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SVD-Based Methodologies for Fetal Electrocardiogram Extraction[1]

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Abstract

This paper deals with the extraction of the antepartum foetal electrocardiogram (ECG) from multilead cutaneous potential recordings, and the modelling of the transfer to the electrodes. We give an overview of a class of algebraic approaches, based on variants of the singular value decomposition (SVD): the concept, pros and cons of techniques relying on the ordinary SVD, quotient SVD and multilinear SVD are discussed.

SVD-BASED METHODOLOGIES
FOR FETAL ELECTROCARDIOGRAM EXTRACTION

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ABSTRACT

This paper deals with the extraction of the antepartum foetal electrocardiogram (ECG) from multilead cutaneous potential recordings, and the modelling of the transfer to the electrodes. We give an overview of a class of algebraic approaches, based on variants of the singular value decomposition (SVD): the concept, pros and cons of techniques relying on the ordinary SVD, quotient SVD and multilinear SVD are discussed.

1. INTRODUCTION

The mechanical action of the heart muscle is initiated by an electrical depolarisation wave, which is followed by repolarisation. This quasi-periodical stimulus involves an electrical current, propagating through the body and resulting in potential differences. The potential differences can be measured between electrodes on the skin (*cutaneous* recordings). The registration of these potential signals, visualized as a function of time, is called the *electrocardiogram* (ECG).

Like for adults, it should be possible to visualize the electrical activity of a fetal heart: the *fetal electrocardiogram* (FECG) contains important indications about the health and condition of the fetus. As long as the membranes protecting the child have not been broken (*antepartum*), the FECG should be obtained from measurements on the mother's skin.

The aim of this paper is to describe a class of algebraic methods, based on variants of the singular value decomposition (SVD), to derive the antepartum FECG from multilead cutaneous recordings.

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2. MATHEMATICAL FORMULATION

In [10] it is shown that, at a considerable distance from the mother heart, its activity as a bioelectric current source can be represented in first order approximation by a three-dimensional vector signal. The three-dimensional vector space, described by the discrete-time evolution of the maternal ECG (MECG) after sampling, will be denoted as the *MECG-subspace*. On the other hand [9] states that the "dimension" of the fetal heart is not necessarily equal to three, but subject to changes during the period of pregnancy. In this paper the term *FECG-subspace* will be used.

In [1, 10] it is argued that the values recorded at the electrodes can be modelled as instantaneous linear combinations of the source signals; noise can be taken into account as an additive perturbation. For a p-channel set-up, and in the presence of q bioelectric sources ($p \geqslant q$), this can be formulated as

$$Y(t) = \mathbf{M}X(t) + N(t), \tag{1}$$

where $Y(t) = (y_1(t) \dots y_p(t))^T$ contains the potential recordings, $X(t) = (x_1(t) \dots x_q(t))^T$ contains the signal values of the bioelectric sources (FECG, MECG, ...) and the noise on each channel is represented by $N(t) = (n_1(t) \dots n_p(t))^T$. $\mathbf{M}$ is the mixing matrix, describing the propagation from source to electrode. As a good approximation the different bioelectric sources can be considered as statistically independent. The noise components $N_i(t)$ $(1 \leqslant i \leqslant q)$ are assumed to be Gaussian, with variance σ_N^2, mutually independent as well as independent from the source signals.

As a conclusion, the derivation of the antepartum FECG from multilead cutaneous recordings can be considered as an example of *Blind Source Subspace Separation* (BSSS): given a dataset of p-channel measurements, we have to estimate the underlying source signals or source subspaces according to the linear model (1), assuming statistical independence of the corresponding bioelectric phenomena.

3. SVD-BASED APPROACH

A first approach is based on the *Singular Value Decomposition* (SVD) of the data-matrix [1].

The measurement data from time instant 1 to T are collected in a $(p \times T)$ data-matrix $\mathbf{Y} = (Y(t_1) \dots Y(t_T))$. Analogously $\mathbf{X} = (X(t_1) \dots X(t_T))$ contains the source signal values. Taking into account that different bioelectric sources are assumed to be mutually uncorrelated, and that they can only be determined up to an arbitrary scaling factor, it can be assumed that $\mathbf{X}$ has mutually orthonormal rows.

