the same, so state estimation will not be considered. The addition of a state estimator is considered as future research.

Constraint and cost function strategy

As pointed out earlier there are two types of constraints, namely hard and soft constraints. Hard constraints are constraints that can never be violated and are typically physical limitations on the gate movements. On the other hand, soft constraints are constraints that can only be violated in case no solution can be obtained that satisfies them all. In water systems the upper limits on the water levels are considered as soft constraints. During periods of heavy rainfall the situation can occur where it is impossible to keep each water level under its corresponding flood level meaning that the constraints are too stringent. In order to obtain a feasible solution some of the soft constraints need to be relaxed or omitted from the optimization problem. In order to do this there needs to be defined a strategy that determines which soft constraints need to be relaxed. Simultaneously there also needs to be a strategy for the cost function as the objective is to steer the water levels corresponding to the relaxed soft constraints back into the feasible region.

Another reason to incorporate a variable constraint and cost function strategy is that the objective of MPC depends on the state of the water system. More specifically, during periods of light rainfall the main goal of MPC is to steer the water levels to a specified reference level. However, during periods of heavy rainfall steering the water levels to refence levels becomes less important as the focus shifts to flood prevention. This change in objectives has as consequence that the constraints as well as the objective function of MPC needs to change during operation. In the remainder of this subsection the chosen constraint and cost function strategy is outlined in more detail.

At first, during normal regulation the objective of MPC is to steer the water levels to the reference levels that are provided by the local water administration. It is important the physical limitations on the gates are satisfied. The constraints on the water levels don't play an important role here as the rainfall is too low to cause floodings. Besides this it is also important that during this period of time the water reservoirs are emptied and reach the lowest level that is allowed. Any unnecessary volume inside the reservoirs can lead to unncessary floodings in the future. Therefore during normal regulation the cost function is adjusted to steer the most important water levels to their reference levels and to empty the reservoirs as much as possible.

During periods of heavier rainfall the expert knowledge of the operators controlling the gates in the field is incorporated in the MPC. At first sight this approach might seem restrictive as MPC is not allowed to use all possible degrees of freedom that are available. However, this is a necessary step in order for the MPC to avoid flooding properly. The reason for this is that rainfall events causing floods can last more than five days. This means that the prediction horizon N_p of MPC should be taken longer than five days in order to be able to properly avoid flooding. But the prediction horizon can't be taken that long for the following reasons:

- The optimal control problem to be solved at each time step would be untractable as the number of unknown variables would be too large.
- Rain predictions are not accurate enough to predict so much in advance. Typically rain predictions are only reliable to predict 2 days ahead.

Therefore it is necessary to incorporate some predefined rules containing expert knowledge in order to prevent flooding. Within these rules it is the task of MPC to find the optimal control strategy. This expert knowledge contains the following rules:

1. For each water level a guard level is defined by the local wated administration. As long as the water levels don't violate their corresponding guard level, it is not allowed to fill the water reservoirs. The reason behind this is to keep the reservoirs as empty as possible for as long as possible. Only when MPC detects that a guard level will get violated within the prediction horizon it is allowed to use some of the storage capacity from the reservoirs to keep the water levels beneath their guard level. This can be achieved by adding the guard levels as constraints of the optimization problem and putting a high value in the cost function for deviations of the reservoir levels from their corresponding reference level. This will ensure the water reservoirs only get filled if this is absolutely necessary to avoid the violation of the guard level constraints.

2. Water reservoirs can be used to avoid violation of the guard levels. However, it is not allowed to use the complete storage volume available in the reservoirs. For each reservoir there is defined a safety limit. Once this safety limit is reached the reservoirs may not be used anymore and the guard levels will be violated. The reason for this is to ensure there is enough storage volume available in case of further future rainfall. Therefore, in the optimization the guard levels are replaced by their flood level and constraints are added to ensure the reservoir levels stay beneath their safety level.

3. If it continues raining untill the point where MPC can't to keep the water levels beneath their flood level within the prediction window, the safety levels of the

reservoirs are removed by their flood levels. By doing this MPC can use all the available storage volume in the reservoirs in order to keep all water levels beneath their flood level.

4. If the rainfall is really excessive then flooding is unavoidable and some or all water levels will violate their corresponding flood levels and the optimization proposed in step 3 will be unfeasible. In order to be able to get a usefull solution from MPC soft constraints need to be removed from the optimization untill a feasible solution is obtained. If a soft constraint on a specific water level is removed, the weight in the cost function corresponding to that water level is increased. This is done in order to minimize the soft constraint violation.

