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CONTRIBUTION

TITLE: **On the Optimality of Iterative Waterfilling**

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ABSTRACT

The *iterative waterfilling* algorithm was recently presented and demonstrates the large gains that can be achieved through the use of *Dynamic Spectrum Management* (DSM). In this contribution we evaluate the optimality of iterative waterfilling in various scenarios by comparison with the *Optimal Spectrum Management* (OSM) algorithm. OSM determines the theoretically optimal transmit spectra for the modems within a network thereby allowing them to achieve the maximum possible bitrates. We show that in many scenarios, the potential benefits of DSM can be even larger than those already demonstrated by iterative waterfilling. This demonstrates the huge potential of DSM and motivates further work in the area.

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On the Optimality of Iterative Waterfilling

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ABSTRACT

The *iterative waterfilling* algorithm was recently presented and demonstrates the large gains that can be achieved through the use of *Dynamic Spectrum Management* (DSM). In this contribution we evaluate the optimality of iterative waterfilling in various scenarios through a comparison with the *Optimal Spectrum Management* (OSM) algorithm. OSM determines the theoretically optimal transmit spectra for the modems within a network thereby allowing them to achieve the maximum possible bitrates. We show that in many scenarios, the potential benefits of DSM can be even larger than those already demonstrated by iterative waterfilling. Achievable data-rates can be increased by up to 270%. This demonstrates the huge potential of DSM and motivates further work in the area.

1. Introduction

The *iterative waterfilling* (IW) algorithm was recently presented and demonstrates the large gains that can be achieved through the use of *Dynamic Spectrum Management* (DSM) techniques[1].

The basic premise behind DSM is that the modems within a network can adapt their transmit spectra to mitigate crosstalk, and avoid poor parts of the transmission spectrum due to bridged taps, RFI interferers, etc. By dynamically adapting the spectra of the modems within a network DSM can increase the data-rate of services offered to customers and increase coverage (through extended service reach).

Since the modems in a DSM enabled network adapt their behaviour dynamically as new sources of interference appear, DSM can also decrease loss of service, increasing reliability and decreasing customer complaints.

The current state-of-the-art DSM algorithm *iterative waterfilling* (IW) can potentially increase data-rates by up to 300% and typically increases reaches by 5-10% as well. One of the benefits of IW is its relative simplicity and the fact that it can be implemented autonomously. That is, each modem can independently determine its own transmit spectra using measurements of its own background noise and direct channels. So no *dynamic spectrum management centre* (D-SMC) is required, as is the case with centralised algorithms.

Despite these positive aspects IW is a *sub-optimal* algorithm. In IW all modems converge to the so-called *competitively optimal* operating point. There is no guarantee that this competitively optimal point is *globally optimal.*

As will be shown, in some situations IW can be significantly sub-optimal. This implies that, in addition to the already spectacular gains that IW provides, significantly more improvement is possible through the application of more advanced DSM algorithms.

Recently a new DSM algorithm was presented known as *Optimal Spectrum Management* (OSM)[2]. This algorithm determines the theoretically optimal transmit spectra for the modems within a network. This allows the modems to achieve the maximum possible bitrates, operating on the boundary of the so-called *rate region.* OSM is a centralised algorithm and requires the use of a D-SMC.

In this contribution we evaluate the optimality of IW in various scenarios through comparison with OSM. We show that in many scenarios, the potential benefits of DSM can be even larger than those already demonstrated by IW. This demonstrates the huge potential of DSM and motivates further work in the area.

2. Performance: Iterative Waterfilling vs. Optimal Spectrum Management

To evaluate the optimality of IW we compare it with the globally optimal solution that can be found with OSM. More details on the OSM algorithm can be found in the appendix.

We examine a number of scenarios. In all simulations we use 26 AWG (0.4mm) lines. The target bit error probability is 10^{-7} or less. The coding gain and noise margins are set to 3 dB and 6 dB respectively. The background noise is set at −140 dBm/Hz and also includes crosstalk from 10 ISDN, 4 HDSL and 10 legacy *Static Spectrum Management* (SSM) ADSL modems which operate in margin adaptive mode.

In the ADSL scenarios we focus on downstream transmission. The maximum transmit power is limited to 20.4 dBm. No spectral masks are applied to the IW or OSM techniques. In the VDSL scenario we consider upstream transmission and the maximum transmit power is limited to 11.5 dBm. We employ the 998 FDD bandplan. A spectral mask at −60 dBm/Hz is applied to the reference PSD method. No spectral masks are applied to the IW or OSM techniques.

We assume that the modems are not synchronised. As a result sidelobes will cause spectral leakage between neighbouring tones and *inter-carrier interference* (ICI). In ADSL this causes some performance loss. In VDSL the performance loss is minimal since the transmitter and receiver windowing helps to suppress the sidelobes. It should be noted that the OSM algorithm does not take the effect of sidelobes into account when determining the optimal spectra. To do this would result in a computationally intractable problem with huge complexity. Despite this, OSM still yields significant gains over the other spectrum management algorithms in many scenarios. We do take the effects of sidelobes into account when evaluating the performance of the different algorithms, so a fair comparison is made.

