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Subspace angles for fault detection

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Abstract

In this paper we present subspace angles between two models and subspace angles within a model. We show how they can be used in fault detection problems. From these angles, model norms and distance measures are derived that are related to the cepstrum of the models. We show that one of the norms is equivalent to the mutual information of the past and the future output process. Finally, we show how the distance between a model and observations can be defined and computed.

1 Introduction

The notion of principal angles between subspaces of linear vector spaces is due to Jordan [16] in the nineteenth century. This notion was translated by Hotelling [15] into the statistical quantities of canonical correlations, which are widely applied (see e.g. [11]). In the area of systems and control, principal angles between two subspaces are used in subspace identification methods [22] and also in model updating [8] and damage location [9]. In the latter two applications, one starts from a finite element model and observations of a certain mechanical structure and one tries to find the subset of parameters of the model that should be adapted to explain the observations, which is done by computing the principal angles between a certain observation space and the parameterized space. In that way, damage to the structure can be located. The subspace-based fault detection algorithm of Basseville et al. [2], on the other hand, is based on linear dynamical models, the type of models that we deal with here. Changes in the eigenmodes of the observed system are determined by monitoring the difference between the column spaces of the observability matrix of the nominal linear dynamical model and the observability matrix of the model that can be identified from the observations. The difference between the column spaces can be quantified by the principal angles between the subspaces.

In this paper we present subspace angles between two models and subspace angles within a model and show how they can be used in fault detection problems. From these angles, model norms and distance measures are derived that are related to the cepstrum of the models. We show that one of the norms is equivalent to the mutual information of the past and the future output process. Finally, we show how the distance between two measured signals, between a model and a signal and between two models can be computed.

The paper is organized as follows. First, in Section 2 we describe the models that we will work with: single-input single-output (SISO), linear stochastic time-invariant models. In Section 3 we briefly recall the notion of principal angles between two subspaces. This geometrical concept is in Section 4 applied to linear stochastic models, so that we obtain subspace angles between two models. In Section 5 we deal with subspace angles within a model. Section 6 gives an overview of possible distance measures and norms based on subspace angles. The cepstrum and a related cepstral norm and cepstral distance are defined in Section 7, where we also give the relation of the cepstral distance and the subspace angles between two models and the relation of the cepstral norm and the subspace angles within a model. Furthermore, we show that for Gaussian processes, the cepstral norm is closely related to the information theoretic notion of mutual information. In Section 8, finally, we indicate how these angles and the derived norms and distances can be used for fault detection.

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2 Linear stochastic models

We describe in this section the model class that we will work with, namely single-input single-output (SISO) linear time-invariant stochastic models. The *forward innovation state space description* of such an nth order model has the following form:

$$\left\{\begin{array}{rcl} x(k+1) & = & Ax(k)+Ku(k)\ , \\ y(k) & = & Cx(k)+u(k)\ , \end{array}\right. \qquad (1)$$

where $\{y(k)\}_{k\in\mathbb{Z}} \in \mathbb{R}$, the output of the model, is the stochastic process that is being modeled, $\{u(k)\}_{k\in\mathbb{Z}} \in \mathbb{R}$ is the (white) innovation process of $\{y(k)\}_{k\in\mathbb{Z}}$ (henceforth, $\{u(k)\}_{k\in\mathbb{Z}} \in \mathbb{R}$ will also be called the input process) and $\{x(k)\}_{k\in\mathbb{Z}} \in \mathbb{R}^n$ is the state process. The matrix $A \in \mathbb{R}^{n\times n}$ is called the system matrix, $C \in \mathbb{R}^{1\times n}$ is the output matrix and $K \in \mathbb{R}^{n\times 1}$ is the Kalman gain. We will denote the model (1) by the threesome (A, K, C).