If the SVD of the mixing matrix $\mathbf{M}$ is given by $\mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^T$, then the noise-free version of Eq. (1) is given by

$$\mathbf{Y} = \mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^T\cdot\mathbf{X}. \tag{2}$$

Eq. (2) demonstrates that the matrices $\mathbf{U}$ and $\mathbf{S}$ can be computed from an SVD of the data-matrix $\mathbf{Y}$, since $\mathbf{V}^T\cdot\mathbf{X}$ is a matrix with mutually orthogonal unit-length rows. Under the noise conditions mentioned in Section 2, $\mathbf{U}$ can still consistently be estimated as the left singular matrix of $\mathbf{Y}$. In addition, in a more-sensors-than-sources set-up $\mathbf{S}$ can be computed after having estimated the noise standard deviation from the smallest singular values of $\mathbf{Y}$. Implicitly, the computation of the SVD (2) amounts to the computation of the Eigenvalue Decomposition (EVD) of the covariance matrix of $Y(t)$; this technique is known as *Principal Component Analysis* (PCA).

From Eq. (2) it is clear that after SVD of the data matrix $\mathbf{Y}$ an orthogonal factor $\mathbf{V}$ remains unidentified. To fix this indeterminacy, one tries to choose the positions of the electrodes in such a way that the mixing mechanism is expected to be orthogonal, i.e. $\mathbf{V} = \mathbf{I}$ [1]. However, this is usually a matter of trial-and-error. Nevertheless, even if orthogonality is not achieved, the right singular matrix often reveals pretty clear FECG-signals. This can be explained resorting to the concept of *oriented energy*:

Definition 1 *The oriented energy of a real $(p\times q)$-matrix $\mathbf{A} = (A_1 \dots A_q)$, in the direction of a real p-dimensional unit vector P, is defined as*

$$\mathcal{E}_P\{\mathbf{A}\} = \sum_{i=1}^{q}(P^T A_i)^2 = \|P^T\mathbf{A}\|^2, \tag{3}$$

with $\|\cdot\|$ the Euclidean norm.

The directions of extremal oriented energy are mutually orthogonal and can be found as the left singular vectors of $\mathbf{A}$. Since large oriented energy values are observed in the MECG-subspace, this subspace is approximately spanned by the first left singular vectors of $\mathbf{Y}$. By considering only the other singular vectors, the MECG can to a large extent be projected out of the dataset. This approach is theoretically motivated in [11, 6].

4. QSVD-BASED APPROACH

A second approach is based on the *Quotient SVD* (QSVD) of two matrices $\mathbf{A}\in\mathbb{R}^{p\times q_1}$ and $\mathbf{B}\in\mathbb{R}^{p\times q_2}$ [8, 1]:

$$\begin{aligned}\mathbf{A} &= \mathbf{T}^{-T}\mathbf{S}_{\mathbf{A}}\mathbf{Q}_{\mathbf{A}}^T,\\ \mathbf{B} &= \mathbf{T}^{-T}\mathbf{S}_{\mathbf{B}}\mathbf{Q}_{\mathbf{B}}^T,\end{aligned}$$

in which $\mathbf{T}\in\mathbb{R}^{p\times p}$ is regular, $\mathbf{S}_{\mathbf{A}}\in\mathbb{R}^{p\times p}$ and $\mathbf{S}_{\mathbf{B}}\in\mathbb{R}^{p\times p}$ are diagonal, and $\mathbf{Q}_{\mathbf{A}}\in\mathbb{R}^{p\times q_1}$ and $\mathbf{Q}_{\mathbf{B}}\in\mathbb{R}^{p\times q_2}$ are column-wise orthonormal matrices. The matrices of which the QSVD is considered, are the data-matrix $\mathbf{Y}$ and a matrix $\tilde{\mathbf{Y}}$, derived from $\mathbf{Y}$, by concatenating intervals in which only the MECG appears. The sources can then be estimated as the columns of $\mathbf{Q}_{\mathbf{Y}}$. This can be explained resorting to the concept of *oriented signal-to-signal ratio*:

Definition 2 *The oriented signal-to-signal ratio of a real $(p\times q_1)$-matrix $\mathbf{A}$ and $(p\times q_2)$-matrix $\mathbf{B}$, in the direction of a real p-dimensional unit vector P, is defined as*

$$\mathcal{E}_P\{\mathbf{A},\mathbf{B}\} = \frac{\mathcal{E}_P\{\mathbf{A}\}}{\mathcal{E}_P\{\mathbf{B}\}} = \frac{\|P^T\mathbf{A}\|^2}{\|P^T\mathbf{B}\|^2}, \tag{4}$$

with $\|\cdot\|$ the Euclidean norm.