In all simulations we allow the modems to support any continuous valued bitloading. This is because the optimal PSDs generated with the continuous bitloading give more intuition than with discrete bitloading. A version of OSM which only allows discrete bitloadings also exists[2]. Using this algorithm gives similar results to those obtained here and the main conclusions are the same. We use the Shannon capacity formulas to relate SINR to bitrate, and an SNR gap to capacity Γ such that the BER is $< 10^{-7}$.

2.1 ADSL CO Distributed

The first scenario we consider consists of two ADSL modems of 3 km and 4 km that are distributed from a *central office* (CO). This is depicted in Fig. 1. We compare the performance of *Margin Adaptive* (MA) mode, IW and OSM. In MA mode the modems transmit at –40 dBm/Hz.

Shown in Fig. 2 are the rate regions supported using the different spectrum management algorithms. Shown in Table 1 are some example operating points that can be achieved by the different algorithms. Fig. 3 and 4 depict the corresponding PSDs. The effect of sidelobes in this scenario is minimal.

As can be seen IW gives moderate gains, around 40% on the 4 km line. The gains in IW come from 2 sources:

- More intelligent allocation of transmit power on the long line towards the lower frequencies which have a higher SNR
- Intelligent avoidance of interference

In the CO distributed case the majority of the gains come from the first point.

We note that IW gives the same data-rates and PSDs as OSM. We found this to be the case *in general* for scenarios with CO distributed ADSL modems. So IW is effectively optimal in that case. Furthermore, every modem that does not adopt an IW PSD adds spectral pollution to the network, limiting the performance of all other lines.

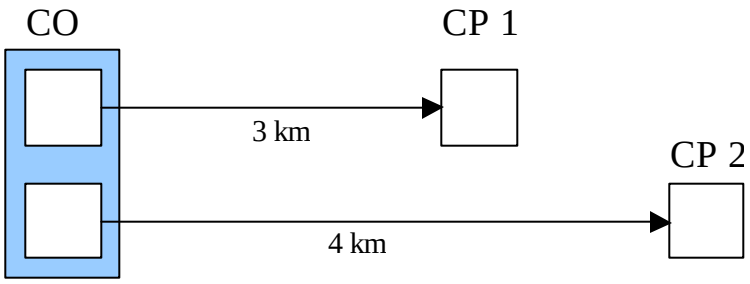

Figure 1 – ADSL CO Distributed Scenario

Technique	4 km Rate (Mbps)	3 km Rate (Mbps)
Margin Adaptive (MA)	1.0	4.4
Iterative Waterfilling (FM)	1.4 (+40%)	4.5
OSM	1.4 (+40%)	4.5