The *transfer function* from $\{u(k)\}_{k\in\mathbb{Z}} \in \mathbb{R}$ to $\{y(k)\}_{k\in\mathbb{Z}}$ is equal to

$$H(z) = C(zI_n - A)^{-1}K + 1\ .$$

It can be developed in a series as $H(z) = \sum_{k=0}^{\infty} h(k)z^{-k} = 1 + CKz^{-1} + CAKz^{-2} + \cdots$, which yields the *impulse response* $h(0), h(1), \ldots$ of the model. The values $h(k)$ ($k \geq 0$), which are equal to

$$\left\{\begin{array}{rcl} h(0) & = & 1\ , \\ h(k) & = & CA^{k-1}K \quad \text{for } k > 0\ , \end{array}\right.$$

are also known as the Markov parameters of the model. The *poles* of the model are the eigenvalues of the system matrix A.

We will make use of the infinite *observability matrix* of the model, which is defined as

$$\Gamma = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \end{pmatrix}\ .$$

From the equations in (1) one can immediately derive the state space equations of the *inverse model*:

$$\left\{\begin{array}{rcl} x(k+1) & = & (A-KC)x(k)+Ky(k)\ , \\ u(k) & = & -Cx(k)+y(k)\ . \end{array}\right. \qquad (2)$$

The *zeros* of the model (A, K, C) are consequently equal to the eigenvalues of $A - KC$. They always lie inside the unit circle, as follows from Kalman filter theory, provided (A, C) is observable. The infinite observability matrix of the inverse model, finally, is denoted by Γ_z:

$$\Gamma_z = \begin{pmatrix} -C \\ -C(A-KC) \\ -C(A-KC)^2 \\ \vdots \end{pmatrix}\ .$$

We will compute principal angles between the row spaces of *input and output Hankel matrices* of linear stochastic models. If s output observations $y(0), y(1), \ldots, y(s-1)$ are given, then the output Hankel matrix is equal to

$$Y = \left(\begin{array}{cccc} y(0) & y(1) & \cdots & y(j-1) \\ y(1) & y(2) & \cdots & y(j) \\ \vdots & \vdots & & \vdots \\ y(i-1) & y(i) & \cdots & y(i+j-2) \\ \hline y(i) & y(i+1) & \cdots & y(i+j-1) \\ y(i+1) & y(i+2) & \cdots & y(i+j) \\ \vdots & \vdots & & \vdots \\ y(2i-1) & y(2i) & \cdots & y(2i+j-1) \end{array}\right), \qquad (3)$$

$$= \left(\begin{array}{c} Y_p \\ \hline Y_f \end{array}\right), \qquad (4)$$

where $2i + j = s$ and i and j are user defined parameters with $j \gg i$ and i larger than the model order n. The submatrix of the first i rows is denoted by Y_p, where the subscript p stands for 'past', while the submatrix of the last i rows is Y_f, the subscript f referring to the 'future'. The past and future input Hankel matrices, $U_p \in \mathbb{R}^{i\times j}$ and $U_f \in \mathbb{R}^{i\times j}$ are defined in a similar way.

3 Principal angles between subspaces

The principal angles between and principal directions in two subspaces S_1 and S_2 are defined as follows.

Definition 1. *Let S_1 and S_2 be subspaces of dimension p and q, respectively, where $p \leq q$. Then, the p principal angles between S_1 and S_2, denoted by $\theta_1, \ldots, \theta_p$, and the corresponding principal directions $u_i \in S_1$ and $v_i \in S_2$ are recursively defined as*

$$\cos\theta_1 = \max_{u\in S_1}\max_{v\in S_2} |u^T v| = u_1^T v_1$$

$$\cos\theta_k = \max_{u\in S_1}\max_{v\in S_2} |u^T v| = u_k^T v_k \quad (k = 2, \ldots, p)$$

subject to $\|u\| = \|v\| = 1$, and for $k > 1$: $u^T u_i = 0$ and $v^T v_i = 0$, where $i = 1, \ldots, k-1$.

Let $A \in \mathbb{R}^{p\times n}$ be of rank r_a and $B \in \mathbb{R}^{q\times n}$ of rank r_b, where $r_a < r_b$. Then, the ordered set of r_a principal angles between the row spaces of A and B is denoted by

$$(\theta_1, \theta_2, \ldots, \theta_{r_a}) = [A \sphericalangle B] \ .$$

Numerically stable methods to compute the principal angles and vectors via a singular value decomposition have been proposed in [3, 14] and can also be found in [13, pp. 603–604].