The directions of extremal oriented signal-to-signal ratio correspond to the columns of $\mathbf{T}$. The non-trivial extrema are found in the direction of the columns of the pseudo-inverse $\mathbf{M}^\dagger$ [1]; the identification of the mixing matrix is important in itself, because it provides insight in the characteristics of the propagation of the bioelectric signals through the body. E.g. if the FECG is not present in $\tilde{\mathbf{Y}}$, some of these directions must be in the FECG-subspace. Here, orthogonality of the mixing matrix is in principle not required, but it makes the approach more robust w.r.t. the presence of noise. A disadvantage of the method is that it requires a supervised learning stage. It is in fact not really necessary that the FECG is not present in $\tilde{\mathbf{Y}}$, but, with a view to the robustness, it is to be recommended. We also remark that the computation of the QSVD is more expensive than the computation of an ordinary SVD.

5. HOS-BASED APPROACH

Whereas the right singular matrix $\mathbf{V}$ cannot be identified from the second-order statistics of $Y(t)$ (Sect. 3), it can be determined from the Higher-Order Statistics (HOS) of the recorded data, when the full statistical independence of the sources is exploited. This way of performing BSS is also referred to as *Independent Component Analysis* (ICA) [4].

If the noise is Gaussian, the element-wise relationship between the fourth-order cumulants of $Y(t)$ and $X(t)$ is given by

$$(\mathbb{C}_4^Y)_{ijkl} = \sum_{i'j'k'l'}(\mathbb{C}_4^X)_{i'j'k'l'}\mathbf{M}_{ii'}\mathbf{M}_{jj'}\mathbf{M}_{kk'}\mathbf{M}_{ll'}. \tag{5}$$

We will write Eq. (5) as

$$\mathbb{C}_4^Y = \mathbb{C}_4^X \times_1 \mathbf{M} \times_2 \mathbf{M} \times_3 \mathbf{M} \times_4 \mathbf{M}. \tag{6}$$

Combined with the SVD of $\mathbf{M}$ this yields

$$\mathbb{C}_4^Z = \mathbb{C}_4^X \times_1 \mathbf{V}^T \times_2 \mathbf{V}^T \times_3 \mathbf{V}^T \times_4 \mathbf{V}^T, \tag{7}$$

in which $\mathbb{C}_4^Z$ is defined as

$$\mathbb{C}_4^Z = \mathbb{C}_4^Y \times_1 (\mathbf{S}^\dagger\mathbf{U}^T) \times_2 (\mathbf{S}^\dagger\mathbf{U}^T) \times_3 (\mathbf{S}^\dagger\mathbf{U}^T) \times_4 (\mathbf{S}^\dagger\mathbf{U}^T). \tag{8}$$

Eq. (7) is a fourth-order generalization of the EVD of matrices: $\mathbb{C}_4^Z$ is super-symmetric (i.e. invariant under all permutations of its indices), $\mathbf{V}$ is orthogonal and the off-diagonal elements of $\mathbb{C}_4^X$ are theoretically zero since they correspond to mutually independent sources. Several techniques have been developed for the computation of this decomposition — see e.g. [3, 4, 5].

This "multilinear SVD" approach has the advantage that the mixing matrix can be estimated in an unsupervised way. In addition, the separation of the measurements into statistically independent source signals (with a physical meaning) is easier to interpret than a decomposition in time-orthogonal principal components. On the other hand, the algorithm is computationally more expensive than the SVD- and QSVD-algorithms. It can also be difficult to estimate the higher-order cumulant accurately.

