FETAL ELECTROCARDIOGRAM EXTRACTION BY SOURCE SUBSPACE SEPARATION

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ABSTRACT

A high-precision method is presented for the extraction of the fetal electrocardiogram from multi-channel potential recordings on the mother's skin. First the problem is appropriately formulated in terms of blind source separation, where the sources can have a multi-dimensional nature. As a summary of preceding work it is shown how the fetal electrocardiogram can be reconstructed by means of second-order tools. However an extra higher-order processing step enhances the accuracy and allows to identify the transfer from source to electrode. The method is illustrated with an example.

1. INTRODUCTION

The mechanical action of the heart muscle is initiated by an electrical depolarisation wave, which is followed by repolarisation. This quasi-periodical stimulus involves an electrical current, propagating through the body and resulting in potential differences. The potential differences can be measured between electrodes on the skin (*cutaneous* recordings). The registration of these potential signals, visualized as a function of time, is called the *electrocardiogram* (ECG)[1].

Like for adults, it should be possible to visualize the electrical activity of a fetal heart: the *fetal electrocardiogram* (FECG) contains important indications about the health and condition of the fetus. As long as the membranes protecting the child have not been broken (*antepartum*), the FECG should be obtained from measurements on the mother's skin.

The aim of this paper is to describe a high-precision method to derive the antepartum FECG from multi-lead cutaneous recordings.

2. MATHEMATICAL FORMULATION

Potential measurements on the mother's skin contain contributions from several bioelectric phenomena and are affected by various kinds of noise. Two aspects have to be examined here: first, the nature of the occurring signals and secondly, the characteristics of the propagation from bioelectric source to electrode.

In [12] it is shown that, at a considerable distance from the mother heart, its activity as a bioelectric current source can be represented in first order approximation by a three-dimensional vector signal. The three-dimensional vector space, described by the discrete-time evolution of the maternal ECG (MECG) after sampling, will be denoted as the *MECG-subspace*. On the other hand [11] states that the "dimension" of the fetal heart is not necessarily equal to three, but subject to changes during the period of pregnancy. In this paper the term *FECG-subspace* will be used. In comparison with the low-voltage range of the FECG other electrical signals can play an important role too: *electromyographic* activity (electrical potentials generated by the muscles), 50 Hz net-interference, ...

The transfer from bioelectric current source to electrode can be assumed to be linear and resistive [12]. On the other hand the frequency at which the cutaneous potential distribution is sampled (typically 500 Hz) can be considered as low, taking into account the

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[1] In medical practice this term additionally implies that the electrodes are placed at standard locations.

high propagation velocity of the electrical signals.

We may conclude that at time instant t the obtained measurement values can be modelled as instantaneous linear combinations of the source signals; noise can be taken into account as an additive perturbation. For a p-channel set-up, and in the presence of q bioelectric sources ($p \geqslant q$), this can be formulated as:

$$Y(t) = \mathbf{M}X(t) + N(t) \tag{1}$$

where $Y(t) = (y_1(t) \dots y_p(t))^t$ contains the potential recordings, $X(t) = (x_1(t) \dots x_q(t))^t$ contains the signal values of the bioelectric sources and the noise on each channel is represented by $N(t) = (n_1(t) \dots n_p(t))^t$. $\mathbf{M}$ is the transfer matrix, describing the propagation from source to electrode. As a good approximation the different bioelectric sources can be considered as statistically independent. The noise components $N_i(t)$ ($1 \leqslant i \leqslant q$) are assumed to be Gaussian, with variance σ_N^2, mutually independent as well as independent from the source signals.

As a conclusion, the derivation of the antepartum FECG from multilead cutaneous recordings can be considered as an example of blind *Source Subspace Separation* (SSS): given a dataset of p-channel measurements, estimate the underlying source signals or source subspaces according to the linear model (1), assuming statistical independence of the corresponding bioelectric phenomena. We stress the fact that the FECG-extraction is formulated as a *blind identification* problem, since it is impossible in practice to adhere to a more parametric approach:

- The transfer coefficients are subject to a large uncertainty because of the significant differences from patient to patient.
- The geometrical and resistivity parameters of the body of a single patient are not constant in time, e.g. because of the fetal growth.
- For the application in medical diagnosis and treatment it is crucial that *un*expected electrocardiogram patterns can be detected and examined.