4 Subspace angles between two models

By looking at the principal angles between certain output spaces derived from two linear stochastic models that are driven by the same white noise, we can define subspace angles between two models. Let $M^{(1)}$ and $M^{(2)}$ be linear stochastic models that are driven by the same white noise process. The order of the models is $n^{(1)}$ and $n^{(2)}$, respectively, and their state space matrices $(A^{(1)}, K^{(1)}, C^{(1)})$ and $(A^{(2)}, K^{(2)}, C^{(2)})$. The largest $n^{(1)} + n^{(2)}$ principal angles between the row spaces of the output Hankel matrices (see (3)) can then be expressed in terms of the model parameters, while the other principal angles are all equal to 0.

Theorem 4.1. *The largest $n^{(1)} + n^{(2)}$ principal angles between the row spaces of the output Hankel matrices of $M^{(1)}$ and $M^{(2)}$, denoted by $Y^{(1)}$ and $Y^{(2)}$, are equal to the principal angles between the column spaces of $\begin{pmatrix}\Gamma^{(1)} & \Gamma_z^{(2)}\end{pmatrix}$ and $\begin{pmatrix}\Gamma^{(2)} & \Gamma_z^{(1)}\end{pmatrix}$, provided the number of rows and columns of the Hankel matrices goes to ∞. The other principal angles are equal to 0:*

$$\left[Y^{(1)} \sphericalangle Y^{(2)}\right] = \left[\begin{pmatrix}\Gamma^{(1)} & \Gamma_z^{(2)}\end{pmatrix}^T \sphericalangle \begin{pmatrix}\Gamma^{(2)} & \Gamma_z^{(1)}\end{pmatrix}^T\right], \quad 0, 0, \ldots, \tag{5}$$

where $\Gamma^{(1)}$ and $\Gamma^{(2)}$ are the observability matrices of the two models and $\Gamma_z^{(1)}$ and $\Gamma_z^{(2)}$ the observability matrices of the inverse models.

The proof for the equality in (5) is given in [5, pp. 133–134]. The principal angles between the column spaces of $\begin{pmatrix}\Gamma^{(1)} & \Gamma_z^{(2)}\end{pmatrix}$ and $\begin{pmatrix}\Gamma^{(2)} & \Gamma_z^{(1)}\end{pmatrix}$ indicate the difference between the models, and we call them the *subspace angles between the two models*.

5 Subspace angles within a model

Besides subspace angles between two models, one can also define subspace angles within one model. We discuss two groups of angles: in Section 5.1 the principal angles between the future input and future output space of a linear stochastic model and in Section 5.2 the principal angles between the past and future output spaces.

5.1 The principal angles between future input and future output

In an analogous way as for the subspace angles between two models, we can express the principal angles between the future input and output spaces of a linear stochastic model of order n in terms of the model parameters as follows.

Theorem 5.1. *The largest n principal angles between the row spaces of the future input and future output Hankel matrices, U_f and Y_f (see (3-4)), of a model M are equal to the principal angles between the column spaces of the observability matrix of M and the observability matrix of the inverse model M^{-1}, provided the number of rows and columns of the Hankel matrices goes to ∞. The other principal angles are equal to 0:*

$$[U_f \sphericalangle Y_f] = \left[\Gamma^T \sphericalangle \Gamma_z^T\right], 0, 0, \ldots \ ,$$

where Γ is the observability matrix of the model and Γ_z is the observability matrix of the inverse model.

The proof of Theorem 5.1 is given in [5, p. 137]. Only n principal angles between the input and output spaces differ from 0. These angles are called the *subspace angles of the model*.

Note that the subspace angles of an nth order model M with transfer function $\frac{b(z)}{a(z)}$ are equal to the subspace angles between the autoregressive model with transfer function $\frac{z^n}{a(z)}$ and the autoregressive model with transfer function $\frac{z^n}{b(z)}$. They consequently indicate a certain 'correlation' of the pole- and zero-part of the transfer function. The closer the zeros lie to the poles, the smaller the subspace angles of the model. Conversely, the subspace angles between two models with transfer function $H^{(1)}$ and $H^{(2)}$, respectively, are equal to the subspace angles of the model with transfer function $\frac{H^{(1)}}{H^{(2)}}$.