Table 1 – ADSL CO Distributed: Example Achievable Rates

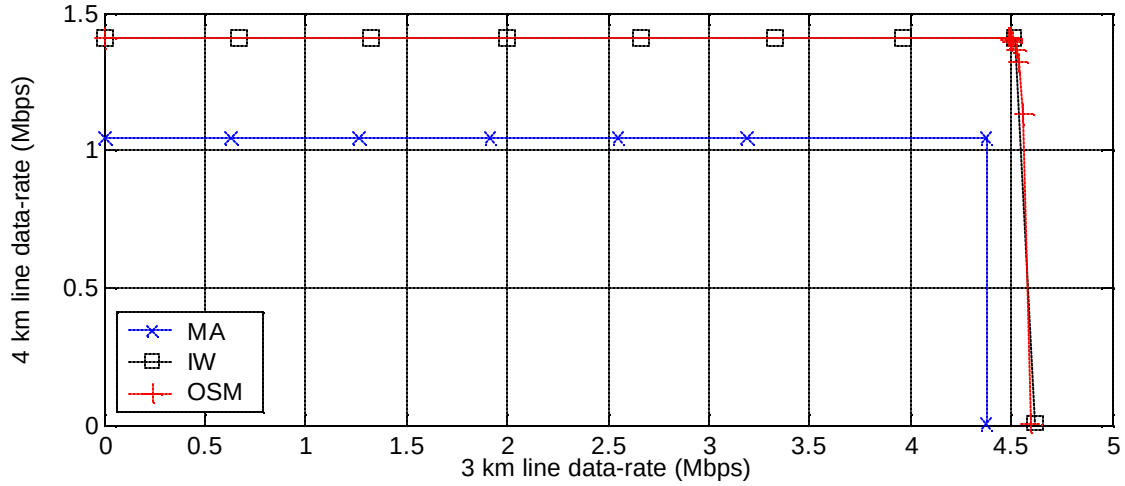

Figure 2 – ADSL CO Distributed: Rate Regions

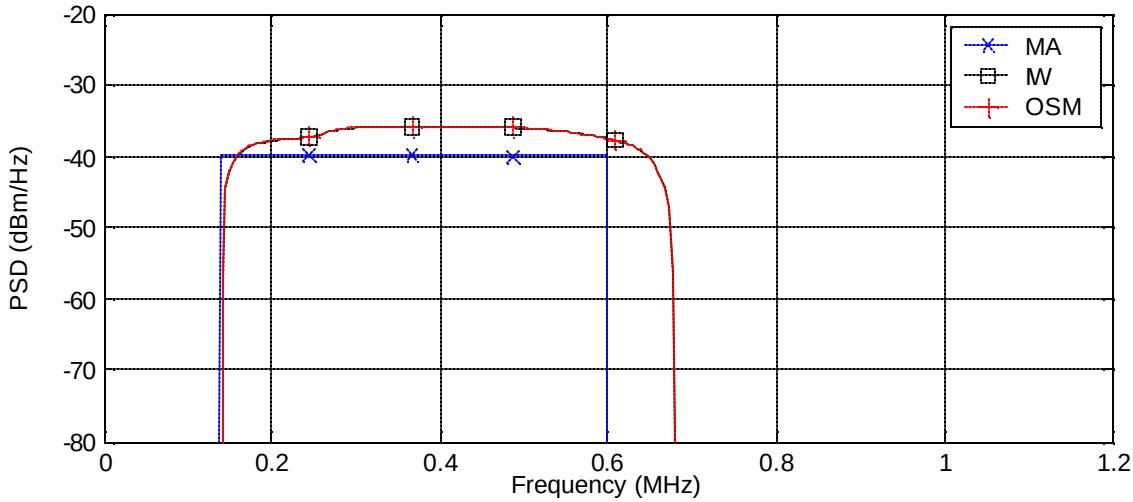

Figure 3 – ADSL CO Distributed: 4 km line PSDs

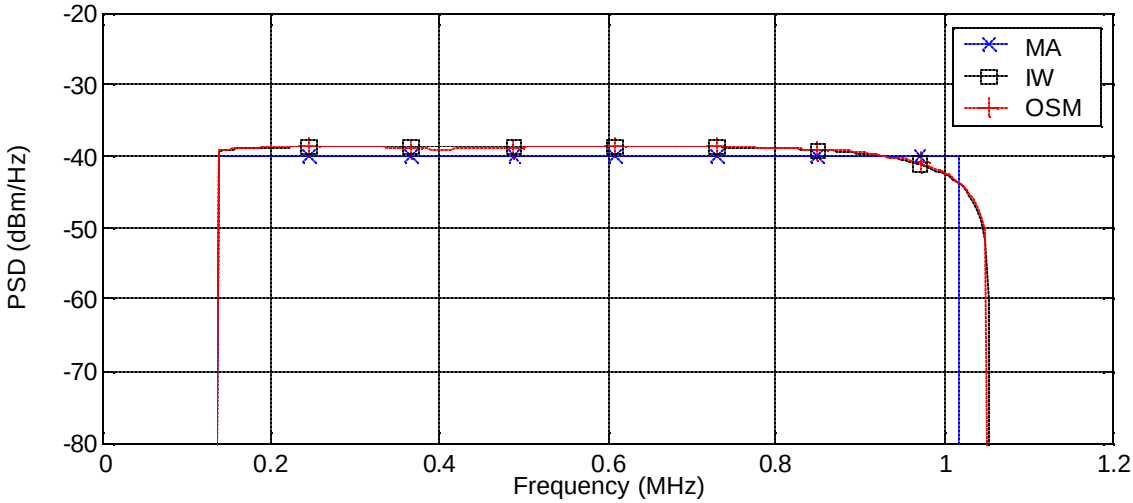

Figure 4 – ADSL CO Distributed: 3 km line PSDs

2.2 ADSL CO/RT Mix

We now consider scenarios with a mixture of CO and RT distributed ADSLs. In this case we consider a CO distributed ADSL of 4 km and a *remote terminal* (RT) distributed ADSL of 3 km. The RT is located 3 km from the CO as depicted in Fig. 5. We compare the performance of MA, IW and OSM. In MA mode the CO modems transmit at –40 dBm/Hz. In MA mode we assume that the RT modems transmit at the minimal PSD (maximum power-backoff) which is –52 dBm/Hz.

Shown in Fig. 6 are the rate regions supported using the different spectrum management algorithms. We assume that the CO ADSL must provide a minimal service of 1 Mbps. The RT ADSL does the necessary amount of *power back-off* (PBO) to ensure that this service is provided. Table 2 show the RT ADSL rates that can be achieved by the different algorithms. Fig. 7 and 8 depict the corresponding IW and OSM PSDs that support a 1 Mbps target rate on the CO ADSL. Note that with IW more PBO is required on the RT ADSL to achieve the target rate on the CO ADSL.

With MA it is not possible to achieve a 1 Mbps service on the CO ADSL, and only 0.5 Mbps is provided. With IW it is possible to support a 1 Mbps service on both CO and RT lines. As shown in Table 2, using the OSM algorithm allows even further gains to be achieved, pushing the RT ADSL rate to 3.3 Mbps whilst still supporting 1 Mbps on the CO ADSL. This corresponds to a gain of 230% relative to IW.

Note that this example is only given to illustrate the potential gains of OSM and IW. The gains achieved in practice will vary depending on the state of the operator's network, background noise, modem configuration and many other factors. These examples should in no way be seen as an obligation on the part of Alcatel to provide such capabilities.

To understand where these large gains come from we examine the PSD of the RT ADSL, which is depicted in Fig. 8. OSM exploits the following facts:

- At low frequencies crosstalk coupling is small, thus the RT ADSL can transmit at a high PSD with minimal degradation to the CO ADSL service.
- At high frequencies, in this case above 400 kHz, the CO ADSL can not effectively use its channel since the channel attenuation is so large. So at high frequencies the CO ADSL is not active and the RT ADSL can transmit with a high PSD. For this reason we see an increase in the PSD with OSM around 440 kHz.