6. EXAMPLE

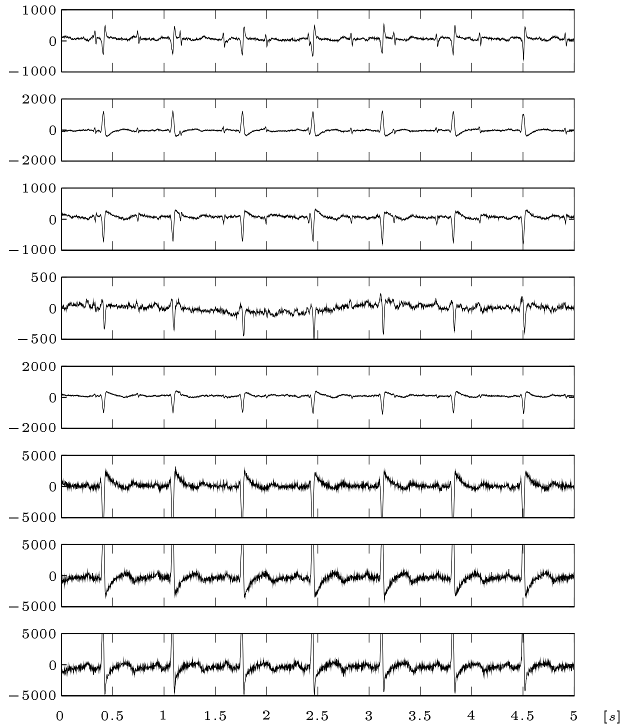

Figure 1: 8-channel set of cutaneous data recordings.

Fig. 1 shows the first 5 seconds of a set of potential signals measured in a one-minute 8-channel experiment. The horizontal axis displays the time in seconds; with respect to the vertical axes only the relative values are important. The sampling frequency was 500 Hz. For details about the data acquisition we refer to [2]. Channels 1 to 5 show signals that were measured on the belly; for channels 6 to 8 the electrodes have been placed further away from the fetus, e.g. on the thorax. Channels 1 to 3 clearly contain weak fetal contributions. Due to the large amplitudes of the MECG in the thoracic signals, the FECG is less visible there.

The source estimates after PCA are displayed in Fig. 2. Two MECG-free FECG-components were obtained as the 6th and the 7th right singular vector of the data-matrix respectively. Signals 1 and 2 partially describe the MECG-subspace; the MECG also appears in signals 3 and 5. Channels 4 and 8 mainly show noise contributions.

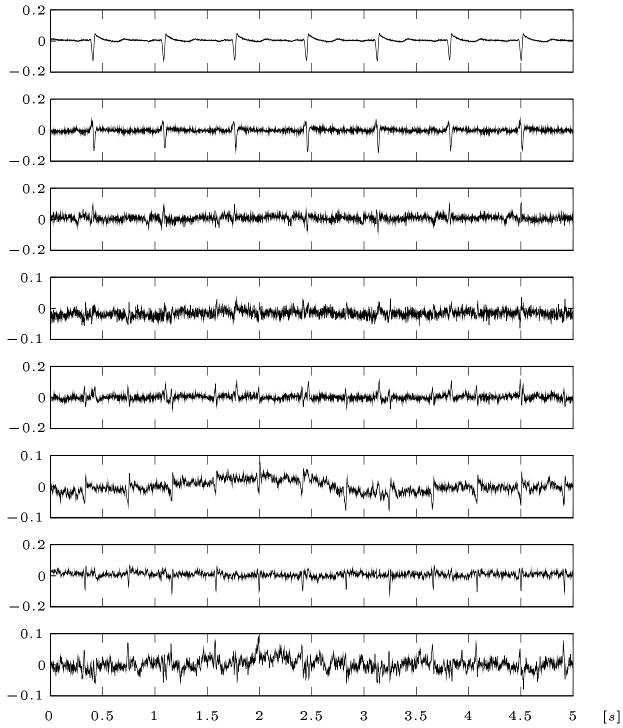

Figure 2: Source estimates obtained by means of PCA.

The source estimates obtained by means of QSVD are displayed in Fig. 3. The matrix $\tilde{\mathbf{Y}}$ was constructed by concatenating the intervals containing an MECG-pulse in the second half of the dataset. Channels 1 and 2 show the MECG. A third MECG-component is displayed in channel 3; this component has a lower Signal-to-Noise Ratio (SNR). Channel 4 mainly contains noise contributions. Channels 6 and 7 contain a clear FECG. Channels 5 and 8 are also FECG-channels, but they have a lower SNR.