5.2 The principal angles between past and future output

It can be proven that the principal angles between the row spaces of the past and future output Hankel matrices, Y_p and Y_f (see (4)), are complementary to the principal angles between the row spaces of the future input and future output Hankel matrices, U_f and Y_f (for the proof, see [5, p. 121]):

$$[Y_p \sphericalangle Y_f] = \frac{\pi}{2} - [U_f \sphericalangle Y_f] \ . \tag{6}$$

Consequently, the principal angles between past and future output of a model of order n can be expressed in terms of the model parameters (A, K, C), as follows:

$$[Y_p \sphericalangle Y_f] = \frac{\pi}{2} - \left[\Gamma^T \sphericalangle \Gamma_z^T\right], \frac{\pi}{2}, \frac{\pi}{2}, \ldots$$

6 Distance measures and model norms

Based on the subspace angles between two models we can derive several distance measures, which have already been reported in [6]. Let $M^{(1)}$ and $M^{(2)}$ be models of order $n^{(1)}$ and $n^{(2)}$, respectively and the subspace angles between them be denoted by $\theta_1^{(12)}, \theta_2^{(12)}, \ldots, \theta_{n^{(1)}+n^{(2)}}^{(12)}$. Then, the following distance measures can be obtained by combining the subspace angles:

$$d_g^2(M^{(1)}, M^{(2)}) = \sin \theta_{\max}^{(12)} \ , \tag{7}$$

$$d_f^2(M^{(1)}, M^{(2)}) = \sum_{i=1}^{n^{(1)}+n^{(2)}} \sin^2 \theta_i^{(12)} \ , \tag{8}$$

$$d^2(M^{(1)}, M^{(2)}) = \sum_{i=1}^{n^{(1)}+n^{(2)}} \log \frac{1}{\cos^2 \theta_i^{(12)}} \ . \tag{9}$$

In agreement with [12, 17], the distance measure in (7) can be called the gap between $M^{(1)}$ and $M^{(2)}$. The distance in (9) is equal to the cepstral distance measure that was defined in [19], which we discuss in the following section.

The subspace angles of a model of order n, denoted by $\theta_1, \ldots, \theta_n$, can be combined in an analogous way, which then leads to the following model norms:

$$\|M\|_g^2 = \sin \theta_{\max} \ ,$$

$$\|M\|_f^2 = \sum_{i=1}^{n} \sin^2 \theta_i \ ,$$

$$\|M\|^2 = \sum_{i=1}^{n} \log \frac{1}{\cos^2 \theta_i} \ . \tag{10}$$

As explained in [5], the term ‘norm’ should be interpreted as follows: it is a measure for the non-whiteness of the output process of the model if it is driven by white noise: the more the model colors the input process, the larger the model norm.

7 A cepstral distance measure and cepstral norm

7.1 The cepstrum

The cepstrum of a stochastic process is defined as the inverse Fourier transform of the logarithm of the spectrum of the process. It can be easily computed by using the fast Fourier transform, as shown in Figure 1. The cepstrum has been introduced for the detection of echoes in seismological data [4]. It is used in speech recognition [21], in fault detection methods for rotating machines [23], but also for measuring the distance between two signals, see [1] and references therein and [19].

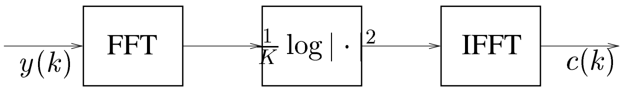

Figure 1: Estimation of the cepstrum of the stochastic process $y(k)$. The Fast Fourier transform (FFT) is applied to the observations $y(k)$ $(k = 0, 1, \ldots, K-1)$. Next, the logarithm of the absolute value squared of the transformed sequence is devided by the number of observations. pAplying the inverse Fast Fourier transform (IFFT) results in an estimate for the cepstrum.