As seen in Fig. 8, IW does not exploit either of these facts, and instead transmits in the intermediate frequencies where the CO ADSL is most active. As a result with IW the RT ADSL must reduce its total transmit power by a lot more (more PBO) to ensure a 700 kbps service for the CO ADSL. This is why OSM yields increased performance.

With IW the PSD on the RT ADSL is quite flat. This fact is discussed in more detail in contribution 295 [4].

Note that with OSM the PSD on the RT ADSL is quite shaped. Furthermore, this shape varies significantly depending on the scenario. So changing the line lengths or background noise can lead to a large change in the optimal RT PSD. Calculating the optimal PSD requires the OSM algorithm to be applied. This algorithm must run from a D-SMC that has knowledge of the direct and crosstalk channels of all active lines within the binder.

The effect of sidelobes in this scenario is significant due to the sudden increase in the PSD of the RT ADSL with OSM around 440 kHz. Fig 8. shows the transmit PSD of the RT ADSL as seen on the line. Note the slow roll-off below 440 kHz which is caused by the sidelobes. This decreases performance, however the gain over IW is still significant.

So using IW we can provide a basic service of 1 Mbps on both lines. Using OSM allows a basic service of 1 Mbps on the CO ADSL and an enhanced 3.3 Mbps service on the RT ADSL.

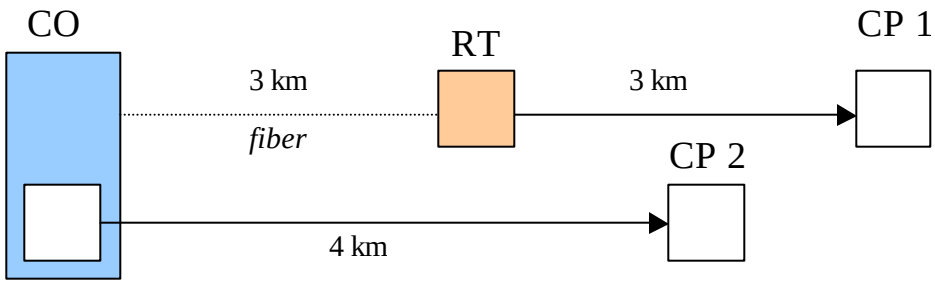

Figure 5 – ADSL CO/RT Mix Scenario

Technique	4 km Rate (Mbps)	3 km Rate (Mbps)
Margin Adaptive (MA)	0.5	1.9
Iterative Waterfilling (FM)	1.0	1.0
OSM	1.0	3.3 (+230%)