The result after BSSS is shown in Fig. 4 (we used the algorithm proposed in [4]; e.g. the methods reported in [3, 5] yield comparable results). The result is an excellent source separation. We remark that, just like in the SVD-approach, the statistics of the non-stationary signals have been estimated "roughly" by simple time-averaging. Whereas the PCA-method obtained only two clear MECG-components (the 3rd signal is heavily perturbated by noise and the fifth component contains important FECG-contributions), BSSS accurately reconstructed the full three-dimensional MECG-subspace (signals 1 to 3 in Fig. 4). As far as the FECG is concerned, the quality of the 7th principal component and the 8th BSSS-signal are comparable, but in the 6th BSSS-signal the SNR is somewhat better than in the 6th PCA-estimate. The off-set in the 6th PCA-signal is found back as an extra source signal (the 7th signal in Fig. 4; this sequence continues as a low-periodic signal and deserves further medical interpretation). The 5th BSSS-signal mainly shows noise contributions.

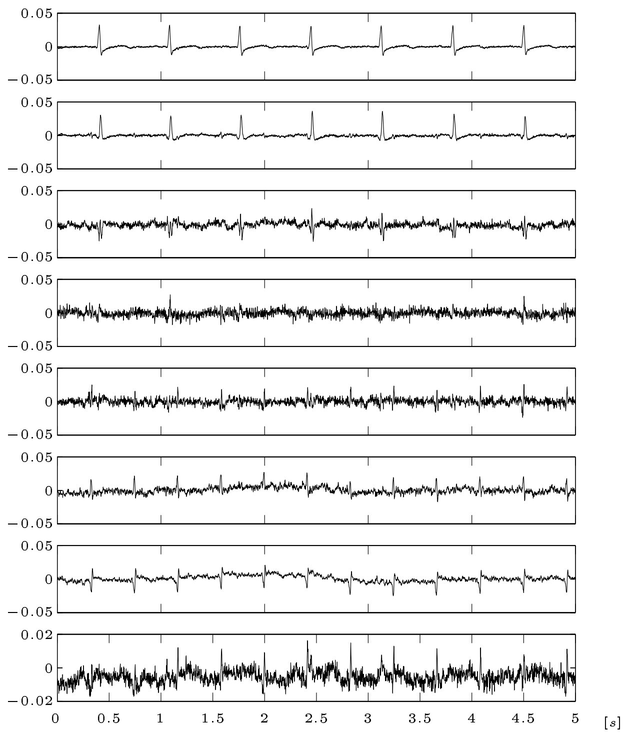

Figure 3: Source estimates obtained by means of QSVD.

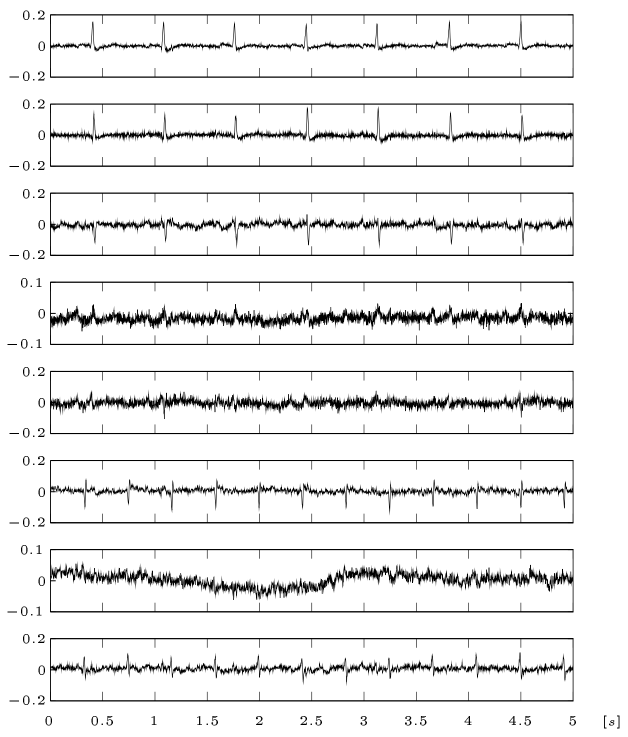

Figure 4: Source estimates obtained by means of BSSS.

7. CONCLUSION

We have discussed an SVD, QSVD and multilinear SVD technique for the extraction of the fetal electrocardiogram from multilead cutaneous potential recordings. These methods have been briefly compared in terms of computational complexity, robustness, required interaction with the user and amount of provided information. Their performance has been illustrated by means of a real-life example.

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