Note that the removal of soft constraints outlined in step 4 is based on a priority based strategy. First, different sets with different priorities are defined. Soft constraints that are more important than others are categorized into sets with higher priorities. Soft constraints of equal importance are categorised into the same priority set. If the MPC defined in step 3 turns out to be infeasible all the soft constraints belonging in the set with the lowest priority are removed from the optimization and in the cost function the weights of the corresponding water levels are increased. If the optimization still turns out to be unfeasible the soft constraints from the second lowest priority set are removed and the corresponding weights are adjusted in the cost function. This procedure is repeated untill a feasible solution is obtained.

Uncontrollable modes

In river control (irrigation channels) the discharges over the gates are typically chosen as control inputs. As a result MPC returns a sequence of future optimal discharge sequences. This sequence is passed as setpoints to local controllers that try to follow their setpoint as close as possible. The advantage of this approach is that the underlying system dynamics turn linear. In literature many examples can be found where this approach has been applied succesfully ([9],[10],[11]). All these examples concern cases where the rainfall is relatively small and where the goal is to steer the water levels to a given reference level. In this work however the main focus is flooding prevention. In this case taking the discharges over the gates as inputs might cause problems:

- It is difficult to take constraints on the gates like upper/lower limits and maximal gate movements into account.
- The relation between the discharge over a gate depends on the gate level and the water level up- and downstream the gate. This dependence is nonlinear and by taking the discharges as inputs this nonlinearity is neglected which can be problematic during heavy rainfall events.

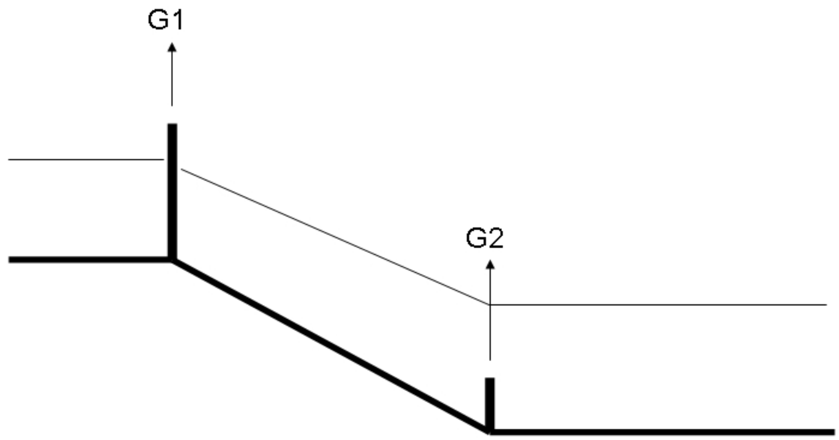


Fig. 15 Two examples of gates that are in an uncontrollable mode.

- The discharge over a gate depends on the water level up- and downstream the gate. But the water levels at their turn also depend on the discharge over the gate. Therefore, if the discharges over the gates are taken as control inputs, it is not sure that the computed discharges can be achieved.

Because of these reasons in this work the gate levels are chosen as control inputs. A difficulty that arises when taking the gate levels as control inputs is that the gates can become locally uncontrollable. This uncontrollability is related to the discharge equations, system equations determining the discharge over a gate. These equations consist of different modes. In some of these modes the discharge equation is independant of the gate level. If a gate is situated in such a mode the gate becomes locally uncontrollable. Because MPC relies on local linear models this means that the controller is not able to control the gate anymore as it thinks that the gate has no influence on the water system.

In Fig. 15 two uncontrollable modes are depicted. Gate $G1$ is closed and therefore the discharge over $G1$ is equal to zero. This is the case for as long as the gate is closed which means that the discharge over a closed gate is independant of the gate level. During normal operation the gates controlling the flow to the flood reservoirs are typically in this mode. This can decrease the performance of the controller because controllability is only recovered whenever the water level upstream of the closed gate raises above the gate level. At the other hand gate $G2$ lies much lower than its up- and downstream water level. Therefore gate $G2$ doesn't influence the discharge going from upstream to downstream which means the gate is also incontrollable. Gate $G2$ can only recover from this uncontrollability if one of the 2 water levels decreases to a level lower than the gate level, or if both water levels decrease to a level at which the gate starts to have an influence again on the discharge.