Table 2 – ADSL CO/RT Mix: Example Achievable Rates

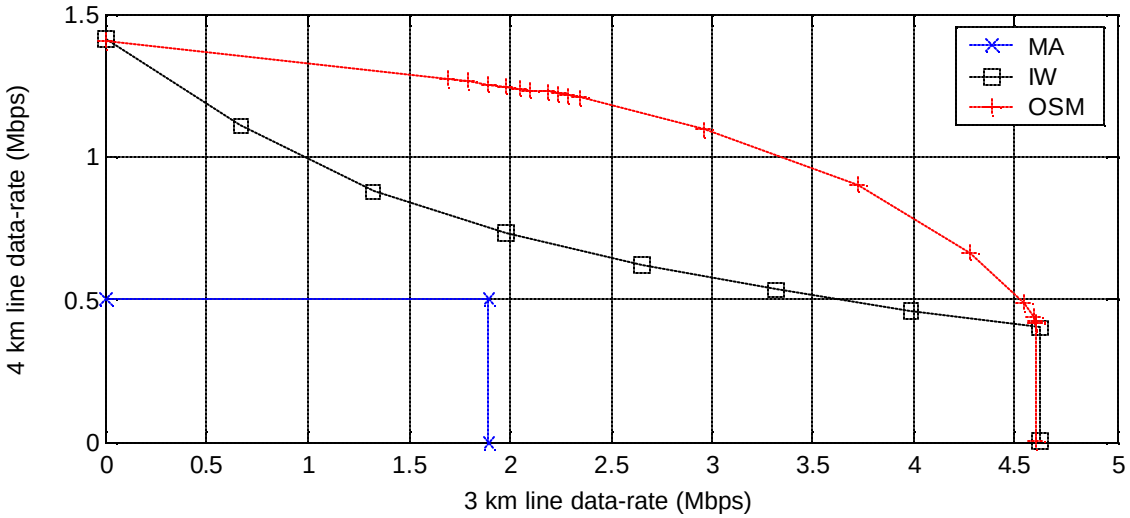

Figure 6 – ADSL CO/RT Mix: Rate Regions

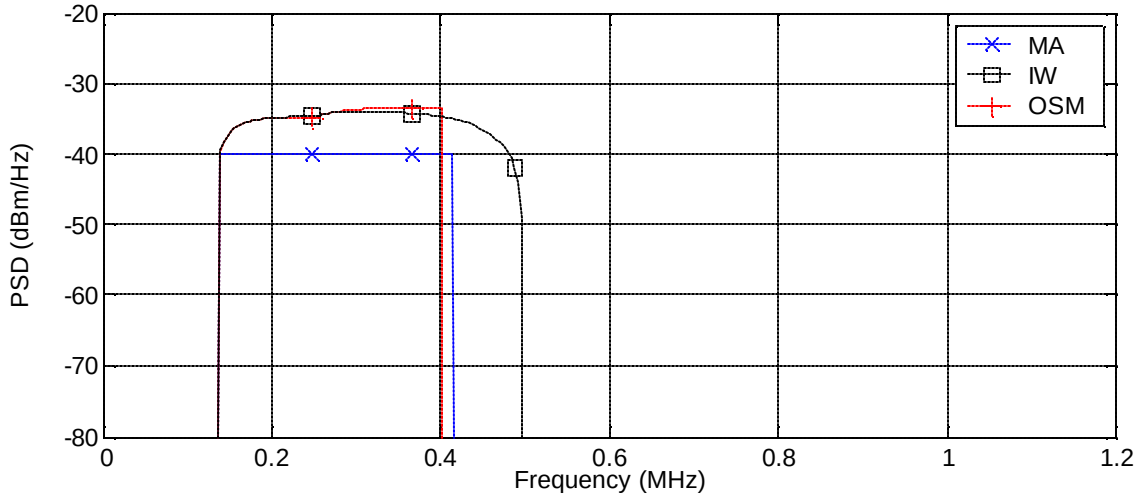

Figure 7 – ADSL CO/RT Mix: 4 km CO ADSL PSDs

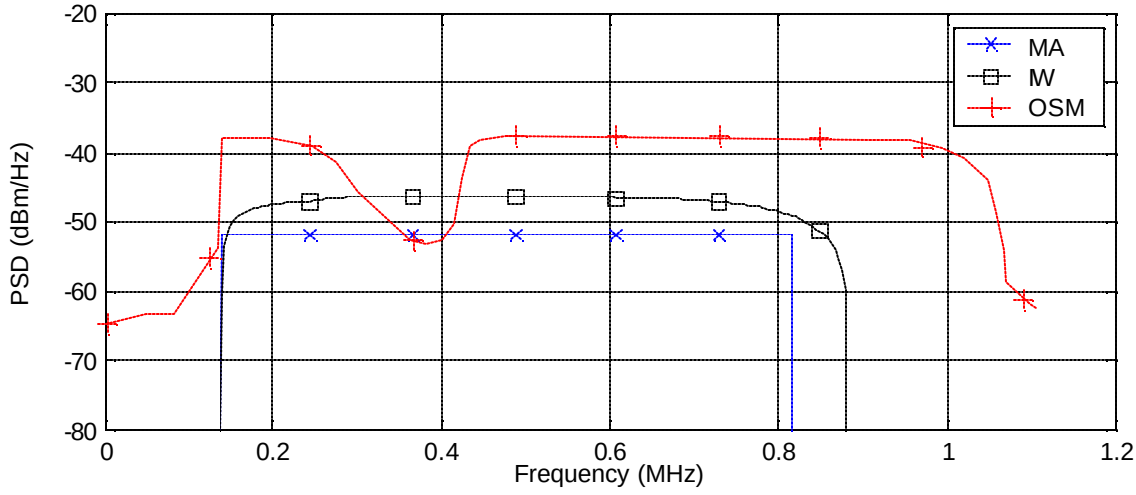

Figure 8 – ADSL CO/RT Mix: 3 km RT ADSL PSDs

2.3 VDSL Upstream

We now consider a scenario with 4 VDSL lines located 600 m from the CO and 4 VDSL lines located 900 m from the CO. This is depicted in Fig. 9. We compare the performance IW and OSM. For comparison we also include the reference PSD method which has been chosen as the PBO technique in VDSL standards[3].

Shown in Fig. 10 are the rate regions supported using the different spectrum management algorithms. Shown in Table 3 are some example operating points that can be achieved by the different algorithms. Fig. 11 and 12 depict the corresponding PSDs. The effect of sidelobes in this scenario is minimal.

We assume that a service of 5 Mbps is required on the 900 m lines. The reference PSD method can provide this service. With IW the service can be provided and the 600 m line rate will be at 10 Mbps. With OSM the service can be provided and the 600 m lines achieve a rate of 14.5 Mbps. Note that this example is only given to illustrate the potential gains of OSM and IW. The gains achieved in practice will vary depending on the state of the operator's network, background noise, modem configuration and many other factors. These examples should in no way be seen as an obligation on the part of Alcatel to provide such capabilities.

Examining Fig. 11 we see that the 900 m lines are not active in the second upstream band (8.5 – 12 MHz) due to their high channel attenuation. Examining Fig. 12 we see that OSM exploits this fact to increase the PSD of the 600 m lines in the second upstream band. This leads to the improved performance over IW, in this case a data-rate gain of 45%.

So only with IW and OSM we can provide a 5 Mbps serviced to all lines in the system. Furthermore with OSM customers on the shorter 600 m lines can be given an enhanced upstream service of 14.5 Mbps.