The cepstrum of the output process of a linear stochastic model can be expressed in terms of the model parameters. Suppose the order of the model is n, its poles are denoted by $\alpha_1, \ldots, \alpha_n$ and the zeros by $\beta_1, \ldots, \beta_n$. Assume that the variance of the white noise input process $u(k)$ is equal to σ^2. Then, the cepstrum coefficients of the output process $y(k)$ are equal to [20, p. 502]:

$$c(k) = \begin{cases} \log \sigma^2 & k = 0 \ , \\ \displaystyle\sum_{i=1}^{p} \frac{\alpha_i^{|k|}}{|k|} - \sum_{i=1}^{q} \frac{\beta_i^{|k|}}{|k|} & k \neq 0 \ . \end{cases}$$

The cepstrum can also be obtained from the input variance and the model's Markov parameters (the impulse response) [20, p. 504]

$$c(k) = \begin{cases} \log \sigma^2 & k = 0 \ , \\ h(|k|) - \frac{1}{|k|} \sum_{n=0}^{|k|-1} nc(n) h\left(|k| - n\right) & k \neq 0 \ . \end{cases}$$

Note that from the cepstrum, the Markov parameters and the input variance can be derived, which can then be used to identify the state space matrices by applying realization algorithms [18, 24] (see [5, pp. 205–208]).

7.2 A cepstral distance measure

Several distance measures for signals that are based on the cepstrum of the signals, have been proposed [1]. We concentrate on the distance that was defined in [19]. Let $M^{(1)}$ and $M^{(2)}$ be two linear stochastic models with cepstrum $c^{(1)}$ and $c^{(2)}$, respectively. Then, the cepstral distance between $M^{(1)}$ and $M^{(2)}$ is defined as

$$d^2(M^{(1)}, M^{(2)}) = \sum_{k=0}^{\infty} k(c^{(1)}(k) - c^{(2)}(k))^2 \ . \quad (11)$$

Assume the models $M^{(1)}$ and $M^{(2)}$ are of order $n^{(1)}$ and $n^{(2)}$, respectively. We have shown in [7] that the cepstral distance in (11) can be expressed in terms of the $n^{(1)} + n^{(2)}$ subspace angles between the models $M^{(1)}$ and $M^{(2)}$, denoted by $\theta_1^{(12)}, \theta_2^{(12)}, \ldots, \theta_{n^{(1)}+n^{(2)}}^{(12)}$, as follows:

$$d^2(M^{(1)}, M^{(2)}) = \sum_{i=1}^{n^{(1)}+n^{(2)}} \log \frac{1}{\cos^2 \theta_i^{(12)}} \ .$$

Consequently, the cepstral distance is equal to the distance measure which we have already mentioned in (9).

7.3 A cepstral norm

Based on the definition in [19] (see (11)), we can derive the corresponding model norm:

$$\|M\|^2 = \sum_{k=0}^{\infty} kc(k)^2 \ ,$$

where $c(k)$ is the cepstrum of the model M. While the cepstral distance between two models is related to the subspace angles between the models, the cepstral norm of a model is related to the subspace angles within the model. Let the model be of order n and let its subspace angles (the non-zero principal angles between future input and future output) be denoted by $\theta_1, \theta_2, \ldots, \theta_n$. Then, the cepstral norm is equal to the norm in (10):

$$\sum_{k=0}^{\infty} kc(k)^2 = \sum_{i=1}^{n} \log \frac{1}{\cos^2 \theta_i} \ .$$

Due to the complementarity property in (6), the norm can also be expressed in terms of the principal angles between the past and future output spaces, which are denoted by $\phi_1, \ldots, \phi_n$:

$$\sum_{k=0}^{\infty} kc(k)^2 = \sum_{i=1}^{n} \log \frac{1}{\sin^2 \phi_i} \ . \quad (12)$$

From the characterization in (12) follows that the cepstral norm of a model is proportional to the mutual information of the past and future output processes (denoted by y_p and y_f), provided the output process is Gaussian. Indeed, the mutual information of the jointly Gaussian stochastic processes y_p and y_f can be written as (see e.g. [10])

$$I(y_p, y_f) = \frac{1}{2} \sum_{i=1}^{n} \log \frac{1}{\sin^2 \phi_i} \ .$$

The mutual information of the past and the future of a Gaussian stochastic output process of a linear model M is consequently also equal to

$$\begin{aligned} I(y_p, y_f) &= \frac{1}{2} \sum_{k=0}^{\infty} kc(k)^2 = \frac{1}{2} \|M\|^2 \ , \\ &= \frac{1}{2} \sum_{i=1}^{n} \log \frac{1}{\cos^2 \theta_i} \ , \end{aligned}$$

where $\theta_1, \ldots, \theta_n$ are the subspace angles of M.