Losing the controllability of a gate can significantly decrease the performance of the controller. Therefore, the controller must try to steer each uncontrollable gate back into a controllable mode. As explained previously, at each time step a simulation is done in order to approximate the nonlinear system 5 by a linear time-varying system 14. At this point a check can be done to determine for each gate whether it will get uncontrollable within the time-window. This can be done by inspecting the colums of the B_k matrix in system equation 14. If a certain column p of B_k consists of only zeros, then the corresponding input $u_k(p)$ is uncontrollable at time step k. Now assume that the water levels uo -and downstream of input $u_k(p)$ are equal to h_u and h_d respectively. In order to steer that gate back into a controllable mode its corresponding value for the reference input u_r at time step k is modified into a value that lies between h_u and h_d. Because the controller ”thinks” the gate doesn’t has an effect on the water levels at that specific time step, it will try to steer the gate to the new reference input and recover its controllability.

In Fig. 16 a schematical overview is given of the complete nonlinear MPC scheme used in this study.

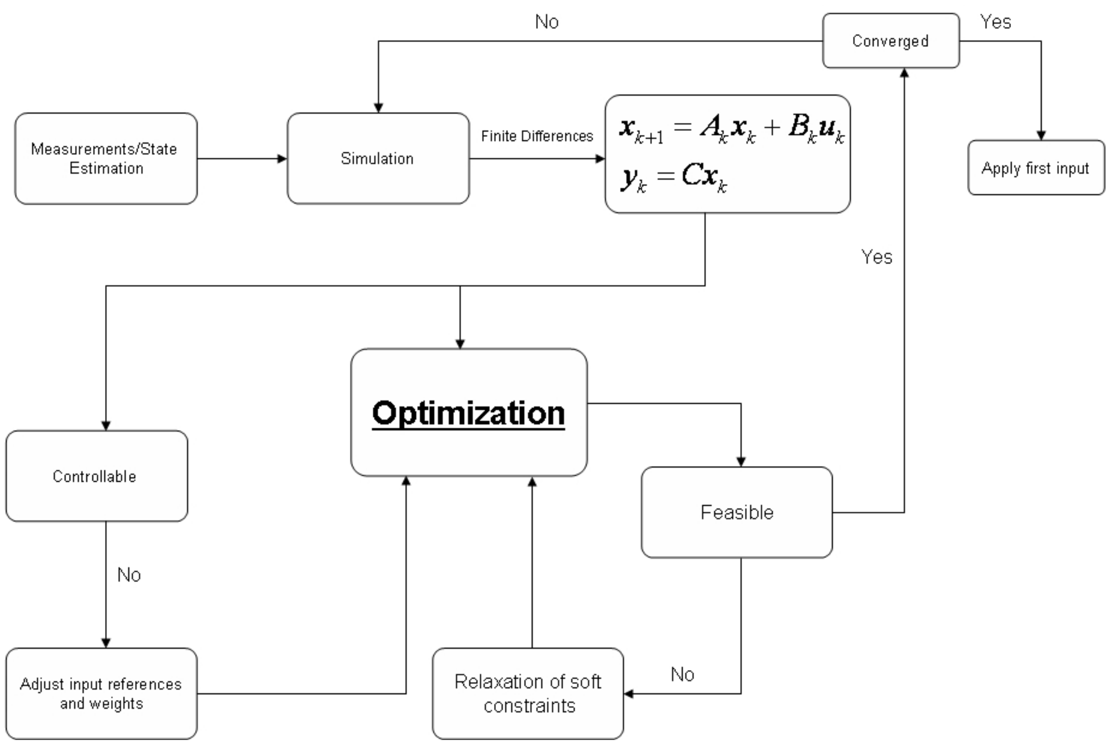


Fig. 16 Schematical overview of the presented nonlinear NMPC scheme.

4 Experimental Results

As mentioned in section 2 this study was divided in two parts. In a first phase a ”reduced area” around the Schulensmeer reservoir has been considered and a seperate conceptual model was developed for this. In a second phase a much larger region of the Demer basin was considered containing the two flood control reservoirs ”Webbekom” and ”Schulensmeer”. MPC was tested in both phases and compared with the current three-position controllers of the local water administration. In the following these results are presented and discussed.