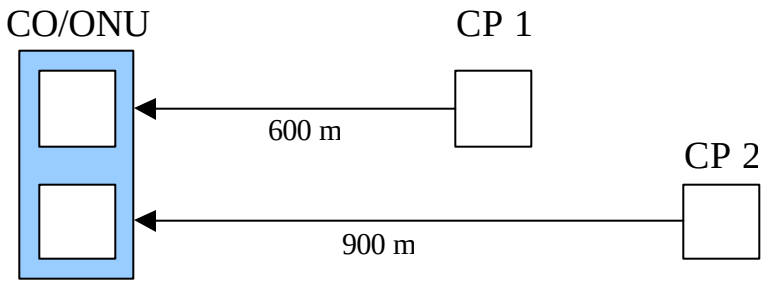

Figure 9 – VDSL Upstream Scenario

Technique	900 m Rate (Mbps)	600 m Rate (Mbps)
Reference PSD	3.9 ✗	12.0
Iterative Waterfilling	5.0 ✓	10
OSM	5.0 ✓	14.5 (+45%)

Table 3 – VDSL Upstream: Example Achievable Rates

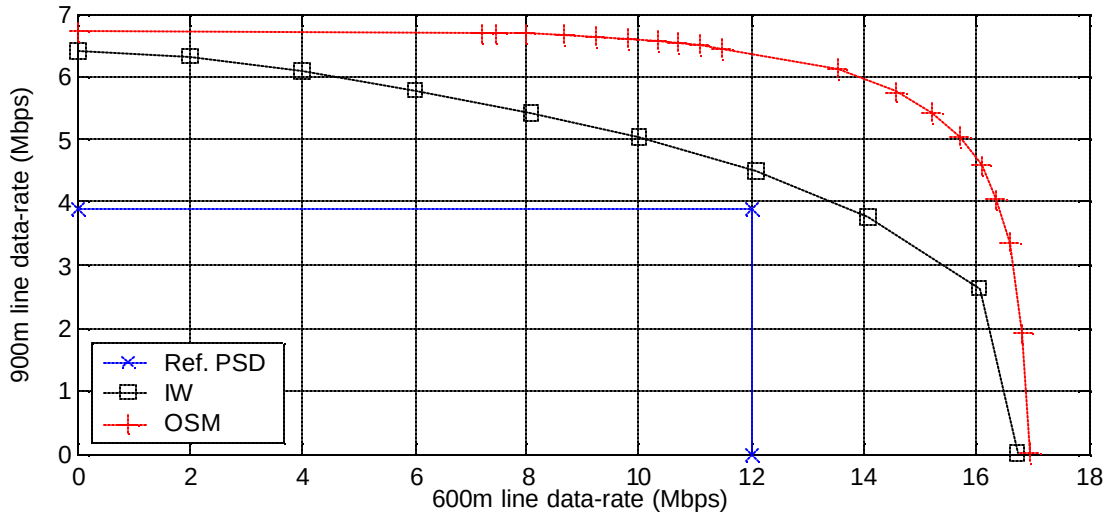

Figure 10 – VDSL US: Rate Regions

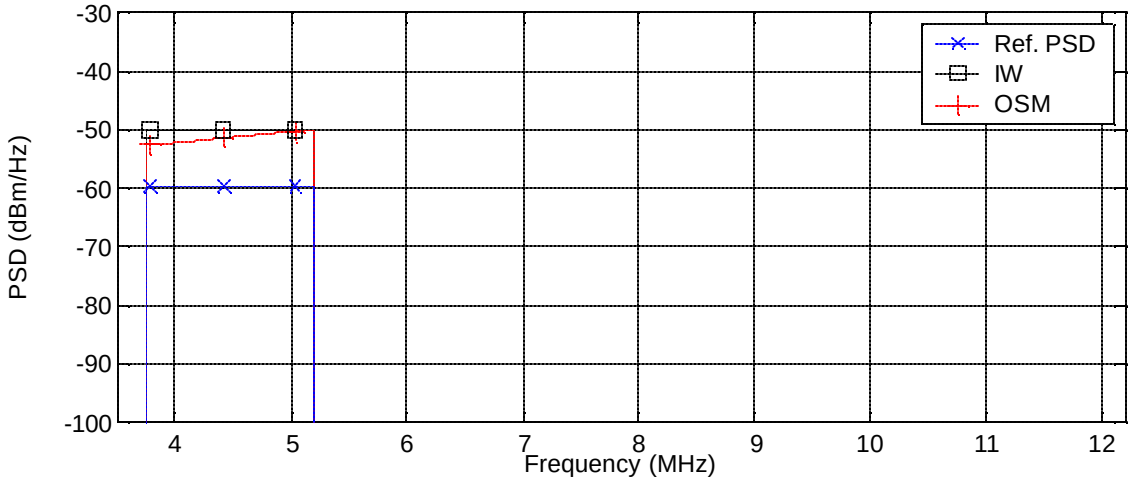

Figure 11 – VDSL US: 900 m line PSDs

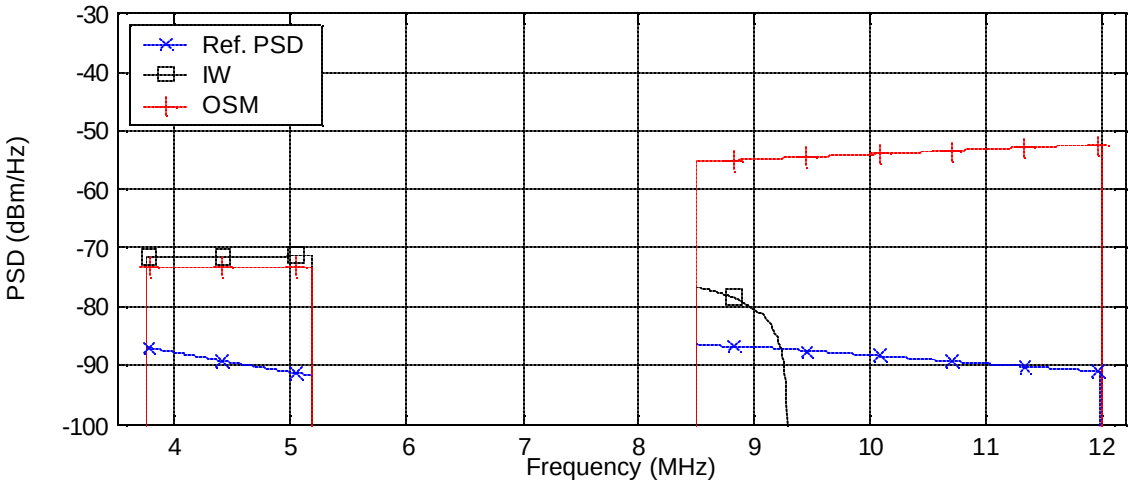

Figure 12 – VDSL US: 600 m line PSDs

3. Conclusions

In this contribution we examined the optimality of IW in a number of scenarios through comparison with the *Optimal Spectrum Management* (OSM) algorithm. We saw that in CO distributed ADSL, IW is effectively optimal, and achieves the maximum possible rates in the network.

In scenarios without co-located TXs e.g. mixtures of CO and RT distributed ADSLs, or upstream VDSL, we saw that in addition to the already spectacular gains that IW provides, significantly more improvement is possible through the application of OSM.

In the immediate future with CO distributed ADSL the most commonly deployed system, IW is the optimal solution. In the near-term as RT distributed ADSL and VDSL are deployed more advanced algorithms can help unlock the true potential of the twisted-pair medium, allowing high-speed, symmetric services to be delivered to the maximum possible number of homes and businesses.

4. Proposals

We propose that the OSM algorithm be included (informative) in Appendix A of the DSM report for centralised (Level 2) operation.

Acknowledgments

A special thanks is made to George Ginis and John Cioffi for their kind remarks.

References

[1] W. Yu, G. Ginis, J. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," in *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1105 – 1115, June 2002.

[2] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, T. Bostoen, "Optimal Multiuser Spectrum Management for Digital Subscriber Lines," *submitted to IEEE Transactions on Communications, IEEE Intl. Conf. on Communications (ICC) 2004.*
Available at http://www.esat.kuleuven.ac.be/~rcedrill/research/publications.html

[3] Very-high bit-rate Digital Subscriber Lines (VDSL) Metallic Interface, ANSI Trial-use Std. T1E1.4/2003-210R1, 2003.