3. A SECOND-ORDER APPROACH

Up till now the most effective method to tackle the FECG-reconstruction has been derived by Callaerts [2]. It is based on the *Singular Value Decomposition* (SVD) of the data-matrix.

The measurement data from time instant 1 to T are collected in a $(p \times T)$ data-matrix $\mathbf{Y} = (Y(t_1) \dots Y(t_T))$. Analogously $\mathbf{X} = (X(t_1) \dots X(t_T))$ contains the source signal values. Without loss of generality it can be assumed that $\mathbf{X}$ has mutually orthonormal rows with respect to the common Euclidean inner product:

- Eq. (1) shows that the unit-length scaling of the rows simply involves the inverse scaling of the columns of $\mathbf{M}$, which is irrelevant.
- The rows corresponding to different bioelectric sources are mutually orthogonal. This boils down to the fact that the calculation of the inner product of two rows can be interpreted, up to a scaling factor, as the computation by time-averaging of the cross-correlation.
- The rows describing the electrical activity of the maternal (resp. fetal) heart may be *chosen* mutually orthogonal: only the final contribution of the MECG (resp. FECG) to the recordings is important - the mutual position of the MECG (resp. FECG) source components has no physical meaning.

If the SVD of the transfer matrix $\mathbf{M}$ is given by $\mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^t$, then the noise-free version of Eq. (1) shows as:

$$\mathbf{Y} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^t \cdot \mathbf{X} \tag{2}$$

Eq. (2) demonstrates that the matrices $\mathbf{U}$ and $\mathbf{S}$ can be computed from an SVD of the data-matrix $\mathbf{Y}$, since $\mathbf{V}^t \cdot \mathbf{X}$ is a matrix with mutually orthogonal unit-length rows. This SVD-approach is a numerically reliable way [10] of performing the *Principal Component Analysis* (PCA) on the covariance $\mathbf{C}_2^Y$ of $Y(t)$:

$$\begin{aligned} \mathbf{C}_2^Y &= E\{Y(t)Y(t)^t\} && (3)\\ &\simeq \frac{1}{T}\,\mathbf{Y} \cdot \mathbf{Y}^t && (4)\\ &= \mathbf{U} \cdot \tilde{\mathbf{S}}^2 \cdot \mathbf{U}^t && (5) \end{aligned}$$

where $\tilde{\mathbf{S}}$ is the diagonal $(q \times q)$-matrix containing the singular values of $(p \times q)$-matrix $\mathbf{S}$ ($p \geqslant q$).

In [9] it is proved that, under the noise conditions mentioned in Section 2, $\mathbf{U}$ can still be estimated consistently as the left singular matrix of $\mathbf{Y}$. In a more-sensors-than-sources setup the noise standard deviation can be estimated as the mean of the "noise singular values" of $\mathbf{Y}$. $\mathbf{S}$ can then be computed taking into account that Eqs. (3-5) remain valid for the noise-free covariance $\mathbf{C}_2^Y - \sigma_N^2\mathbf{I}$, which means that the squared singular values are shifted in the same way.

From Eq. (2) it is clear that after SVD of the data matrix $\mathbf{Y}$ an orthogonal factor $\mathbf{V}$ remains unidentified. However the right singular matrix usually reveals pretty clear FECG-signals. This can be explained resorting to the concept of *oriented energy*:

Definition 1 *The oriented energy of a real $(p \times q)$-matrix $\mathbf{A} = (A_1 \dots A_q)$ in the direction of a real p-dimensional unit vector P is defined as:*

$$\mathcal{E}_P\{\mathbf{A}\} = \sum_{i=1}^{q} (P^t A_i)^2 = \|P^t \mathbf{A}\|^2 \tag{6}$$

with $\|\cdot\|$ the Euclidean norm.