Experimental results on the ”reduced area” around Schulensmeer

The ”reduced area” considered here is depicted in Fig. 4. There are 3 water levels that need to be kept underneath their corresponding flood level. In order to achieve this MPC needs to control the 3 gates that are available. The control objectives can be summarized as follows:

- Steer h_{opw} as close as possible to 21.5 m.
- Gates can’t move at a rate faster than 0.1 m/hour.
- The upper and lower limit on the gates are the same for the 3 gates and are equal to 23 m and 20 m respectively.
- The guard level of h_{opw} is equal to 23 m; the safety limit for the reservoir level h_s is 23 m.
- The flood levels of the 3 water levels are:
  - $h_{opw} \leq 23.2$ m
  - $h_s \leq 23.2$ m
  - $h_{afw} \leq 22.75$ m

In Fig. 17 and Fig. 18 simulations are depicted with a three-position controller and a MPC controller respectively . Both simulations are based on the same flooding event of 1998. From the figures it can be seen that with MPC the water level h_{opw} lies much closer to the reference level of 21.5 m. Besides this one can notice that with MPC all water levels stay beneath their flood level. Only at hour 800 there is a small floodings of h_{afw}. However, the simulation with the three-position controller shows severe floodings of all the water levels especially around hour 800. The MPC clearly outperforms the three-position controller. (Note: These results were published in [15]).

Experimental results on the large area

The large area considered here corresponds to the area presented in Fig. 3. The objectives of the controller are summarized in the following:

- Steer h_{opw} and hbg_{opw} as close as possible to respectively 21.5 m and 23.8 m during normal operation.

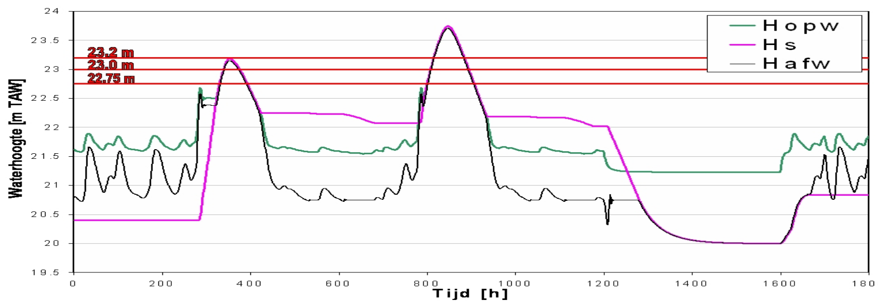


Fig. 17 Regulation of the reduced area conceptual model with a three-position controller

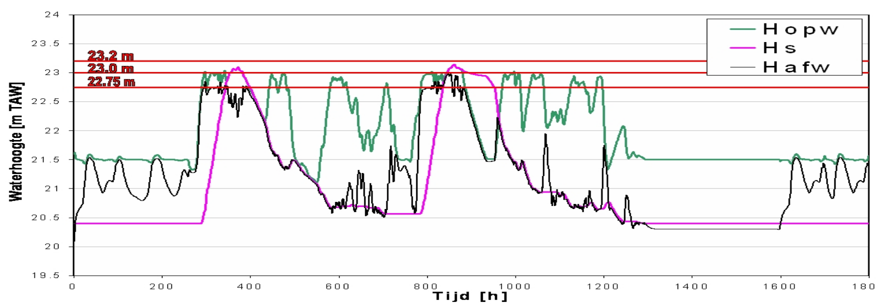


Fig. 18 Regulation of the reduced area conceptual model with MPC

- Gates can't move at a rate faster than 0.1 m/hour.
- For each gate an upper and a lower limit is taken into consideration.
- The guard levels for the reservoirs h_s and h_w are equal to 23 m and 22 m.
- The safety limit for h_{opw} is equal to 23 m.
- The flooding levels for the most important water levels are :
  - $h_{opw} \leq 23.2m$
  - $h_s \leq 23.2m$
  - $h_2 \leq 22.737m$
  - $hbg_{opw} \leq 24.84m$
  - $h_w \leq 22.4m$
  - $h_{afw} \leq 20.46m$
  - $h_{gl} \leq 22m$

Simulations were done with historical rain data from 1998. In Fig. 19 a simulation with the three-position controller is presented. In Fig. 20 the same simulation is performed but with the NMPC scheme displayed in Fig. 16. In both figures the flooding levels are indicated as red lines. Comparing both results leads to following conclusions:

- The NMPC controller manages to steer h_{opw} and hbg_{opw} much closer to their setpoints than the three-position controller.