8 Fault detection based on subspace angles

In this section, we consider the problem of detecting changes in the dynamics of a certain structure or process (e.g. the frequency or damping of vibration modes of a bridge, an airplane, ...). We show how the subspace angles between two models and the subspace angles of a model could be useful for this purpose.

Suppose we have a model of the undamaged structure with transfer function $H(z)$ on the one hand and new observations $y(k), y(k + 1), \ldots$ of the structure on the other hand. The aim is to decide, based on the observations, whether the dynamic properties of the structure have changed significantly, so that one can assume that the structure has been damaged.

A first way to tackle this problem is by identifying a new model, based on the observations. We denote its transfer function by $H'(z)$. The identification could be done recursively, so that there is a continuous monitoring of the structure. The difference between the original model $H(z)$ and the new model $H'(z)$ can then be measured by the subspace angles between the two models (5), or by a distance measure based on the subspace angles (7–9).

Note that if the input process is non-stationary, but still white, then neither the zeros of the models, nor the observability matrices of the inverse models, which show up in (5), can be determined. However, one could still monitor the changes of the poles by computing the principal angles between the column spaces of the observability matrices of the models.

Secondly, the angles/distance between the observations and the model of the undamaged structure can also be computed more directly, namely without having to identify a model from the observations. If the observations are filtered by the inverse model $H^{-1}(z)$, which can be done by mere Kalman filtering (see (2)), we obtain the residuals. In case the structure did not change, these residuals would be white, while they are colored if the dynamics did change. The residuals can be seen as the output process of the series connection of $H^{-1}(z)$ and the model for the observations, $H'(z)$, see Figure 2. As explained in Section 5.1, the subspace angles of this concatenated model with transfer function $\frac{H'(z)}{H(z)}$ are equal to the subspace angles between $H(z)$ and $H'(z)$. They can be computed via the principal angles between the past and future spaces of the residuals (Section 5.2), which can be done recursively.

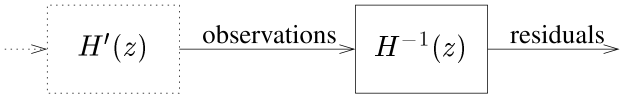

Figure 2: Filtering the observations by the inverse original model gives the residuals. They can be viewed as the output of the series connection of $H'(z)$ and $H^{-1}(z)$.

Based on the subspace angles of $\frac{H'(z)}{H(z)}$ we can compute a certain model norm, e.g. the cepstral norm, which is then equal to the cepstral distance between $H(z)$ and $H'(z)$. Of course, this distance can also be obtained as follows. Compute the cepstrum of the model with transfer function H (see Section 7.1), denoted by c_H, and estimate K cepstral coefficients of the observations, e.g. as in Figure 1, which is denoted by c_y. Then, the cepstral distance can be estimated as

$$d^2(H, H') = \sum_{k=0}^{K-1} k(c_H(k) - c_y(k))^2 \,.$$

As said before, when the input is non-stationary, one is only interested in the changes of the poles. For this case, too, one can determine principal angles directly from the observations and the given model, without having to identify a new model. The principal angles that we want to estimate are those between the column spaces of the observability matrix of the given model $H(z)$, denoted by Γ, and the observability matrix of $H'(z)$, denoted by Γ'. We do not need to identify the latter model since the column space of Γ' is equal to the column space of the orthogonal projection of the future output process (observations) and the past output process. This projection can again be computed in a recursive way.

9 Conclusions

We have presented subspace angles between two models and subspace angles within a model. A cepstral distance measure can be expressed in terms of the subspace angles. We have indicated how the subspace angles and the cepstral distance could be used in fault detection problems.

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