The directions of extremal oriented energy are mutually orthogonal and can be found as the left singular vectors of $\mathbf{A}$ [8]. With respect to the data-matrix $\mathbf{Y}$ it can be expected that large values of oriented energy will be observed in the MECG-subspace. In directions perpendicular to this subspace the oriented energy will be much smaller since the MECG is much stronger than the FECG (e.g. a factor 10 in amplitude). It follows that the first left singular vectors of $\mathbf{Y}$ will form an orthonormal basis for the MECG-subspace. By considering only the other singular vectors the MECG can to a large extent be projected out of the data-set.

4. FROM PCA TO ICA

From Eq. (5) it is clear that a mere second-order approach leaves the right singular matrix $\mathbf{V}$ unidentified. However $\mathbf{V}$ cán be determined from the higher-order statistics of the recorded data, when the full statistical independence of the sources is exploited. This way of performing blind SS is sometimes referred to as *Independent Component Analysis* (ICA) [5].

Our algorithm is based on fourth-order cumulants since the third-order cumulant $\mathbb{C}_3^Y$ of $Y(t)$ is theoretically zero: all the signals involved are zero-mean. The element-wise relationship between the fourth-order cumulants of $Y(t)$ and $X(t)$ is given by:

$$\left(\mathbb{C}_4^Y\right)_{ijkl} = \sum_{i'j'k'l'} \left(\mathbb{C}_4^X\right)_{i'j'k'l'} \mathbf{M}_{ii'}\mathbf{M}_{jj'}\mathbf{M}_{kk'}\mathbf{M}_{ll'} \tag{7}$$

The noise contribution drops out of the equation since it was assumed to be Gaussian. Globally, Eq. (7) will be denoted as:

$$\mathbb{C}_4^Y = \mathbb{C}_4^X \times_1 \mathbf{M} \times_2 \mathbf{M} \times_3 \mathbf{M} \times_4 \mathbf{M} \tag{8}$$

Combined with the SVD of $\mathbf{M}$ this yields:

$$\Phi = \mathbb{C}_4^X \times_1 \mathbf{V}^t \times_2 \mathbf{V}^t \times_3 \mathbf{V}^t \times_4 \mathbf{V}^t \tag{9}$$

in which Φ is defined as:

$$\Phi = \mathbb{C}_4^Y \times_1 (\mathbf{S}^\dagger \mathbf{U}^t) \times_2 (\mathbf{S}^\dagger \mathbf{U}^t) \times_3 (\mathbf{S}^\dagger \mathbf{U}^t) \times_4 (\mathbf{S}^\dagger \mathbf{U}^t) \tag{10}$$

with † denoting the pseudo-inverse.

Eq. (9) is the fourth-order equivalent of the Eigenvalue Decomposition: Φ is super-symmetric (i.e. invariant under all permutations of its indices), $\mathbf{V}$ is orthogonal and the off-diagonal elements of $\mathbb{C}_4^X$ are zero when they correspond to mutually independent sources. For the computation of this decomposition one can resort to a number of techniques, in recent years developed in the framework of multilinear algebra:

- In [7] the decomposition is computed as the symmetric case of the *Higher-Order Singular Value Decomposition* (HOSVD) [6], which is a convincing extension of the matrix SVD to tensors of higher order. It is shown that $\mathbf{V}$ is revealed by the second-order SVD of a $(q \times q^3)$-matrix with Kronecker structure, constructed from Φ.
- In [5] a very promising technique is developed by Comon: the orthogonal matrix $\mathbf{V}$ is obtained as the product of elementary Jacobi-rotations minimizing the off-diagonal energy (i.e. the sum of squared values) of Φ. Each elementary rotation involves the computation of the zeros of a fourth-order polynomial.
- Also interesting is the approach by Cardoso, who formulates the decomposition of Eq. (9) as a simultaneous diagonalization problem [4]. The "simultaneous eigenmatrix" $\mathbf{V}$ can also be obtained as the product of elementary Jacobi-rotations, where each rotation involves the computation of the best rank-1 approximation of a real symmetric (3×3)-matrix.

For all these methods the computational cost can substantially be reduced without loss of medical information: the fact that only the different source subspaces have to be separated, instead of all the source components, limits the number of Jacobi-rotation angles that have to be identified.