- The three-position controller violates the flood level constraints for water levels hbg_{opw}, h_w, hgl and h_2. The NMPC controller only violates the flood level constraints of water level h_2. Also note that the violation of h_2 is smaller than the violation that occurs with the three-position controller.
- The simulation with the NMPC controllers show that the peaks of the water levels are all lower than the peaks of the water levels in the simulation with the three-position controller. So in case of NMPC there is still some extra storage volume available. This is not the case for the three-position controller.
- It can be seen clearly that NMPC manages to steer the water levels to their limits in order to use the available capacity as much as possible. This can be seen by inspecting how the water levels h_w and h_{gl} are pushed to their flood level constraint.

The simulations in the large area again show that NMPC outperforms the existing three-position controller.

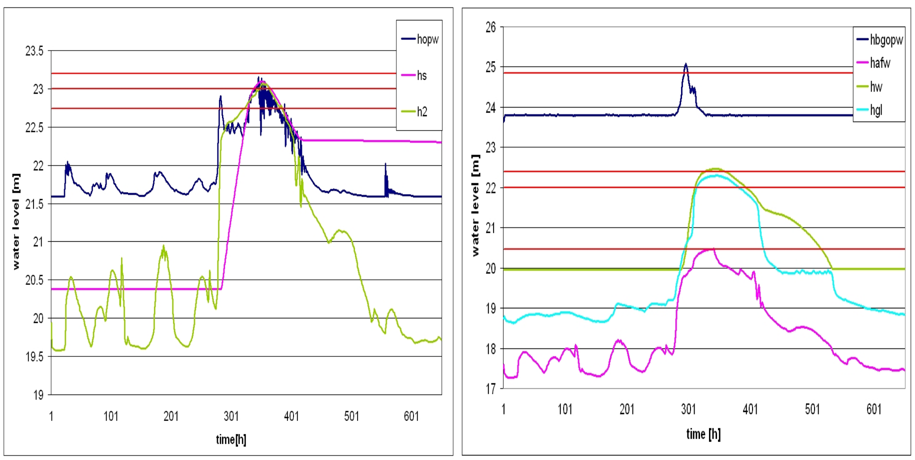


Fig. 19 Regulation with three-position controller

5 Future Research

The main purpose of this study was to proof that NMPC can be used for real-time flood regulation and that it works better than the current three-position controller. The results in this chapter show that MPC is capable to work closer to the limits of the constraints which resulted into improved performance with respect to flooding regulation. However, there are still some issues that should be considered before

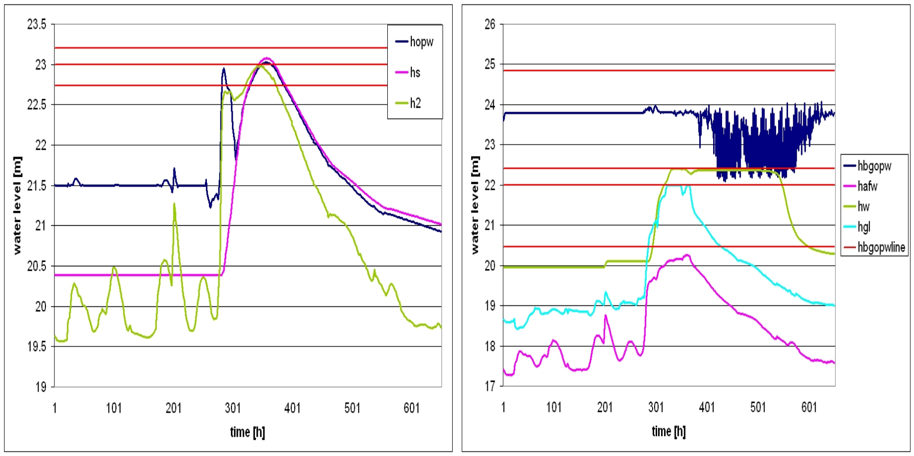


Fig. 20 Regulation with NMPC controller

implementing NMPC on the field:

- State estimation: In this chapter it was assumed that the full state of the system was known. In practice this is never the case. In practice only the most important water levels are measured; discharges are not measured. Therefore, it is necessary to perform a state estimation at each time step. The Kalman filter is the standard choice for state estimation of linear systems when measurements are noisy and process disturbances unknown. Often additional insights about the process can be obtained in the form of inequalities. In this case better results are obtained if the estimation problem is formulated as a quadratic program. These kind of state estimators are called moving horizon estimators (MHE) [7]. MHE can be seen as the dual of MPC. Similar to MPC, MHE uses a finite-time window of past measurements. Based on these measurements the estimation problem is optimized and an optimal state estimation is performed. MHE for nonlinear models also exists and is called nonlinear MHE (NMHE) [8]. Typically NMHE outperforms Kalman filter approaches when the underlying process models are nonlinear. In case of flooding regulation the underlying models are highly nonlinear and therefore NMHE seems the most suitable option for the state estimation.