[4] The target PSD obtained with iterative waterfilling is almost flat, ANSI Contribution. T1E1.4/2003-295, 2003.

Appendix. Optimal Spectrum Management

The OSM algorithm calculates the theoretically optimal spectra for a network of communication systems. This is done through the application of *optimisation theory* and a technique known as the *dual decomposition*. For ease of explanation we will describe the algorithm as applied to a system with 2 users. Extensions to more than 2 users follow in a straightforward manner.

A.1 The Spectrum Management Problem

The basic problem of spectrum management is to maximize the rate of a user (in this case user 2), subject to minimum service rates for the other users within the network (in this case user 1). Mathematically we need to maximize the rate of user 2 over all of the possible transmit PSDs for user 1 and user 2

$$\max_{\mathbf{s}_1,\mathbf{s}_2} R_2(\mathbf{s}_1,\mathbf{s}_2)$$

$$\text{s.t. } R_1(\mathbf{s}_1,\mathbf{s}_2) \geq R_1^{\text{target}} \quad (1)$$

$$\sum_k s_k^1 \leq P_{\max}$$

$$\sum_k s_k^2 \leq P_{\max}$$

Target rate constraint for user 1

Power constraints for users 1 and 2

Here s_k^n is the transmit PSD of user n on tone k, $\mathbf{s}_n = [s_1^n \cdots s_K^n]$ is a vector containing the transmit PSD of user n on all tones, and $P_{\max}$ is the maximum transmit power supported by a modem. $R_n(\mathbf{s}_1,\mathbf{s}_2)$ is the data-rate achieved by user n when transmit spectra $\mathbf{s}_1$ and $\mathbf{s}_2$ are used by user 1 and user 2 respectively. R_1^{target} is the target service data-rate for user 1.

Unfortunately (1) is a non-convex optimisation and requires complexity $O(e^{KN})$ to solve where K is the number of tones in the system and N the number of users. With K=256 in ADSL and K=4096 in VDSL this leads to a computationally intractable problem.

Using a technique from optimization theory known as the *dual decomposition* allows us to solve the spectrum management problem (1) with a linear complexity in K. This leads to the OSM algorithm which is computationally tractable and can be solved on a standard PC in a number of minutes.