The higher-order processing step adds the following advantages to the second-order approach:

- It is possible to enhance the quality of separation. A first reason is that more available information is resorted to, by exploiting the higher-order statistical properties of the data-set. On the other hand part of the medical information concerning the fetus is lost in the second-order method: the computation of the SVD of the data-matrix implies that the FECG-subspace components that are parallel to the MECG-subspace, are orthogonally projected out of the FECG-estimate.
- The propagation of the electrical signals can be characterized in an essentially unique way. We mention two important implications:

- The transfer vectors indicate how strongly the different electrodes capture each source signal, from which better measurement positions may be deduced. It should be mentioned that the positioning of the electrodes is still the most crucial factor for the success of the PCA-method [3].

- An important aspect in the evaluation of the fetal well-being is the quantification of fetal movements [2]. The number of significant changes in the FECG-subspace, which can be obtained from an on-line adaptive implementation of our method, can be very useful information here.

- The physician can resort to a more intuitive interpretation of the results: the separation of the measured signals into statistically independent source signals with a physical meaning, is easier to interpret than a decomposition in time-orthogonal principal components.

5. EXAMPLE

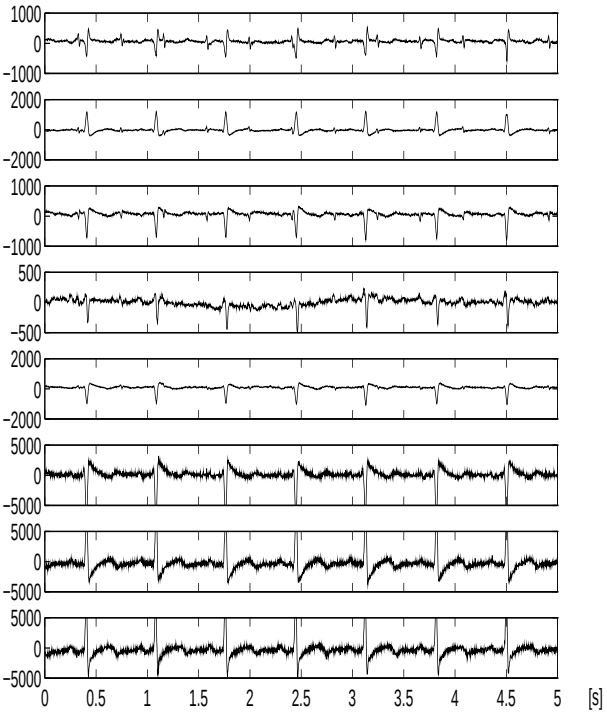

Figure 1: 8-channel set of cutaneous data recordings.

Fig. 1 shows the potential signals measured during an 8-channel experiment. The horizontal axis displays the time in seconds; concerning the vertical axes only the relative values are important. The sampling frequency was 500 Hz. Channels 1 to 5 show *abdominal* signals, measured near the fetus. Channels 1 and 3 clearly contain weak fetal contributions. For channels 6 to 8 the electrodes had been placed near the mother's heart, e.g. on the *thorax* (chest). Due to the large amplitudes of the MECG in the thoracic signals, the FECG is less visible. However the strong pick-up of MECG allows an accurate modelling of the electrical activity of the mother heart and realizes a large gap between the oriented energy in the MECG-subspace and the FECG-subspace (see Section 3).