- Robustness: MPC makes use of an internal model in order to make future predictions of the output. An inherent property of models is that their predictions are uncertain by definition. Models are a simplification of the real world and as a consequence the predictions made with a model are never going to be followed exactly in the real world. This means that each output returned by the model is subject to uncertainty. In case of water systems an extra uncertainty on the predictions made by MPC comes from the uncertainty of the rain predictions. These uncertainties can deteriorate the on-line performance of MPC if they are not considered in the optimal control problem. Therefore future research could consider to quantify these uncertainties. In a next step these uncertainties can be taken into consideration into the optimization problem. This can be done by extending the multiple model configuration of [9] to flooding regulation.

- Consensus between different water authorities: In this study the main focus was to avoid flooding in the Demer basin. However, each action taken in the Demer basin will also affect water systems that lie downstream of the Demer basin. At their turn actions taken in water systems downstream of the Demer basin can have an effect on the water levels of the water system in the river Demer basin. What must be avoided is that actions taken in order to avoid flooding in one basin leads to severe floodings in other basins. Therefore, there is a need of cooperation and coordination between the different local water management bodies. A possible way to achieve this is by using a multi-agent model predictive controller [12].

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References

1. Chow, V.T., Maidment, D.R., Mays, L.W., 1988. Applied hydrology. McGraw-Hill International Editions.
2. OBM-Demer, 2003. Operational river basin model Demer - Technical documentation v.2.0 (in Dutch), Flemish Environment Agency, Aalst, Belgium.
3. Vaes, G., Willems P., and Berlamont, J. 2002. The use of reservoir models for the assessment of the input from combined sewer overflows into river models, 9th Intern. Conference on 'Urban Drainage', Portland, 8-13 september 2002, CD-ROM proceedings: p. 16
4. WS (2008a). InfoWorks-RS, version 8.5. Wallingford Software & Halcrow, UK.
5. WS (2008b). FloodWorks, version 9.5. Wallingford Software & Halcrow, UK.
6. Rogers, D. C., and Goussard, J. (1998). "Canal control algorithms currently in use.", J. Irrig. Drain. Eng., 124(1), 1115.
7. Rao, C.V. and Rawlings, J.B. and Lee, J.H. (2001),"Constrained Linear State Estimation - A Moving Horizon Approach",Automatica 37(10),1619-1628.
8. Rao, C.V. and Rawlings, J.B. and Mayne, D. (2003), "Constrained State Estimation for Nonlinear Discrete-time systems: Stability and Moving Horizon Approximations",IEEE Transactions on Automatic Control,48,246-258.
9. Overloop, P.J. van (2006), "Model Predictive Control of Open Water Systems", PhD thesis, Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department of Water Management, Delft, The Netherlands.
10. Bautista, E., A.M.ASCE; Clemmens, A.J., M.ASCE; and Strand, R.J. (2006), "Salt River Project Canal Automation Pilot Project: Simulation Tests", Journal of irrigation and drainage engineering, 132(2), 143-152.
11. M. Marinaki and M. Papageorgiou, "Optimal Real-Time Control of Sewer Networks", Springer, 2005.

12. Negenborn, R.R. (2007), ”Multi-Agent Model Predictive Control with Applications to Power Networks”, PhD thesis, Delft University of Technology, Delft, The Netherlands.
13. Camacho, E.F. and Bordons, C. (1999), ”Model Predictive Control.”, Springer, London.
14. Rossiter, J.A. (2000), ”Model Predictive Control: A Practical Approach.”, CRC press.
15. Barjas Blanco, T., Willems, P., De Moor, B. and Berlamont, J., ”Flooding prevention of the Demer using Model Predictive Control”, IFAC World Congress, Volume 17, part 1.
16. Overloop, P.J. van, Mursi-Batt, A., Heeringen, K.J. van, Thabet, R.A.H.A. (2001), ”Real-Time-Control of Water Quantity and Quality in a Re-Use of Drainage Water Project” , Proceedings International ICID-Conference, Seoul, S-Korea.