A.2 The OSM Algorithm

The OSM algorithm is based on maximising the so-called *Lagrangian* on each tone. The Lagrangian on tone k is defined

$$L_k = wb_k^1(s_k^1, s_k^2) + (1-w)b_k^2(s_k^1, s_k^2) - \lambda_1 s_k^1 - \lambda_2 s_k^2$$

where $b_k^n(s_k^1, s_k^2)$ denotes the bitloading achieved by user n on tone k when user 1 and user 2 adopt transmit PSDs s_k^1 and s_k^2 respectively. The optimal transmit spectra on tone k are found by maximising L_k

$$s_k^{1,\text{opt}}, s_k^{2,\text{opt}} = \arg\max_{s_k^1, s_k^2} L_k$$

The weight w determines the desired trade-off of data-rates between user 1 and user 2. Setting w = 1 gives full priority to user 1 and user 2 is switched off. Setting w = 0 gives full priority to user 2 and user 1 is switched

off. The terms λ_1 and λ_2 are the *Lagrangian multipliers* and enforce the power constraints on modems 1 and 2 respectively.

During operation the OSM algorithm adjusts w such that the target data-rate of user 1 is just achieved. The algorithm does not give more priority to user 1 than is necessary to achieve their target data-rate, thereby maximising the data-rate of user 2. In a similar fashion λ_1 and λ_2 are adjusted such that the power constraints on both modems are enforced.

The complete algorithm is depicted in Fig. 13. For further details see [2].

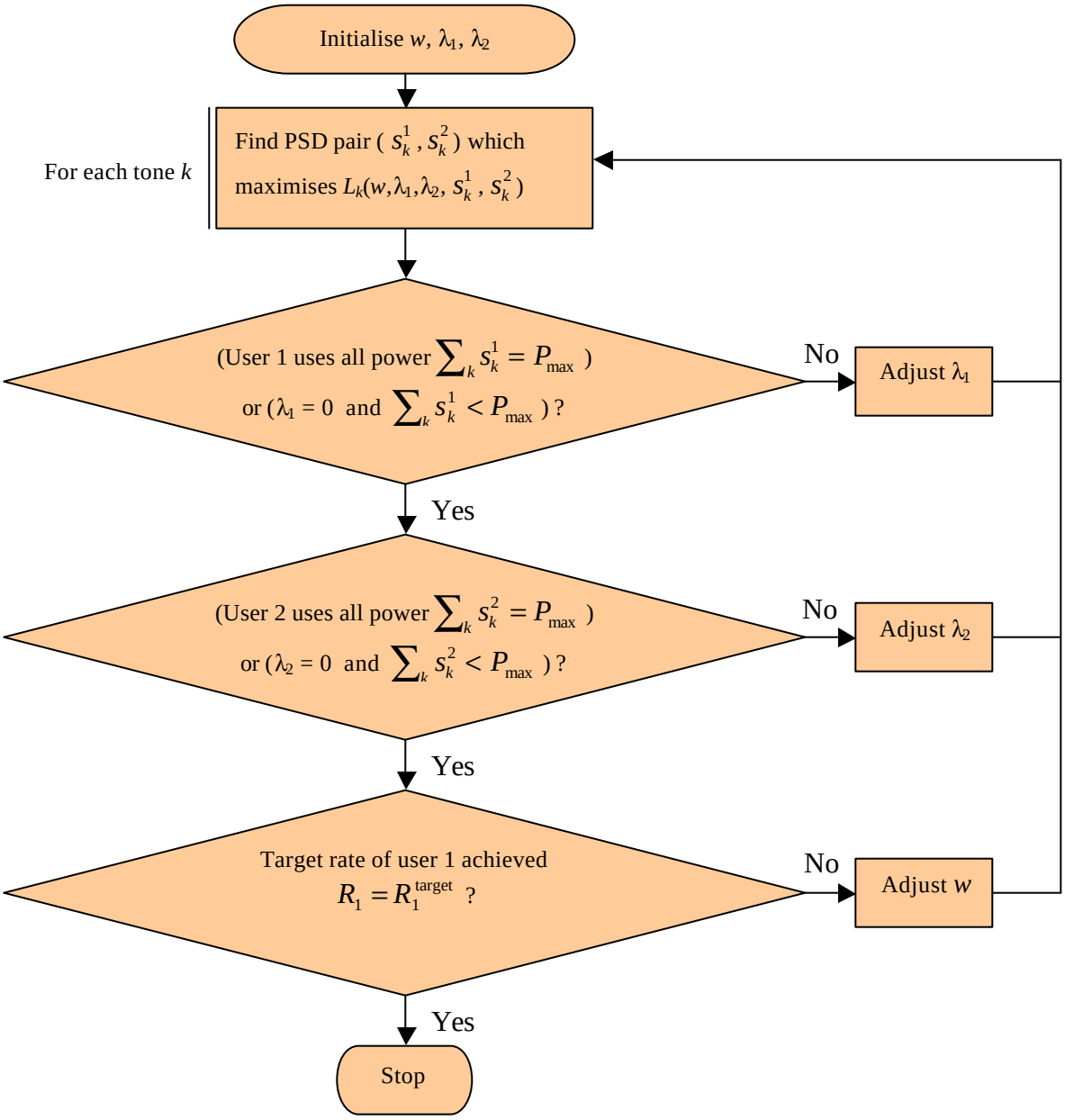

Figure 13 – OSM Algorithm