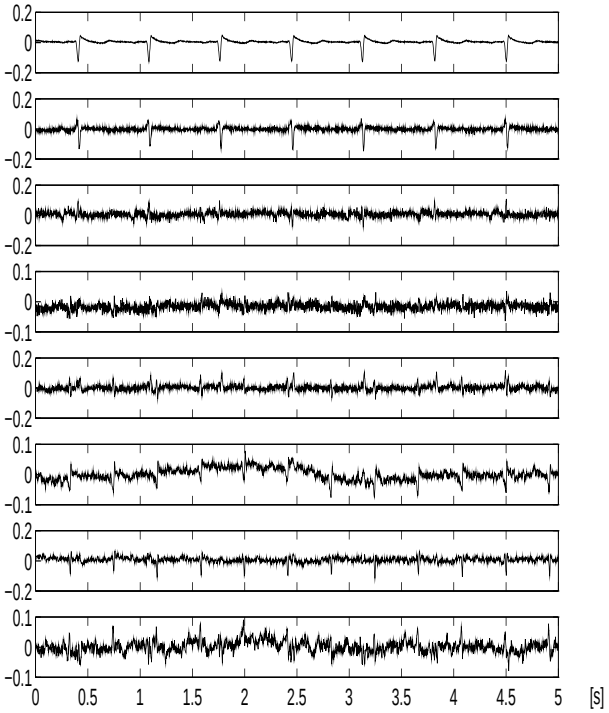

Figure 2: Source estimates obtained via PCA.

The source estimates after PCA are displayed in Fig. 2. Two MECG-free FECG-components have been obtained as resp. the 6th and the 7th right singular vector of the data-matrix. The signals 1 and 2 partly describe the MECG-subspace; the MECG also appears in signals 3 and 5. Channels 4 and 8 mainly show noise contributions.

The result after SSS is shown in Fig. 3. Although the statistics of the non-stationary signals have been estimated by time-averaging, the result is an excellent source separation. Where the PCA-method obtains only two clear MECG-components (the 3rd signal is heavily perturbated by noise and the fifth principal component contains important FECG-contributions), SSS accurately reconstructs the full three-dimensional MECG-subspace (signals 1 to 3 in Fig. 3). As far as the FECG is concerned, the quality of the 7th princi-

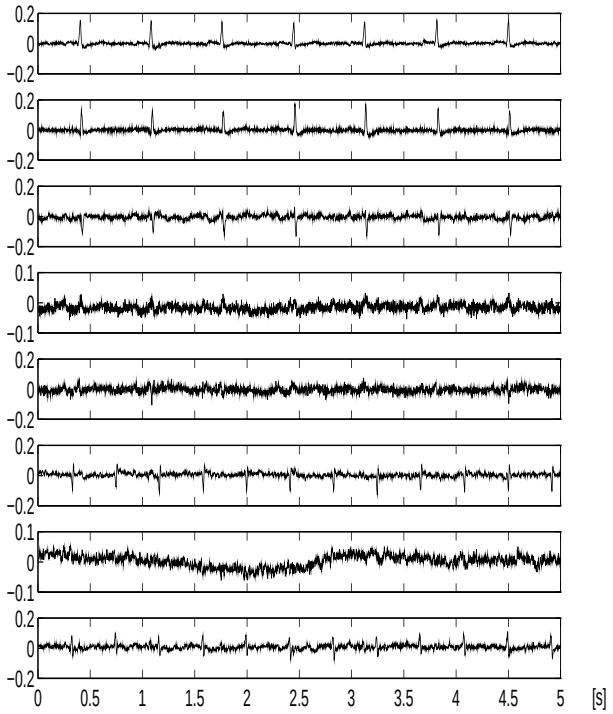

Figure 3: Source estimates obtained via SSS.

pal component and the 8th SSS-signal are comparable, but in the 6th SSS-signal the signal-to-noise ratio is somewhat higher than in the 6th PCA-estimate. In addition, the off-set in the 6th PCA-signal is identified as an extra source signal (the low-periodic 7th signal in Fig. 3), which probably arises from respiration. The 5th SSS-signal mainly shows noise contributions.

6. CONCLUSIONS

The extraction of the fetal electrocardiogram from multilead potential recordings on the mother's skin has been tackled by a combined use of second-order and higher-order techniques. The higher-order processing requires that the fourth-order cumulant tensor of the data is estimated, and consists of the computation of an orthogonal matrix that partially diagonalizes a related fourth-order tensor.

The resulting method yields very accurate results and therefore promises to be a valuable help in the medical diagnosis. Moreover it allows to reconstruct other significant bioelectric source signals, as well as the characteristics of their propagation to the electrodes.

The method is already partly implemented in a real-time data acquisition and signal processing platform, which will allow a further evaluation of its qualities with respect to the application in practice.

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